RISK AVERSION AND STOCK PRICE VOLATILITY
WHEN DIVIDENDS ARE DIFFERENCE STATIONARY

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The relationship between stock price volatility and agents' risk aversion is studied within a recursive equilibrium asset pricing model in which the endowment growth rate is assumed to follow a two-state stationary process. In contrast to a model that is stationary in the level of the endowment, the volatility of stock prices is not generally an increasing function of agents' risk aversion. The specific relationship between risk aversion and stock price volatility is shown to depend on the autocorrelation properties of the growth rate.

1. Introduction

The conclusive rejection of the present-value model of stock prices reported in Shiller (1981) and LeRoy and Porter (1981) represented a serious challenge to fundamental-based theories of asset prices. It is not surprising that, given the intuitive appeal of the present-value theory (and the lack of a well-articulated alternative), these papers generated an extensive literature that examined both the empirical and theoretical foundations of Shiller's and LeRoy and Porter's analyses.

While papers by Flavin (1983) and Kleidon (1986) have revealed potentially serious econometric problems in the empirical tests employed, concurrent developments in the theory of asset prices by Lucas (1978) demonstrated that the present-value model is valid only under the assumption that agents are risk-neutral. Using a representative agent recursive equilibrium exchange economy, Lucas additionally showed that if output (the equivalent of dividends in the model) is serially uncorrelated, then the elasticity of stock prices with respect to a dividend realization will be equal to (and therefore an increasing function of) agents' Arrow-Pratt measure of relative risk aversion. The relationship between risk aversion and stock price volatility was analyzed further in LeRoy and LaCivita (1981) through the use of a two-state version of Lucas's model. Within this context they demonstrated that stock price volatility is an increasing function of agents' risk aversion. Furthermore, the establishment of a monotonic relationship did not depend (as in Lucas) on the serial correlation properties of dividends. Together these theoretical results suggested that the empirical rejection of the present-value model may be a rejection of the implicit assumption of risk neutrality rather than that of a fundamental-based theory of stock prices.

While these papers importantly demonstrated a functional relationship between stock price volatility and agents' preferences toward risk, the specific relationship was derived within a highly unrealistic technological setting. Namely, in both models it was assumed that the endowment

1 Harris Dallas and Robert Rich provided invaluable comments and suggestions.
(aggregate output) was stationary in levels; an assumption at odds with casual observation and the statistical characterization of GNP by Nelson and Plosser (1982). Consequently, the question is raised whether the monotonic relationship between stock price volatility and risk aversion is maintained in a more plausible technological environment – one in which the growth rate of the endowment follows a stationary Markov process.

The analysis presented below demonstrates that the monotonic relationship described by Lucas and LeRoy and LaCivita is not, in general, maintained. Furthermore, the relationship between stock price volatility and risk aversion is shown to depend critically on the autocorrelation properties of the endowment growth rate. That the information content implied by a current dividend realization could influence stock prices has, of course, long been recognized. Indeed, Lucas identifies and discusses the conflicting ‘income’ and ‘information’ effects that a dividend realization has on the demand for stock [Lucas (1978, p. 1441)]. An attractive feature of the model used here [originally developed by Mehra and Prescott (1985)] is that it is easy to assess the relative magnitudes of these effects.

2. The model

To demonstrate the above statements, I use a model identical to that presented in Mehra and Prescott (1985). It is assumed that the motion of the non-storable endowment, \( x \), is given by

\[
x_{t+1} = x_t (1 + g_{t+1}).
\]

(1)

where \( g \) is the endowment growth rate. It is assumed that \( g \) follows a stationary two-state Markov process with possible realizations \([g(h) > g(l)]\) and transition probability matrix \( \Pi \). An element of \( \Pi \) is denoted as

\[
\pi(s, s') = \text{prob}[g_{t+1} = g(s') | g_{t} = g(s)]; \quad s, s' = (h, l).
\]

(2)

To simplify the exposition, it is assumed that \( \pi(h, h) = \pi(l, l) = \pi \) so that \( \text{cov}(g_{t}, g_{t-1}) \geq 0 \) as \( \pi \geq 1/2 \). Agents are identical and maximize

\[
E \left[ \sum_{t=0}^{\infty} \beta^t U(c_t) \right]; \quad \beta \in (0, 1),
\]

(3)

where \( E \) denotes the expectations operator, \( c \) is consumption, and \( \beta \) represents agents’ subjective discount factor. In order to ensure a stationary solution, it is assumed that \( U(\cdot) \) exhibits constant relative risk aversion (denoted by \( \gamma \)), i.e.,

\[
U(c_t) = \frac{c_t^{1-\gamma} - 1}{1 - \gamma}, \quad \gamma \neq 1,
\]

\[
= \ln c_t, \quad \gamma = 1.
\]

