This example is from Stanley Fischer's article, "Dynamic Inconsistency and the Benevolent Dissembling Government," *(Journal of Economic Dynamics and Control (1980) 93-107.)*

Off[General::"spell1", General::"spell"]
This model presents a good illustration of dynamic inconsistency within the context of optimal taxation. We examine three cases: a model with no distortionary taxes, a model with distortionary taxes and precommitment and a model with distortionary taxes and no precommitment.

First the model with no distortionary taxes. This introduces the baseline model: The economy lasts for two periods. In the first period agents have an endowment and face a simple consumption/savings decision. In the second period, agents receive income through the returns on capital (i.e. savings) and work effort. This is used to consume and pay taxes. The taxes are used to finance government expenditures. The details are:

The utility function is:

\[ U(c_1, c_2, 1 - n, g) = \ln c_1 + \beta (\ln c_2 + \delta \ln (1 - n) + \eta \ln g) \]

where

- \( c_1 \) is first period consumption,
- \( c_2 \) is second period consumption,
- \( n \) is labor supply (the household has one unit of time so \((1-n)\) is leisure) and
- \( g \) is government expenditures.

The parameter \( \beta \) represents agents' time preference while \( \delta \) and \( \eta \) represent the importance of utility from leisure and government expenditures respectively.

We first solve the problem as a social planner problem - the government knows the preferences of the individuals and chooses the level of consumption, labor and government expenditures to maximize utility.

The first period resource constraint is:

\[ (1) \quad c_1 + k = y \]

where \( k \) is capital and \( y \) is the endowment in the first period.

The second period resource constraint is:

\[ (2) \quad c_2 + g = \alpha n + R k \]

where \( R \) is the rate of return on capital (this is exogenous) and \( \alpha \) is the productivity of labor (the production function is linear). Note that taxes are ignored since in this example they are lump sum and do not influence agents' decisions.

The intertemporal budget constraint is obtained by solving (1) for \( k \) and substituting into (2). The maximization problem (written as a Lagrangian is):

\[
L = \log[c_1] + \beta (\log[c_2] + \delta \log[1 - n] + \eta \log[g]) + \lambda (\alpha n + R y - R c_1 - c_2 - g) + \lambda (-g + R y + n \alpha - R c_1 - c_2)
\]

Next we obtain the first-order conditions
focs = {D[L, c1] = 0, D[L, c2] = 0, D[L, n] = 0, D[L, g] = 0, D[L, λ] = 0};

{ -R λ + \frac{1}{c_1} = 0, -λ + \frac{β}{c_2} = 0, -\frac{β δ}{1-n} + α λ = 0, \frac{β η}{g} - λ = 0, -g + R y + n α - R c_1 - c_2 = 0 }

Then we solve for the optimal choices: (c_1, c_2, n, g)

sol = Flatten[ Simplify[ Solve[fo, {c_1, c_2, n, g}, λ] ] ];

{ c_2 \to \frac{(R y + α) β}{1 + β (1 + δ + η)}, g \to \frac{(R y + α) η}{1 + β (1 + δ + η)},
  c_1 \to \frac{R y + α}{R (1 + β (1 + δ + η))}, n \to \frac{-R y β δ + α (1 + β + β η)}{α (1 + β (1 + δ + η))} }

\{c_1^*, c_2^*, n^*, g^*\} = {c_1, c_2, n, g} /. sol

Let's check the solution. Note eqs. (5)-(7) in Fischer's article - we check to see if these hold.

\{c_1^* = β R c_1^*, Simplify[ n^* = 1 - \frac{δ}{α} β R c_1^*], g^* = η c_2^*\}

{True, True, True}

- Now we introduce distortionary taxes. Since taxes are applied on labor and capital income in period 2, we need to acknowledge through our notation that the tax rate that prevails in the second time period may not be the same as announced in the first period.

- First we examine the optimal solution when there is precommitment, that is when some mechanism exists so that announced (in period 1) and actual tax rates are the same. Solving this is done in two steps.
  1. The individual makes choices based on the expected tax rates - this produces demand functions for (c_1, c_2, n) that are functions of the announced tax rates. Expected government expenditures do not affect household decisions since utility is separable - hence we will ignore g in period 1.
  2. The government then maximizes household indirect utility by choosing the tax rates subject to the government budget constraint.

