Output Dynamics in Real-Business-Cycle Models

By Timothy Cogley and James M. Nason*

The time-series literature reports two stylized facts about output dynamics in the United States: GNP growth is positively autocorrelated, and GNP appears to have an important trend-reverting component. This paper investigates whether current real-business-cycle (RBC) models are consistent with these stylized facts. Many RBC models have weak internal propagation mechanisms and must rely on external sources of dynamics to replicate both facts. Models that incorporate labor adjustment costs are partially successful. They endogenously generate positive autocorrelation in output growth, but they need implausibly large transitory shocks to match the trend-reverting component in output. (JEL E32, CS2)

There is an extensive empirical literature on the time-series properties of aggregate output. For example, prominent univariate studies include papers by Charles R. Nelson and Charles I. Plosser (1982), Mark W. Watson (1986), John Y. Campbell and N. Gregory Mankiw (1987), John H. Cochrane (1988), and James D. Hamilton (1989). Multivariate analyses include papers by Olivier Jean Blanchard and Danny Quah (1989), Robert G. King et al. (1991), and Cochrane (1994). This literature documents two stylized facts about output dynamics in the United States. First, GNP growth is positively autocorrelated over short horizons and has weak and possibly insignificant negative autocorrelation over longer horizons (e.g., Nelson and Plosser, 1982; Cochrane, 1988). Second, GNP appears to have an important trend-reverting component that has a hump-shaped impulse-response function (e.g., Blanchard and Quah, 1989; Cochrane, 1994).

This paper links the empirical literature on output dynamics with the theoretical literature on real-business-cycle (RBC) models. In particular, it considers whether various RBC models are consistent with these stylized facts. Our approach is similar in spirit to the "test of the Adelmans" (e.g., Irma Adelman and Frank L. Adelman, 1959; King and Plosser, 1994; Scott P. Simkins, 1994), except that we concentrate on a different set of stylized facts.1,2 We ask how often an econometrician armed with the techniques used in the time-series literature would observe the same kind of stylized facts in data generated by RBC models. To

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1 These authors consider whether various models can replicate Burns-Mitchell stylized facts.
2 There is also an extensive literature on testing for unit roots. All the models that we study replicate the univariate persistence found in U.S. GNP. This follows directly from the specification of technology shocks. In models where technology shocks are difference-stationary, output has a unit root. In models where technology shocks are trend-stationary, output has a near unit root that conventional tests cannot distinguish from unity.
be specific, we generate artificial data by simulating a variety of RBC models, we compute autocorrelation and impulse-response functions for each artificial sample, and then we count the fraction of artificial samples that yield results like those found in U.S. data. Formally, our procedure can be regarded as a specification test of a particular RBC model. Alternatively, one can regard this as an informal guide to model reformulation.  

We find that standard RBC models must rely heavily on exogenous sources of dynamics in order to replicate both stylized facts. Many RBC models have weak endogenous propagation mechanisms and do not generate interesting output dynamics via their internal structure. For example, in models that rely on capital accumulation and intertemporal substitution to spread shocks over time, output dynamics are essentially the same as impulse dynamics. Hence, these models must rely on impulse dynamics to replicate observed autocorrelation and impulse-response functions.

Other RBC models incorporate gestation lags or costs of adjusting the capital stock. For example, time-to-build models assume that it takes several quarters to install new capital (e.g., Finn E. Kydland and Edward C. Prescott, 1982), while \( q \)-theoretic models assume that the marginal cost of installing new capital is an increasing function of the rate of investment (e.g., Marianne Baxter and Mario J. Crucini, 1993). Gestation lags and quadratic adjustment costs have little effect on output dynamics. Although these factors alter the flow of investment, the change in the flow is very small relative to the stock of capital. Hence gestation lags and capital adjustment costs have little effect on the trajectory of the capital stock. Since the capital stock is what matters for production, these factors do not generate business-cycle dynamics in output. Therefore, time-to-build and simple \( q \)-theoretic models must also rely on impulse dynamics to replicate observed output dynamics.

Finally, we examine RBC models that incorporate lags or costs of adjusting labor input. For example, Craig Burnside et al. (1993) assume that firms are subject to a one-quarter lag in adjusting employment. We also consider dynamic labor-demand models in which the marginal cost of adjusting employment is an increasing function of the rate of change in employment. These models are partially successful. They endogenously generate positive autocorrelation in output growth and a small hump in the transitory impulse-response function. However, they must rely on implausibly large transitory shocks in order to match the large hump found in the transitory component of GNP.

