Risk Structure of Interest Rates

Interest rates and yields on credit market instruments of the same maturity vary because of differences in default risk, liquidity, information costs, and taxation. These determinants are known collectively as the risk structure of interest rates.

1.   Default Risk

Default risk is the probability that a borrower will not pay in full the promised interest, principal, or both. The risk premium on a financial instrument is the difference between its yield and the yield on a default-risk-free instrument of comparable maturity. Generally, the larger the default risk, the larger the risk premium, the higher the interest rate.

2.   Liquidity

Because investors care about the cost required to convert a financial instrument into cash, an increase in liquidity can make an instrument more desirable to investors, who will then accept a lower rate of return. Thus a less liquid asset, called an illiquid asset, must pay a higher yield in order to compensate savers for their sacrifice of liquidity. This liquidity premium is commonly combined with default risk as part of the risk premium.

3.   Information Costs

Gathering information requires resources and reduces the expected return on a financial asset. Government bonds such as Treasury bills have the lowest information costs because all savers know with certainty that the principal and interest will be repaid. Information costs increase if the borrowers are not that well known and research needs to be conducted before potential lenders are willing to buy bonds from such borrowers. In short the higher the information costs, the higher the interest rate.

4.   Taxation

Whether the financial instrument is exempted from taxation for interest payments or capital gains affects the interest rate of the instrument. Generally instruments that are tax exempt, are able to offer lower interest rates.

__Term Structure of Interest Rates__

This section focuses on variations in yields among financial instruments with common default risks, liquidity, information cost, and taxation characteristics but with different maturities. The variation in yield for related instruments differing in maturities is known as the term structure of interest rates. Because the risk structure factors can be held constant most easily for U.S. government obligations, term structure usually is defined with respect to yields on those securities.

In order to obtain information about investors' expectations of credit market conditions, market analysts often study the yields to maturity on different default-risk free instruments as a function of maturity. The graph of this relationship is known as a <u>yield curve</u>.

In principle, yield curves can be upward sloping, flat, or downward sloping. An upward sloping yield curve indicates that long-term yields are higher than short-term yields. A flat yield curve indicates that yields on short-term and long-term obligations are the same. A downward sloping yield curve indicates that short-term yields are higher than long-term yields.

Since World War II, yield curves for U.S. Treasury securities have exhibited two interesting patterns. First, the yield curve usually slopes upward, so that long-term yields are generally higher than short-term yields. Second, yields on default-free instruments with different maturities typically move together, increasing or decreasing together. Hence the yield curve typically shifts up or down rather than twisting. Understanding these patterns can help you use yield curves to forecast economic variables such as interest rates and inflation rates.

Economists have advanced three theories as explanations of patterns in the yield curve: the segmented markets theory, the expectations theory and the preferred habitat theory.

The Expectations Theory of Term Structure

The expectations theory states that investors view assets of all maturities as perfect substitutes at the same levels of default risk, liquidity, information costs and taxation. This view implies that a long-term bond rate equals the average of short-term rates covering the same investment period. In addition, it suggests that the yield curve shape depends on the expected pattern of short-term rates.

The perfect substitutability assumption of the expectations theory implies that expected returns for a holding period must be the same for bonds of different maturities. Otherwise, investors would change their relative demand for instruments with different maturities to take advantage of differences in yields. In addition, the perfect substitutability assumption also implies that the yield on a long-term bond will equal an average of expected short-term yields over the life of the bond (see below).

Investment Strategies

Let i_{nt} be the annual interest rate for a *n* year bond beginning in year *t*.

Let i_{nt}^{e} be the expected annual interest rate for a *n* year bond beginning in year *t*.

Suppose that you would like to invest \$1 in bonds for a time period of two years, going from year *t* to year *t+2*. There are basically two strategies one could use to invest for a period of two years:

a.   Buy and Hold Strategy

This strategy implies that you buy a two-year bond and hold it until maturity. The annual interest rate on a two-year bond beginning in year *t* would be i_{2t}. Thus the return on the two-year bond at maturity would be:

$$(1+i_{2t})(1+i_{2t}) = \quad 1+i_{2t}+i_{2t}+i_{2t}^{2}$$
$$\approx \quad 1+2i_{2t}$$

The last line of the equation is obtained as a first-order approximation in which any term which is the result of the multiplication of two interest rates, usually extremely small in numerical value, is assumed to be zero.

b.   <u>Roll-over Strategy</u>

This strategy consists of buying a one-year bond beginning in period *t* with an interest rate of i_{1t} and holding it until maturity. Then after the one-year bond matures in year

t+1, you buy another one-year bond and hold it to maturity. The precise interest rate on the second bond will be unknown when you begin your investment in period *t*. You will only be able to guess the interest rate on a one-year bond that begins in period *t+1,* thus it will be denoted as $i^e_{1,t+1}$.

The return on such a roll-over strategy at the end of the two years to a first order approximation will be:

$$(1+i_{1t})(1+i^e_{1,t+1}) \qquad \approx \qquad 1+i_{1t}+i^e_{1,t+1}$$

Because of the perfect substitutability assumption discussed above, the expected return on both strategies must be the same:

$$
\begin{array}{ccc}
\text{Return on buy and hold strategy} & = & \text{Return on roll over strategy} \\
1+2i_{2t} & = & 1+i_{1t}+i^e_{1,t+1}
\end{array}
$$

This implies that

$$i_{2t} \qquad = \qquad \frac{i_{1t}+i^e_{1,t+1}}{2}$$

More generally, for an *n*-period bond:

$$i_{nt} = \frac{i_{1t}+i^e_{1,t+1}+...+i^e_{1,t+n-1}}{n}$$

showing that the yield on a long-term bond will equal an average of expected short-term yields over the life of the bond.

Numerical Example

Suppose that in period *t* the interest rate on a one-year bond is 6%, the interest-rate on a two-year bond is 7%, the interest rate on a three-year bond is 8%, and the interest rate on a four-year bond is 9%. Using the above notation :

$$i_{1t} = 6\%$$
$$i_{2t} = 7\%$$
$$i_{3t} = 8\%$$
$$i_{4t} = 9\%$$

Suppose that we want to find the expected future one-year interest rate that begins in year *t+1*, from above we know that:

$$i_{2t} \quad = \quad \frac{i_{1t} + i^e_{1,t+1}}{2}$$

This implies that

$$i^e_{1,t+1} \quad = \quad 2i_{2t} - i_{1t}$$

Solving the above leads to the solution that $i^e_{1,t+1}$ is 8%.

Similarly, one can infer the expected future one-year rate beginning in year *t+3*, $i^e_{1,t+3}$, by looking at the four-year bond rate. However, one should note that the return on a four-year bond, to a first approximation, equal to:

$$(1 + i_{4t})^4 \quad = \quad 1 + 4i_{4t}$$

This implies that:

$$i_{4t} \quad = \quad \frac{3i_{3t} + i^e_{1,t+3}}{4}$$

which leads to the solution $i^e_{1,t+3}$ is 12%