To simplify the notation, define the net after-tax returns from capital and labor as:
Re_2 = (1 - τ_2^e) R and αe_2 = (1 - τ_2^e) α where the notation 'e' denotes expected level and the "2" is used to denote Economy 2.

First we express the household's problem as the following Lagrangian (convince yourself that this is correct). Note the differences - the returns on labor and capital are affected by the expected tax rates and government expenditures are no longer a choice variable).

L_2 = Log[c_1] + β (Log[c_2] + δ Log[1 - n]) + λ (αe_2 n + Re_2 y - Re_2 c_1 - c_2);
\[
\text{focs2} = \{D[L2, c_1] = 0, D[L2, c_2] = 0, D[L2, n] = 0, D[L2, \lambda] = 0\}
\]
\[
\begin{align*}
\frac{1}{c_1} - \lambda R_2 &= 0, \\
-\lambda + \frac{\beta}{c_2} &= 0, \\
\frac{-\beta \delta}{1 - n} + \lambda \alpha e_2 &= 0, \\
-c_2 + y R_2 - c_1 R_2 + n \alpha e_2 &= 0
\end{align*}
\]
\[
\text{sol2} = \text{Flatten}[\text{Simplify}[\text{Solve}[\text{focs2}, \{c_1, c_2, n, \lambda\}]]]
\]
\[
\{c_2^*, c_2^*, n^*\} = \{c_1, c_2, n\} /. \text{sol2}
\]
\[
\left\{ \frac{y R_2 + \alpha e_2}{(1 + \beta + \beta \delta) R_2}, \frac{\beta (y R_2 + \alpha e_2)}{1 + \beta + \beta \delta}, \frac{-y \beta \delta R_2 + (1 + \beta) \alpha e_2}{(1 + \beta + \beta \delta) \alpha e_2} \right\}
\]

Now use the households indirect utility (we now incorporate \( g \) since the government will need this when maximizing households' utility). The indirect utility function is obtained by substituting the demand functions for the choice variables in agents' (direct) utility function. Then utility is a function of tax rates rather than consumption and labor

\[
\text{IU2} = \log(c_1) + \beta (\log(c_2) + \delta \log(1 - n) + \eta \log(g^2)) /. \text{sol2}
\]
\[
\log\left(\frac{-y R_2 + \alpha e_2}{(1 + \beta + \beta \delta) R_2}\right) + \beta \left(\eta \log(g^2) + \log\left(\frac{\beta (y R_2 + \alpha e_2)}{1 + \beta + \beta \delta}\right) + \delta \log\left(1 - \frac{-y \beta \delta R_2 + (1 + \beta) \alpha e_2}{(1 + \beta + \beta \delta) \alpha e_2}\right)\right)
\]

We need to define capital and labor in terms of the demand functions - these will be used in the government budget constraint

\[
k_2 = y - c_1 /. \text{sol2} ;
\]
\[
n_2 = n /. \text{sol2} ;
\]

Now express the government’s Lagrangian and (try to) do the same steps. (Note that \( R - R_2 = r^* R \) and similarly for the tax on labor.)

\[
\text{GL2} = \text{IU2} + \gamma ((R - R_2) k_2 + (\alpha - \alpha e_2) n_2 - g^2)
\]
\[
\log\left(\frac{-y R_2 + \alpha e_2}{(1 + \beta + \beta \delta) R_2}\right) +
\beta \left(\eta \log(g^2) + \log\left(\frac{\beta (y R_2 + \alpha e_2)}{1 + \beta + \beta \delta}\right) + \delta \log\left(1 - \frac{-y \beta \delta R_2 + (1 + \beta) \alpha e_2}{(1 + \beta + \beta \delta) \alpha e_2}\right)\right) +
\gamma \left(-g^2 + \frac{(\alpha - \alpha e_2) (-y \beta \delta R_2 + (1 + \beta) \alpha e_2)}{(1 + \beta + \beta \delta) \alpha e_2} + (R - R_2) \left(y - \frac{y R_2 + \alpha e_2}{(1 + \beta + \beta \delta) R_2}\right)\right)
\]