Our discussion is organized as follows. Section I replicates the time-series evidence and motivates our interest in it. Section II discusses models that abstract from gestation lags, employment lags, and costs of adjustment. Section III examines the effects of incorporating gestation lags and capital adjustment costs, and Section IV investigates employment lags and labor adjustment costs. The final section summarizes our results.

I. Stylized Facts about Output Dynamics

According to Prescott (1986), the “business-cycle phenomenon” has three dimensions: the periodicity of output, comovements of other variables with output, and the relative volatilities of various series. This paper provides information on the first dimension by comparing output dynamics in RBC models with stylized facts reported in the time-series literature.

Before proceeding to this comparison, we undertake two preliminary tasks. First, since the data used in the empirical literature differ in a number of ways from the customary treatment in RBC models, we replicate the time-series evidence so that it is conformable with the RBC literature. Second,
we explain why students of the business cycle might care about these stylized facts.

A. The Autocorrelation Function for Output Growth

Nelson and Plosser (1982) and Cochrane (1988) report that output growth is positively and significantly autocorrelated over short horizons and negatively but insignificantly autocorrelated over longer horizons. While their results are based on annual data, RBC models are typically calibrated to generate quarterly data. There are two ways to reconcile the difference in sampling frequency. One is to time-aggregate data generated by RBC models, and the other is to replicate the time-series evidence using quarterly data. Since temporal aggregation involves a loss of information, we prefer to do the latter.

The upper left panel of Figure 1 reports the autocorrelation function (ACF) for real per capita GNP growth, 1954:1–1988:4, along with bands that mark plus and minus two standard errors. At lags of 1 and 2 quarters, the sample autocorrelations are positive and statistically significant. At higher lags, the autocorrelations are mostly negative and statistically insignificant. This is basically the same pattern as found in annual data.

\[\text{Y-C Model}\]

\[\text{Y-H model}\]

\[\text{95\% Confidence Band}\]

\[\text{Lags}\]

\[\text{ACF for Output Growth}\]

\[\text{Spectrum for Output Growth}\]

\[\text{Transitory Impulse-Response Function}\]

\[\text{Permanent Impulse-Response Function}\]

\[\text{Lags}\]

\[\text{Quarters per Cycle}\]

\[\text{Figure 1. Stylized Facts about GNP Dynamics}\]

\[\text{Y-C model}\]

\[\text{Y-H model}\]

\[\text{Lags}\]

\[\text{ACF for Output Growth}\]

\[\text{Spectrum for Output Growth}\]

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\[\text{Lags}\]

\[\text{ACF for Output Growth}\]

\[\text{Spectrum for Output Growth}\]

\[\text{Transitory Impulse-Response Function}\]

\[\text{Permanent Impulse-Response Function}\]

\[\text{Lags}\]
The economic significance of the autocorrelation function becomes apparent when it is transformed into frequency domain. The lower left panel of Figure 1 shows the spectrum for output growth, which was estimated by smoothing the periodogram using a Bartlett window. The spectrum decomposes the variance of output growth by frequency. A peak in the spectrum indicates that the corresponding periodic components have greater amplitude than other components and therefore contribute a greater portion of the variance. The spectrum for output growth has a broad peak that ranges from approximately 2.33 to 7 years per cycle, with maximum power at roughly 3.2 years per cycle. Thus a relatively large proportion of the variance of output growth occurs at business-cycle frequencies.

B. Impulse-Response Functions for Output

While the autocorrelation function provides some information about business-cycle periodicity, it also masks differences in the dynamic response of output to various kinds of shocks. This problem does not arise in one-shock RBC models. However, in multi-shock RBC models, the autocorrelation function is a complicated function of the various impulse-response functions. Since these models imply that output responds differently to different kinds of shocks, the impulse-response functions contain additional useful information about output dynamics.

To estimate impulse-response functions, we use the structural VAR technique developed by Blanchard and Quah (1989). They use information on output growth and the unemployment rate to decompose GNP into permanent and transitory components. In particular, they assume that there are two kinds of orthogonal shocks, one that has a permanent effect on output and another that has only a transitory effect, and these assumptions are sufficient to identify the two components.