As in all representative agent asset-pricing models, the equilibrium price of stock in period \( t \) and state \( s \), \( p_t(s) \), must satisfy the intertemporal marginal condition

\[
p_t(s)[x_t(s)]^{-\gamma} = \beta \sum_{s'} \pi(s, s')[x_{t+1}(s')]^{-\gamma} [p_{t+1}(s') + x_{t+1}(s')].
\]

(4)
where I have used the functional form for $U(\cdot)$ and imposed the equilibrium condition that $c_t = x_t$. While equilibrium stock prices will be a function of both the level and the growth rate of the endowment, the restriction on preferences implies (as shown in Mehra and Prescott (1985)) that this function [denoted $\rho(x_t, s)$] will be homogeneous of degree one in the endowment. That is

$$p_i(s) = \rho(x_t, s) = x_t \rho(s).$$

(6)

Using eq. (6) and eq. (1) allows eq. (5) to be written as

$$\rho(s) = \beta \sum_{s'} \pi(s, s') [1 + g(s')]^{1-\gamma} [\rho(s') + 1].$$

(7)

The two equations implied by eq. (7) are linear in the two equilibrium values [$\rho(h)$, $\rho(l)$] and, hence, guarantee a unique solution. Furthermore, the analysis of these equations is straightforward and implies the characterization of equilibrium in table 1. These equilibrium characteristics reflect the income and information effects implied by a dividend realization. For instance, suppose growth rates are positively autocorrelated ($\pi > 1/2$) and the current growth rate is high. This implies a high income today which increases the demand for stock but also signals a high income tomorrow which decreases stock demand. If $\gamma < 1$, the income effect dominates [$\rho(h) > \rho(l)$] while if $\gamma > 1$, the information effect dominates [$\rho(l) > \rho(h)$].

3. Stock price volatility and risk aversion

The percentage change in stock prices if the state goes from $s$ to $s'$ is denoted $\delta(s, s')$ and defined by

$$[1 + \delta(s, s')] = \frac{p_{t+1}(s')}{p_t(s)} = [1 + g(s')]^{\rho(s')/\rho(s)}.$$

(8)

Due to the equilibrium characteristics described in table 1, the volatility of stock prices relative to that of dividends implied by eq. (8) are as follows:

**Case 1.** $\text{Cov}(g_t, g_{t-1}) = 0$ and/or $\gamma = 1$.

Since these parameter values imply $\rho(h) = \rho(l)$, we have

$$\delta(h, s') = \delta(l, s') = g(s').$$

(9)

\[2\text{ In this analysis, parameter values are restricted so that the equity premia (conditional and unconditional) are positive. For a detailed analysis of the behavior of the equity premia implied by this model, see Salyer (1988).} \]
Or, stock price movements are proportional to dividends. Hence, if endowment growth rates are serially uncorrelated, stock price volatility is not a function of agents' risk aversion.

**Case 2.** \( \text{Cov} (g_r, g_{r-1}) > 0. \)

If \( \gamma < 1, \rho(h) > \rho(l) \) so that

\[
\delta(l, h) > \delta(h, h) = g(h) > g(l) = \delta(l, l) > \delta(h, l). \tag{10}
\]

Hence, stock price volatility is greater than that of dividends. But if \( \gamma > 1, \) then

\[
g(h) = \delta(h, h) > \delta(l, h) > \delta(h, l) > \delta(l, l) = g(l). \tag{11}
\]

Consequently, stock prices exhibit less volatility than dividends implying that an increase in relative risk aversion causes a reduction in stock price volatility.

**Case 3.** \( \text{Cov} (g_r, g_{r-1}) < 0. \)

If \( \gamma < 1, \) then

\[
g(h) = \delta(h, h) > \delta(l, h) > \delta(h, l) > \delta(l, l) = g(l). \tag{12}
\]

While if \( \gamma > 1, \) then

\[
\delta(l, h) > \delta(h, h) = g(h) > g(l) = \delta(l, l) > \delta(h, l). \tag{13}
\]

The implication is that if the growth rate of the endowment is negatively autocorrelated, then stock price volatility is an increasing function of risk aversion. Hence, only under this pattern of serial correlation of endowment growth rates does this model duplicate the monotonic relationship between stock price volatility and risk aversion described in LeRoy and LaCivita (1981).

In summary, this paper has demonstrated that, not surprisingly, the equilibrium characteristics of a model will depend on the technological assumptions in addition to those describing preferences. These results suggest that perhaps more attention should be given to a model's technological setting before drawing inferences and/or policy recommendations.

**References**