This will get messy fast...
gfocs2 = {D[GL2, Re2] = 0, D[GL2, ae2] = 0, D[GL2, g2] = 0, D[GL2, y] = 0} \\
{(1 + \beta + \beta \delta) \frac{\text{Re2} \left( \frac{\gamma}{(1 + \beta + \beta \delta) \text{Re2}} - \frac{\gamma \text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right)}{\text{Re2} + \alpha e_2} + \gamma \left(-\frac{\alpha}{(1 + \beta + \beta \delta) \text{Re2}} + \frac{\text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right) \left( \frac{y}{(1 + \beta + \beta \delta) \text{Re2}} + \frac{\text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right) + \\
\left( \frac{\gamma \text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} + \left( \frac{y}{(1 + \beta + \beta \delta) \text{Re2}} + \frac{\text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right) \left(1 - \frac{y \beta \delta (\alpha - \alpha e_2)}{(1 + \beta + \beta \delta) \text{Re2}} + \frac{\text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right) \right) + \\
\beta \left( \frac{\gamma \text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} + \left( \frac{y}{(1 + \beta + \beta \delta) \text{Re2}} + \frac{\text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right) \left(1 - \frac{y \beta \delta (\alpha - \alpha e_2)}{(1 + \beta + \beta \delta) \text{Re2}} + \frac{\text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right) \right) = 0, \\
\frac{1}{\text{Re2} + \alpha e_2} + \gamma \left(-\frac{\gamma}{(1 + \beta + \beta \delta) \text{Re2}} + \frac{\text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right) \left( \frac{y}{(1 + \beta + \beta \delta) \text{Re2}} + \frac{\text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right) \left(1 - \frac{y \beta \delta (\alpha - \alpha e_2)}{(1 + \beta + \beta \delta) \text{Re2}} + \frac{\text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right) = 0, \\
-\gamma + \frac{\beta \eta}{g2} = 0, -g2 + \left( \frac{(1 - \alpha e_2) \left( -\frac{\gamma \beta \delta (\alpha - \alpha e_2) + (1 + \beta) \text{ae2}}{(1 + \beta + \beta \delta) \text{ae2}} \right)}{(1 + \beta + \beta \delta) \text{ae2}} \right) + (R - Re2) \left( \frac{y}{(1 + \beta + \beta \delta) \text{Re2}} + \frac{\text{Re2} + \alpha e_2}{(1 + \beta + \beta \delta) \text{Re2}} \right) = 0}

This defines three equations in the three unknowns (Re2, ae2, g2) - but it cannot be solved analytically. So let's use some numbers.

{R \rightarrow 1.5, y \rightarrow 3, \alpha \rightarrow 1, \beta \rightarrow 0.9, \delta \rightarrow 0.25, \eta \rightarrow 0.5}