RBC models do not generate data on the unemployment rate. To make the Blanchard-Quah model conformable with RBC models, we substitute per capita hours worked for the unemployment rate. Balanced-growth RBC models imply that per capita hours are stationary, so the modified VAR satisfies the Blanchard-Quah assumptions. A second-order VAR was estimated for per capita output growth and hours over the period 1954:1–1988:4, and impulse-response functions were estimated using the Blanchard-Quah technique. The solid lines in the right-hand panels of Figure 1 illustrate the results. In response to a permanent shock, output rises gradually and reaches a plateau after about six years. In response to a transitory shock, output rises for a few quarters and then returns to its stochastic trend. A substantial portion of the variation in output growth is due to transitory fluctuations. Thus, output appears to have an important trend-reverting component.

As a robustness check, we also estimated impulse-response functions by applying the Blanchard-Quah technique to a vector error-correction model for output and consumption. The results are shown as dotted lines in the right-hand side of Figure 1, and they are quite similar to those obtained from the output-hours model. For example, the contemporaneous correlation between
the permanent innovations in the two models is 0.89. Output rises more quickly in response to a permanent shock, and the transitory impulse-response function is a bit smaller in magnitude, but these differences do not affect any of the results reported below. To save space, we focus on results derived from the output-hours model.

Finally, when applied to data generated by two-shock difference-stationary RBC models, the Blanchard-Quah decomposition extracts reasonable estimates of the population impulse-response functions. For example, Figure 2 compares the population impulse-response functions from the model of Christiano and Martin Eichenbaum (1992) with the mean estimate from 1,000 Monte Carlo replications. The estimated impulse-response functions exhibit the right qualitative pattern, but they are biased downward due to the usual small-sample bias in time series.\(^6\) Nonetheless, for the multishock models that we consider, the Blanchard-Quah method allows us to estimate reasonable sample analogues to the population impulse-response functions.

II. Baseline Real-Business-Cycle Models

Broadly speaking, RBC models rely on three kinds of propagation mechanisms: capital accumulation, intertemporal substitution, and various kinds of adjustment lags or costs. This section studies a number of models that abstract from adjustment lags or costs and that rely entirely on capital accumulation and intertemporal substitution to spread shocks over time. Our list includes the models of King et al. (1988b), Jeremy Greenwood et al. (1988), Gary D. Hansen (1989), Jess Benhabib et al. (1991), Christiano and Eichenbaum (1992), and R. Anton Braun (1994).

These models have similar structures and parameter values, and they generate similar output dynamics. For expositional purposes, it will be convenient to discuss our results in terms of a single model and then to indicate how the results for other models differ. We begin by outlining the Christiano-Eichenbaum model. In this model, there is a representative agent whose preferences are given by

\[
E_t \left( \sum_{j=0}^{\infty} \beta^j \left[ \ln(c_{t+j}) + \gamma(N - n_{t+j}) \right] \right)
\]

where \(c_t\) is consumption, \(N\) is the total endowment of time, \(n_t\) is labor hours, and \(\beta\) is the subjective discount factor. Following Christiano and Eichenbaum, we assume that \(\beta = 1.03^{-0.25}\) and \(\gamma = 0.0037\).

There is also a representative firm that produces output by means of a Cobb-Douglas production function:

\[
y_t = k_t^\theta (a_t n_t)^{1-\theta}
\]

where \(y_t\) is output, \(k_t\) is the capital stock, and \(a_t\) is a technology shock. The capital stock obeys the usual law of motion:

\[
k_{t+1} = (1 - \delta) k_t + i_t
\]

where \(\delta\) is the depreciation rate and \(i_t\) is gross investment. Christiano and Eichenbaum (1992) estimate that \(\theta = 0.344\) and \(\delta = 0.021\).

The model is driven by technology and government spending shocks. Technology shocks are assumed to be difference-stationary, but none of our results depends on this assumption.\(^7\) We initially assume that technology shocks follow a random walk with drift and that government spending shocks evolve as a persistent AR(1) process

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\(^6\)The reduced-form VAR has a near unit root, and the estimated root is biased downward in finite samples. This causes the estimated impulse-response functions to decay faster than the population impulse-response functions. We obtain similar results for the other two-shock models studied in this paper.

\(^7\)We have also studied trend-stationary representations for each of the models discussed in the paper, and the results are essentially the same.
around the stochastic technology trend:

\begin{align}
(4) \quad (1 - L) \ln(a_t) &= \mu + \varepsilon_{at} \\
(5) \quad \ln(g_t) - \ln(a_t) &= \bar{g} + \varepsilon_{gt}/(1 - \rho L)
\end{align}

where \( g_t \) denotes government spending and \( \varepsilon_{at} \) and \( \varepsilon_{gt} \) are the technology and government spending innovations, respectively. Later we consider models with more complicated impulse dynamics. Christiano and Eichenbaum (1992) estimate \( \mu = 0.004 \), \( \bar{g} = 0.177 \), and \( \rho = 0.96 \). We assume that the relative volatility of the two shocks is the same as in Christiano and Eichenbaum (1992), but we rescale the innovation variances so that the model matches the sample variance of per capita GNP growth. This yields \( \sigma_{a} = 0.0097 \) and \( \sigma_{g} = 0.0113 \).

The model has a balanced-growth equilibrium. The natural logarithm of per capita output inherits the trend properties of total factor productivity and is therefore difference-stationary. Preferences are restricted so that technical progress has no long-run effect on labor supply. Hence per capita hours follow a stationary process.

The other models in the group differ in various ways. The King et al. (1988b), Hansen (1989), and Greenwood et al. (1988) models assume that technology shocks are the only source of fluctuations. Greenwood et al. also assume that technology shocks affect new capital goods but not existing capital and that firms vary capacity utilization in response to variation in the user cost of capital. Braun (1994) extends the Christiano-Eichenbaum model by including distortionary taxes on labor and capital. Finally, Benhabib et al. (1991) study a two-sector model in which goods are produced at home as well as in the market. These variations have important effects on co-movements and relative volatilities, but in most cases they have little influence on output dynamics.\(^8\)

The remainder of this section considers whether these models are consistent with the stylized facts discussed in the previous section. Our statistical approach is based on Monte Carlo simulation. The models were used to generate artificial data over a time horizon of 140 quarters, which matches the length of the sample period, 1954:1–1988:4. Each model was simulated 1,000 times. Autocorrelation and impulse-response functions were estimated for each artificial sample, and the results were collected into empirical probability distributions. The empirical distributions were then used to calculate the probability of observing the statistics estimated from U.S. data under the hypothesis that the data were generated by a particular RBC model.

### A. Autocorrelation Functions

Our first question is whether the models replicate the autocorrelation function for output growth. To test the match between sample and theoretical autocorrelation functions, we compute generalized \( Q \) statistics, which are defined as follows:

\begin{equation}
(6) \quad Q_{acf} = (\hat{\varepsilon} - c)^\hat{\nu}_c^{-1}(\hat{\varepsilon} - c).
\end{equation}

The vector \( \hat{\varepsilon} \) is the sample autocorrelation function, and \( c \) is the model-generated autocorrelation function. The latter was estimated by averaging autocorrelations across

\(^8\)In replicating these models, we follow the original construction as closely as possible. For example, we use the same preference and technology parameter values as in the original. However, in the Greenwood et al. (1988) model, the original specification generates time series that are stationary around a steady-state equilibrium. We transform this to a sustained-growth economy by adding nonstationary technical progress. Similarly, in the Benhabib et al. (1991) model, the original specification generates series that are stationary around a deterministic trend. We transform this to a difference-stationary model by assuming that market technology shocks follow a random walk with drift. Finally, we rescale the shocks in all the models so that they match the sample variance of output growth. A technical appendix, which is available from the authors upon request, provides details about the model specifications and parameter values, and it discusses the numerical techniques used to solve the models.
the ensemble of artificial samples:

$$c = (1/N) \sum_{i=1}^{N} c_i$$ \hspace{1cm} (7)

where $c_i$ is the autocorrelation function on replication $i$, and $N = 1,000$ is the number of replications. The covariance matrix, $\hat{V}_c$, is estimated by taking the ensemble average of the outer product of the autocorrelation function for simulated data:

$$\hat{V}_c = N^{-1} \sum_{i=1}^{N} [c_i - c][c_i - c]^T$$ \hspace{1cm} (8)

Generalized $Q$ statistics are approximately chi-square with degrees of freedom equal to the number of elements in $c$. As with an ordinary $Q$ statistic, some judgment is required in choosing the number of lags. Since the autocorrelations die out at high-order lags, too high a choice will diminish the power of the test. We report results for a lag order equal to 8, but the results are not sensitive to this choice.

The first column of Table 1 reports $Q_{acf}$ statistics for each model, with chi-square probability values shown in parentheses. A large value of $Q_{acf}$ indicates that the theoretical autocorrelation function is a poor match for the sample autocorrelation function. The models are all rejected at roughly the 1-percent level or better.

The upper left panel of Figure 3 illustrates the results for the model of Christiano and Eichenbaum (1992). The solid line shows the sample autocorrelation function, and the dotted line shows the model-generated autocorrelation function. The dotted line is hard to see because it lies on top of the horizontal axis. Since the model-generated autocorrelations are all close to zero, output growth is close to being white noise. The main discrepancy between the sample and theoretical autocorrelations is the absence of positive dependence at lags 1 and 2.