ngfocs2 = gfocs2 /. pars

\{2.125 \text{Re2} \left( \frac{1.41176 - 0.470588 \left(3 \text{Re2} + \alpha e_2\right)}{\text{Re2}} \right) + \gamma \left(-3 - 0.317647 \left(1 - \alpha e_2\right) + \frac{0.470588 \left(3 \text{Re2} + \alpha e_2\right)}{\text{Re2}} + (1.5 - \text{Re2}) \left( \frac{1.41176}{\text{Re2}} + \frac{0.470588 \left(3 \text{Re2} + \alpha e_2\right)}{\text{Re2}} \right) \right) + \\
0.9 \left( \frac{3}{\alpha e_2} + \frac{0.0794118}{\alpha e_2 \left(1 - \frac{0.470588 \left(-0.675 \text{Re2} + 1.9 \alpha e_2\right)}{\alpha e_2}\right)} \right) = 0, \\
\frac{1}{\alpha e_2} + \gamma \left(-\frac{0.470588 \left(1.5 - \text{Re2}\right)}{\alpha e_2} + \frac{0.894118 \left(1 - \alpha e_2\right)}{\alpha e_2} - \frac{0.470588 \left(1 - \alpha e_2\right) \left(-0.675 \text{Re2} + 1.9 \alpha e_2\right)}{\alpha e_2} - \frac{0.470588 \left(-0.675 \text{Re2} + 1.9 \alpha e_2\right)}{\alpha e_2} \right) + \\
0.9 \left( \frac{1}{\alpha e_2} + \frac{0.25 \left(-0.894118 + 0.470588 \left(-0.675 \text{Re2} + 1.9 \alpha e_2\right)\right)}{1 - \frac{0.470588 \left(-0.675 \text{Re2} + 1.9 \alpha e_2\right)}{\alpha e_2}} \right) = 0, \frac{0.45}{g2} - \gamma = 0, \\
-g2 + \frac{0.470588 \left(1 - \alpha e_2\right) \left(-0.675 \text{Re2} + 1.9 \alpha e_2\right)}{\alpha e_2} + (1.5 - \text{Re2}) \left(3 - \frac{0.470588 \left(3 \text{Re2} + \alpha e_2\right)}{\text{Re2}} \right) = 0\}

tax2 = NSolve[ngfocs2, {Re2, ae2, g2}, y] // Last

\{Re2 \rightarrow 0.999441, \alpha e2 \rightarrow 0.668364, g2 \rightarrow 0.776475\}

That's better. Now compute the actual tax rates and the choice variables. After we have these, we can compute household utility.
\begin{verbatim}
taxrates = (R - Re2, α - ae2) /. parms /. tax2
{0.500559, 0.331636}

choices2 = (nc2*, nk2*, nc2*, nn2*, g2*) = {c2*, k2, c2*, n2*, g2} /. parms /. tax2
{1.72646, 1.27354, 1.55295, 0.419122}

utility2 = IU2 /. parms /. tax2
0.706147

- For comparison, let's go back to the first case and get numerical values

choices1 = (nc1*, nk1*, nc1*, nn1*, gl1*) = {c1*, y - c1*, cl1*, n1*, gl1} /. parms
{1.42395, 1.57605, 1.92233, 0.519417, 0.961165}

utility1 = Log[c1*] + β (Log[c1*] + δ Log[1 - n1*] + η Log[gl1*]) /. parms
0.758923

Note that utility is higher in the economy with no distortionary taxes...precisely what you expect.

- Now we examine the case with no precommitment. This involves two economies: one
  which is a time-consistent equilibrium and one which is a time inconsistent solution
  (but can not be a rational expectations equilibrium).

- The time consistent example (Economy 3) - as in the previous example, we require in
  equilibrium that the expected and actual tax rates are the same. But here there is
  no precommitment, so to determine the path of taxes, we need to work backwards:
  First determine the optimal set of taxes in the last period given the capital stock
  inherited from period 1 and then households, in the first period, choose capital
  knowing that these are the taxes they will face.

  For notational purposes we define the net after tax return on capital as
  \[ R3 = (1 - τ3) R \] and \[ a3 = (1 - τ3) a \] where the "3" denotes
  that we are analyzing Economy 3.

  The government maximizes second period utility taking as given k. First, we need to
  analyze the optimal consumer behavior to obtain the demand functions for \( c_2 \) and \( n \).
  Then the government maximizes the household's indirect utility function subject to
  the government budget constraint. The households problem is (note k3 is exogenous since
  it was chosen in period 1 and again g is ignored by the household):

  \[
  \begin{align*}
  L3 &= \log[c2] + δ \log[1 - n] + λ (R3 k3 + a3 n - c2); \\
  focs3 &= \{D[L3, c2] = 0, D[L3, n] = 0, D[L3, λ] = 0\}; \\
  sol3 &= Flatten[Simplify[Solve[focs3, {c2, n}, λ]]];
  \end{align*}
  \]
\end{verbatim}
\{c_{3z}^*, n^3\} = \{c_2, n\} / . \text{sol3}