Table 1—Baseline Real-Business-Cycle Models

<table>
<thead>
<tr>
<th>Model</th>
<th>$Q_{acf}$</th>
<th>$Q_{irf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>King et al. (1988b)</td>
<td>23.0</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Hansen (1989)</td>
<td>20.5</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Greenwood et al. (1988)</td>
<td>25.7</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Christiano and</td>
<td>22.1</td>
<td>50.1</td>
</tr>
<tr>
<td>Eichenbaum (1992)</td>
<td>(0.005)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Benhabib et al. (1991)</td>
<td>31.2</td>
<td>23.0</td>
</tr>
<tr>
<td>(0.0001)</td>
<td>(0.064)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Braun (1994)</td>
<td>19.9</td>
<td>25.2</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.054)</td>
</tr>
</tbody>
</table>

Notes: This table reports test statistics for the autocorrelation and impulse-response functions. The statistics $Q_{acf}$ and $Q_{irf}$ are defined by equations (6)–(8) and (9)–(11), respectively. The variable $y_p$ refers to the permanent component of output, and $y_T$ refers to the transitory component. The autocorrelation and impulse-response functions are truncated at lag 8, and probability values are in parentheses.

The theoretical $Q$ statistics are shown by the solid line, with the model-generated spectrum, which is shown by the dotted line. The dashed lines show upper and lower 2.5-percent probability bounds implied by the model.\(^9\) The Christiano-Eichenbaum model does not generate business-cycle periodicity in output growth. Instead, its spectrum is quite flat, which indicates that business-cycle components are no more important than other periodic components. Further, the disparity between sample and theoretical spectra cannot be dismissed as sampling error, since the business-cycle peak in the sample spectrum lies well above the 2.5-percent upper probability bound implied by the model. Data generated by this model rarely exhibit business-cycle peaks of this magnitude.

These results also apply to four of the other five models in this group. The Benhabib et al. (1991) model is the only one

\(^9\)The theoretical spectrum was estimated by smoothing the ensemble averaged periodogram with a Bartlett window. Upper and lower probability bounds were computed using the approximation given by theorem 5.5.3 in David R. Brillinger (1981).
that endogenously generates serial correlation in output growth, but it generates negative autocorrelation and is rejected even more strongly than the others (see the fifth row of Table 1).

**B. Impulse-Response Functions**

Our second question is whether the baseline models replicate observed impulse-response functions. To test the match between sample and model-generated impulse-response functions, we compute the following statistic:

\[
Q_{\text{irf}} = (\hat{\mathbf{r}} - \mathbf{r}) \hat{\mathbf{V}}^{-1}_{\text{r}} (\hat{\mathbf{r}} - \mathbf{r}).
\]

The vectors \(\hat{\mathbf{r}}\) and \(\mathbf{r}\) are the sample and model-generated impulse-response functions, respectively. Theoretical impulse-response functions are estimated by averaging across the ensemble of artificial samples,

\[
\mathbf{r} = (1/N) \sum_{i=1}^{N} \mathbf{r}_i,
\]

where \(\mathbf{r}_i\) is the impulse-response function on iteration \(i\). The covariance matrix, \(\hat{\mathbf{V}}_{\text{r}}\), is estimated by taking the ensemble average of the outer product of the simulated impulse response functions:

\[
\hat{\mathbf{V}}_{\text{r}} = N^{-1} \sum_{i=1}^{N} [\mathbf{r}_i - \mathbf{r}] [\mathbf{r}_i - \mathbf{r}]'.
\]
Again, since we are interested in short-run dynamics, we truncate at lag 8.\textsuperscript{10} The second and third columns of Table 1 report impulse-response statistics for the Christiano-Eichenbaum, Benhabib et al., and Braun models, with Monte Carlo probability values shown in parentheses. (The other models in this group are driven by a single shock, so their bivariate VAR's have stochastic singularities.) These models have some success matching the permanent impulse-response function but do not match the transitory impulse-response function. On this dimension, the models are rejected at better than the 1-percent level.

The right-hand panels of Figure 3 illustrate the results for the model of Christiano and Eichenbaum. The solid lines show the sample impulse-response functions, and dotted lines show model-generated impulse-response functions. The model strongly damps transitory shocks, so most of the variation in output growth is due to permanent movements. Furthermore, while GNP first rises and then falls in response to a transitory shock, the model generates monotonic decay. Thus the model does not generate an important trend-reverting component in output. Similar results apply to the Braun (1994) and Benhabib et al. (1991) models.

C. Impulse and Propagation

These results can be interpreted in terms of the model's impulse dynamics and propagation mechanisms. First consider the information in the autocorrelation function. In the Christiano-Eichenbaum model, technology shocks account for most of the variation in output growth. So far, we have assumed that technology shocks follow a random walk, which implies that the autocorrelations for growth in total factor productivity are zero. Figure 3 shows that the autocorrelations for output growth are also close to zero. Thus the dynamics of output growth are essentially the same as the dynamics of total factor productivity growth. This suggests that the model has weak propagation mechanisms.

The impulse-response functions provide additional information. In the Christiano-Eichenbaum model, the permanent component of output can be written as:

\begin{equation}
\begin{aligned}
y_p(t) &= \frac{e^a(t)}{1 - L} \left[ (1.04) \left( \frac{1 - 0.87L}{1 - 0.94L} \right) \right]
\end{aligned}
\end{equation}

\text{(12)}

(see Cogley and Nason, 1993). The right-hand side is partitioned so that the first term shows shock dynamics and the second term shows propagation effects. The permanent component inherits a random-walk term from the technology shock and an ARMA(1,1) term from the propagation mechanisms. However, the ARMA(1,1) term contains near common factors. Since these roughly cancel, the propagation effects nearly vanish. Hence the model weakly propagates technology shocks.

The transitory component of output can be written as

\begin{equation}
\begin{aligned}
y_t(t) &= \left[ \frac{e^g(t)}{1 - 0.96L} \right] (0.16).
\end{aligned}
\end{equation}

\text{(13)}

This is also partitioned so that the first term shows shock dynamics and the second term shows propagation effects. The transitory impulse-response function inherits the AR(1) dynamics of the government spending shock. Propagation mechanisms damp government spending shocks but do not alter their dynamics. Hence there are no dynamic propagation effects on government spending shocks.

In the Christiano-Eichenbaum model, output dynamics are determined primarily by impulse dynamics, with little contribution from propagation mechanisms. Since propagation mechanisms are weak, the model does
not endogenously generate business-cycle dynamics in output. Similar results apply to four of the other five models in this group.

The Benhabib et al. (1991) model is the only one in this group that has a strong propagation mechanism, but it generates negative autocorrelation in output growth. This arises from intersectoral labor flows, which enhance intertemporal substitution into and out of market employment. To understand how this works, it is useful to think about the response of market output to Cholesky innovations. A transitory shock is defined as a one-unit increase in home productivity, while a permanent shock consists of a one-unit increase in market productivity plus a fractional increase in home productivity.

In response to a temporary increase in home productivity, labor flows out of the market and into the home sector, causing market output to fall at impact. Home productivity shocks follow an AR(1) process, so home productivity peaks at impact and then begins to decline. As it falls, labor gradually returns to the market, and measured output rises back toward its stochastic trend. Thus, in response to a transitory shock, measured output falls at impact and then rises back toward trend, generating negative autocorrelation in the transitory component of output growth.

The response to a permanent shock is a combination of the responses to home and market innovations. Output rises in response to a positive innovation in market productivity, but the impact effect is partially damped by a positive innovation in home productivity, which draws some labor out of the market sector. Subsequently, as home productivity declines, labor gradually returns to the market, generating further small increases in output. The positive impact effect on market output is followed by a sequence of further small increases, and this generates modest positive autocorrela-

tion in the permanent component of output growth.

The autocorrelation function for output growth depends on both effects. But since the permanent-growth component involves partially offsetting effects, the negative autocorrelation in the transitory-growth component dominates. Therefore the home-production model generates negative autocorrelation in output growth.

D. External Sources of Dynamics

Models that have weak propagation mechanisms must rely on external sources of dynamics to replicate observed output dynamics. This section briefly considers three possible candidates: temporal aggregation, serially correlated increments to total factor productivity, and higher-order autoregressive representations for transitory shocks. While all three are sufficient to generate positive serial correlation in output growth, the first two do not generate a hump-shaped transitory impulse-response function. Furthermore, all three require shock dynamics that are counterfactual in some dimension.