\{k_3 R^3 + a_3, a_3 - k_3 R^3 \delta \over 1 + \delta, a_3 + a_3 \delta \}

IU3 = \text{Simplify} [\text{Log}[c_2] + \delta \text{Log}[1 - n] + \eta \text{Log}[g_3] / . \text{sol3}];

The government chooses \(a_3^*\) and \(R_3^*\) in order to maximize household utility subject to the government budget constraint. The Lagrangian is

\[ GL_3 = IU3 + \gamma ((R - R_3) k_3 + (a - a_3) n_3^* - g_3); \]

\[ \gamma \left(-g_3 + k_3 (R - R_3) + \frac{(a - a_3) (a_3 - k_3 R_3 \delta)}{a_3 + a_3 \delta}\right) + \]

\[ \eta \text{Log}[g_3] + \text{Log}\left[\frac{k_3 R_3 + a_3}{1 + \delta}\right] + \delta \text{Log}\left[\frac{(k_3 R_3 + a_3) \delta}{a_3 (1 + \delta)}\right] \]

\[ \text{gfocs3} = \{D[GL_3, R_3] = 0, D[GL_3, a_3] = 0, D[GL_3, g_3] = 0, D[GL_3, \gamma] = 0\}; \]

\[ \text{gsol3} = \text{Flatten} [\text{Simplify} [\text{Solve}[\text{gfocs3}, (R_3, a_3, g_3), \gamma]]]\]

\[ \{g_3 \rightarrow \frac{(k_3 R + a) \eta}{1 + \delta + \eta}, R_3 \rightarrow \frac{k_3 R (1 + \delta) - a \eta}{k_3 (1 + \delta + \eta)}, a_3 \rightarrow a\} \]

\[ \{g_3^*, R_3^*, a_3^*\} = \{g_3, R_3, a_3\} / . \text{gsol3} \]

Note that the result \(a_3^* = a\) implies that the tax on labor is zero. That is, since capital is already in place, a capital tax is not distortionary so the optimal choice for the government is to not tax labor (given that this tax is distortionary) and tax capital only. Also note that the tax rate on capital is \((R - R_3) = g_3/k_3\) so that the tax rate \((g_3/k_3)\) times the tax base \(k_3\) is equal to total expenditures \(g_3\).

We now turn to the individual's problem in the first period. In the first period, the household has two choice variables: \((c_{3_1}, k_3)\) and, from the indirect utility function, it already knows the level of utility it can achieve for any choice of \(k_3\). Also, the agent makes these choices using its forecast of taxes given by the solution \(\text{gsol3}\) above. The individual's Lagrangian is (note again, we ignore government choices...the individual takes this as given). First define the indirect utility function that does not include government

\[ IU_{33} = \text{Log}[c_2] + \delta \text{Log}[1 - n] / . \text{sol3} \]

\[ \text{Log}\left[\frac{k_3 R^3 + a_3}{1 + \delta}\right] + \delta \text{Log}\left[1 - \frac{a_3 - k_3 R_3 \delta}{a_3 + a_3 \delta}\right] \]

\[ L_{33} = \text{Log}[c_{3_1}] + \beta IU_{33} + \lambda (y - c_{3_1} - k_3) \]

\[ \beta \left[\text{Log}\left[\frac{k_3 R^3 + a_3}{1 + \delta}\right] + \delta \text{Log}\left[1 - \frac{a_3 - k_3 R_3 \delta}{a_3 + a_3 \delta}\right]\right] + \text{Log}[c_{3_1}] + \lambda (-k_3 + y - c_{3_1}) \]

\[ \text{focs33} = \{D[L_{33}, c_{3_1}] = 0, D[L_{33}, k_3] = 0, D[L_{33}, \lambda] = 0\} \]

\[ \{-\lambda + \frac{1}{c_{3_1}} = 0, \beta \left(\frac{R_3}{k_3 R^3 + a_3} + \frac{R_3 \delta^2}{(a_3 + a_3 \delta) (1 - \frac{a_3 - k_3 R_3 \delta}{a_3 + a_3 \delta})}\right) - \lambda = 0, -k_3 + y - c_{3_1} = 0\} \]
sol33 = Flatten[Simplify[Solve[focs33, \{c31, k3\}, \lambda\}]]