We investigate these issues in the context of the Christiano-Eichenbaum model. We first consider whether temporal aggregation might account for observed output dynamics. Suppose, for example, that households and firms make decisions on a weekly basis. To convert from a quarterly to a weekly model, we followed the procedure of Christiano (1989). The parameters with time dimensions, such as the discount rate, 1 minus the depreciation rate, and the autoregressive coefficients, were adjusted by raising them to the $1/13$ power. The time endowment and means of the disturbances were divided by 13, and the innovation variances were rescaled so that the time-aggregated data match the sample variance of output growth. The modified model was used to generate weekly data, the weekly data were flow-averaged to generate quarterly series, and autocorrelation and impulse-response functions were estimated from the temporally aggregated quarterly data. The first row of Table 2 reports test

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11Home and market technology shocks are positively correlated, and this thought experiment implicitly accounts for the covariance term.
Next we consider whether serial correlation in total factor productivity growth might account for observed output dynamics. We return to the quarterly model but now assume that technology shocks follow an ARIMA(1,1,0) process with autoregressive coefficient equal to 0.2. Output growth inherits the AR(1) structure of total factor productivity growth, so this model also passes the autocorrelation test (see the second row of Table 2). But this specification also generates too much autocorrelation in total factor productivity growth. Furthermore, since technology shocks drive the permanent component of output, this modification does not generate a hump-shaped transitory impulse response (see the second row of Fig. 4). The model is still rejected on these dimensions.

To replicate both stylized facts, the model needs a high-variance transitory shock that has a hump-shaped moving-average representation. Hump-shaped shock dynamics are needed to match the shape of the sample impulse-response function, and a high variance is needed to match its magnitude. For example, suppose that government spending shocks follow an AR(2) process with roots equal to 0.9 and 0.45.12 Also suppose that the standard error of the innovation to government spending is 3.5 times larger than in the baseline model. (To match the sample variance of output growth, the standard error of technology shocks must then be reduced by roughly 30 percent.) In this case, the transitory component of output inherits a large hump-shaped impulse-response function from the transitory shock (see the bottom panels of Fig. 4). This generates positive low-order autocorrelation in output growth, so the model also passes the autocorrelation test (see the third row of Table 2).

While this specification can replicate both stylized facts, it must rely on partially counterfactual shock dynamics to do so. The

\[12\text{These are roughly the same as the AR roots for the transitory component of GNP.}\]
specification for Solow residuals is plausible, but the representation for government spending is problematic. In U.S. data, government spending is better approximated by an AR(1) process around the productivity trend. If one fits an AR(2) model to these data, the second root is equal to 0.07 with a standard error of 0.05. Furthermore, the assumed innovation variance for government spending is much larger than in the data, so the government-spending shocks are implausibly large.

III. Gestation Lags and Capital Adjustment Costs

In the baseline models, there are no costs or time lags associated with adjusting the capital stock. This section studies two models that incorporate these features. The first is a time-to-build model, which assumes that firms face multiperiod gestation lags when installing new capital and that there are no markets for capital goods in process. The second is a $q$-theoretic model, which
assumes that the marginal cost of installing new capital is an increasing function of the rate of investment.

The time-to-build model closely resembles the one studied by K. Geert Rouwenhorst (1991). In this model, technology shocks are the only source of fluctuations. Preferences and technology are restricted so that there is a balanced-growth equilibrium, and firms face a 3-quarter gestation lag when installing new capital. We follow Rouwenhorst’s specification because it isolates the role of the time-to-build technology and because it has a balanced-growth equilibrium when driven by labor-augmenting technical progress.

The \( q \)-theoretic model is a variant of the Christiano-Eichenbaum model in which the production function is modified so that there are quadratic costs of adjusting the capital stock. In particular, the production function becomes

\[
\ln(y_t) = \ln\left[ f(k_t, a, n_t) \right] - (\alpha_k/2) \left[ \Delta k_t/k_{t-1} \right]^2
\]

while all the other aspects of the model remain the same. This specification implies that the marginal cost of adjusting the capital stock is a linear function of the rate of net investment, and it is similar to the speci-
fications used by Matthew D. Shapiro (1986) and Baxter and Crucini (1993).\textsuperscript{13}

The parameter $\alpha_k$ is calibrated from estimates in Shapiro (1986). Using his parameter estimates, the marginal cost of a 1-percent increase in the capital stock amounts to roughly 2.2 percent of a quarter’s output. Hence the baseline value for $\alpha_k$ is 2.2. To check whether the results are sensitive to the choice of $\alpha_k$, we also simulated the model for $\alpha_k$ equal to 2.2/4, 2.2/2, 2×2.2, and 4×2.2, and we found that the results were not sensitive to the size of the adjustment-cost parameter.