\{c31 \rightarrow \frac{R3 y + \alpha \lambda}{R3 + R3 \beta + R3 \beta \delta}, k3 \rightarrow \frac{-\alpha \lambda + R3 y \beta (1 + \delta)}{R3 (1 + \beta + \beta \delta)}\}

Note we have two equations in two unknowns. From the government’s problem, we have that the optimal choice of taxes will be: \(R3 = \frac{k3 R (1 + \delta) - \alpha \lambda}{k3 (1 + \delta + \eta)}\). That is, the tax rate will be a function of the capital stock. Then, agents choose a capital stock based on their expectations of the tax rate: \(k3 = \frac{-\alpha \lambda + R3 y \beta (1 + \delta)}{R3 (1 + \beta + \beta \delta)}\). In equilibrium, we require these to be consistent so use the second expression to eliminate \(k3\) in the first expression.

\[
\text{rhs} = \frac{g sol3[[2, 2]]}{sol33} /. \alpha3 \rightarrow \alpha
\]

Then we require this expression to be equal to \(R3\)...this produces a quadratic expression which can be seen by the analytic solution for \(R3\).

\[
R3soln = \text{Simplify}[\text{Solve}[R3 == \text{rhs}, R3]]
\]

\[
\{\{R3 \rightarrow \frac{\alpha + R y \beta + R y \beta \delta - \alpha \eta - \sqrt{-4 R y \alpha \beta (1 + \delta + \eta) + (\alpha + R y \beta (1 + \delta) - \alpha \eta)^2}}{2 y \beta (1 + \delta + \eta)}\},
\{R3 \rightarrow \frac{\alpha + R y \beta + R y \beta \delta - \alpha \eta + \sqrt{-4 R y \alpha \beta (1 + \delta + \eta) + (\alpha + R y \beta (1 + \delta) - \alpha \eta)^2}}{2 y \beta (1 + \delta + \eta)}\}\}
\]

To get a numerical answer, use the same parameter values we used earlier

\[
nR3 = R3soln /. \text{parms}
\]

\[
\{\{R3 \rightarrow 0.406099\}, \{R3 \rightarrow 0.781732\}\}
\]

Note that the first value implies a very high tax rate on capital since \(R3\) represents the net after tax return. It is likely that this would imply a lower level of utility. If this conjecture is correct, then it can not be an equilibrium - let’s check.

\[
R3^1 = nR3 // \text{Last}
\]

\[
\{R3 \rightarrow 0.781732\}
\]

\[
R3^2 = nR3 // \text{First}
\]

\[
\{R3 \rightarrow 0.406099\}
\]

\[
tax3 = \{R - R3, \alpha - \alpha3\} /. \alpha3 \rightarrow \alpha /. \text{parms} /. \text{R3}^1
\]

\[
\{0.718268, 0\}
\]

First we calculate the choice variables in the low tax state (denoted as "1")

\[
\{c31^*, k3^*, c32^*, n3^*, g31\} =
\{c31, k3, c32^*, n3^*, g3\} /. \frac{(k3 R + \alpha) \eta}{1 + \delta + \eta} /. \text{sol33} /. \alpha3 \rightarrow \alpha /. \text{parms} /. \text{R3}^1
\]

\[
\{2.01375, 0.986254, 1.41679, 0.645803, 0.708394\}
\]
Now we calculate their equivalents in the high tax state (denoted as "2")

\[
\{c32_1^*, k32^*, c32_2^*, n32^*, g32^*\} =
\{c3_1, k3, c3_2^*, n3^*, g3^*\} / . g3 \rightarrow \frac{(k3 R + \alpha) \eta}{1 + \delta + \eta} / . \text{sol33} / . \alpha \rightarrow \alpha / . \text{parms} / . R32^* \\
\{0.257057, 0.429432, 0.939514, 0.765122, 0.469757\}
\]

Note that, as expected, consumption in period 1 is higher in the high tax state since savings (i.e. capital) is low. Also note that labor is higher and govt. consumption is lower in the high tax state...the following shows that this does indeed result in lower utility:

\[
\text{utility31} =
\log[c3_1] + \beta (\log[c3_2^*] + \delta \log[1 - n3^*] + \eta \log[g3^*]) / . g3 \rightarrow \frac{(k3 R + \alpha) \eta}{1 + \delta + \eta} / . \text{sol33} / . \alpha \rightarrow \alpha / . \text{parms} / . R31^* \\
0.624883
\]

\[
\text{utility32} =
\log[c3_1] + \beta (\log[c3_2^*] + \delta \log[1 - n3^*] + \eta \log[g3^*]) / . g3 \rightarrow \frac{(k3 R + \alpha) \eta}{1 + \delta + \eta} / . \text{sol33} / . \alpha \rightarrow \alpha / . \text{parms} / . R32^* \\
0.222025
\]

Utility in the high tax equilibrium is lower so we ignore this as an equilibrium.

- **Finally, we calculate the time inconsistent solution... note that this is NOT a permissible rational expectations equilibrium since agents' expectations of future taxes are wrong (predictably wrong...this is not a stochastic model).**

To solve the model, we assume that agents believe in the first period that the optimal tax policy under precommitment (Economy 2) will be followed in the second period even though there is no commitment mechanism. Hence, we know the path of taxes: For period 1, the agent solves the following problem as in Economy 2 so that all values are the same. We repeat these below:

\[
\text{choices4} = \{c4_1^*, k4^*\} = \{c2_1^*, k2^*\} / . \text{parms} / . \text{tax2} \\
\{1.72646, 1.27354\}
\]

Then in period 2, the government uses the optimal plan described in Economy 3: set labor taxes to zero and tax only capital. This solution was described in the first part of the time consistent equilibrium without precommitment; that is, we already have the government's optimal choice of taxes for a given capital stock and the household's demand functions. First we solve for taxes and government expenditures consistent with the level of capital inherited from the first period:

\[
\{g4^*, R4^*, a4^*\} = \{g3^*, R3^*, a3^*\} / . k3 \rightarrow nk2^* / . \text{parms} \\
\{0.831515, 0.847081, 1\}
\[
\text{tax4} = R - R^4^*/. \text{parms}  \\
0.652919
\]

Then we use the tax rate on capital to determine the optimal amount of consumption and labor in the second period.

\[
\{c_4^*, n_4^*\} = \{c_3^*, n_3^*\} / (R_3 \rightarrow R_4^*, \alpha_3 \rightarrow \alpha_4^*, \ k_3 \rightarrow nk_2^*) / . \text{parms}  \\
\{1.66303, 0.584242\}
\]

Finally, we use these values to compute utility:

\[
\text{utility4} = \text{Log}[nc_1^*] + \beta (\text{Log}[c_4^*] + \delta \text{Log}[1 - n_4^*] + \eta \text{Log}[g_4^*]) / . \text{parms}  \\
0.723354
\]

Note that utility is indeed higher in this solution, hence Fischer's title: the Benevolent Dissembling Government.

The table below summarizes the results:

<table>
<thead>
<tr>
<th>Solution</th>
<th>U**</th>
<th>c_1</th>
<th>k</th>
<th>c_2</th>
<th>n</th>
<th>(\tau_k)</th>
<th>(\tau_c)</th>
<th>(\tau_n)</th>
<th>(\tau_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Planner</td>
<td>0.758</td>
<td>1.42</td>
<td>1.58</td>
<td>1.92</td>
<td>0.519</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Precommitment</td>
<td>0.706</td>
<td>1.73</td>
<td>1.27</td>
<td>1.55</td>
<td>0.419</td>
<td>0.50</td>
<td>0.50</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>Time Consistent Eq.</td>
<td>0.625</td>
<td>2.01</td>
<td>0.986</td>
<td>1.42</td>
<td>0.646</td>
<td>0.718</td>
<td>0.718</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Time Inconsistent Soln.</td>
<td>0.723</td>
<td>1.73</td>
<td>1.27</td>
<td>1.66</td>
<td>0.584</td>
<td>0.653</td>
<td>0.50</td>
<td>0</td>
<td>0.33</td>
</tr>
</tbody>
</table>