Figure 5 summarizes output dynamics in the time-to-build and baseline $q$-theoretic models, and Table 3 reports test statistics. The left-hand panels of Figure 5 report autocorrelation functions and power spectra for output growth, with solid lines showing sample moments and dashed and dotted lines showing model-generated moments. Gestation lags and capital adjustment costs do not generate serial correlation or business-cycle periodicity in output growth. As in the baseline RBC models, time-to-build and $q$-theoretic models imply that output growth is approximately white noise, and this implication is rejected at better than the 1-percent level (see the first column of Table 3).\textsuperscript{14}

\begin{table}[h]
\centering
\begin{tabular}{l|ccc}
\hline
Model & $Q_{acf}$ & $y_F$ & $y_T$ \\
\hline
Time-to-build & 20.6 & — & — \\
& (0.008) & & \\
$Q$-theory, $\alpha_k = 2.2$ & 20.1 & 57.4 & 419.3 \\
& (0.008) & (0.015) & (0.000) \\
Sensitivity analysis: & & & \\
$\alpha_k = 2.2/4$ & 21.8 & 40.1 & 410.8 \\
& (0.005) & (0.028) & (0.000) \\
$\alpha_k = 2.2/2$ & 21.4 & 40.0 & 406.5 \\
& (0.006) & (0.028) & (0.000) \\
$\alpha_k = 2(2.2)$ & 19.0 & 71.1 & 385.9 \\
& (0.015) & (0.008) & (0.000) \\
$\alpha_k = 4(2.2)$ & 16.4 & 95.9 & 387.1 \\
& (0.036) & (0.005) & (0.000) \\
\hline
\end{tabular}
\caption{Gestation Lags and Capital Adjustment Costs}
\end{table}

Notes: This table reports test statistics for the autocorrelation and impulse-response functions. The statistics $Q_{acf}$ and $Q_{uf}$ are defined by equations (6)–(8) and (9)–(11), respectively. The variable $y_F$ refers to the permanent component of output, and $y_T$ refers to the transitory component. The autocorrelation and impulse-response functions are truncated at lag 8, and probability values are in parentheses. The parameter $\alpha_k$ governs the marginal cost of a 1-percent change in the capital stock [see equation (14) and the discussion in Section III].

The $q$-theoretic model is driven by technology and government spending shocks, and the two impulse-response functions are shown in the right-hand panels of Figure 5. For the sake of comparison, the impulse-response functions for the original model of Christiano and Eichenbaum (1992) are also shown. The addition of capital adjustment costs has almost no effect on the model’s impulse-response functions. As in the original Christiano-Eichenbaum model, output dynamics are determined primarily by impulse dynamics. Since capital adjustment costs do not help to propagate shocks over time, the $q$-theoretic model does not endogenously generate business-cycle dynamics in output. Thus, the $q$-theoretic model must also rely on external dynamics to match the sample impulse-response functions.

\textsuperscript{13}There are two superficial differences between our specification and that of Baxter and Crucini. First, Baxter and Crucini subtract the adjustment cost from the flow of investment rather than the flow of output. Second, they start with a general functional form for adjustment costs and then linearize the marginal adjustment-cost function when approximating the model’s first-order conditions. The two formulations yield similar output dynamics.

\textsuperscript{14}The main difference between our specification and that of Rouwenhorst is that he assumes that technology shocks are trend-stationary. This has a minor effect on the results. In the trend-stationary specification, the autocorrelation function for output growth exhibits slowly decaying, low-amplitude, period-3 oscillations (the spectrum has a small peak at 3 quarters per cycle). This arises directly from the gestation lag. A positive technology shock generates an increase in output. The increase is temporary, so the representative consumer saves much of it, thus generating a relatively large increase in new project starts. Three periods hence, the new projects come on line as productive capital, generating a secondary increase in output, and so on. Since gestation lags are typically assumed to be three or four quarters, time-to-build cycles do not account for U.S. business cycles.
This result may seem counterintuitive. Initially, our intuition was that gestation lags and capital adjustment costs would help to propagate shocks by spreading the response of investment over time and that this might generate interesting output dynamics. This intuition is half right. Gestation lags and capital adjustment costs do alter the flow of investment relative to the baseline model, but the change in the flow is small relative to the stock of capital. Quarterly net investment is only 0.4 percent of the capital stock, on average, so gestation lags and capital adjustment costs would have to have huge effects on investment in order to have significant effects on the short-term dynamics of capital. Since these factors have only modest effects on investment, they have very little effect on the path of the capital stock. The capital stock is what matters for production, so gestation lags and capital adjustment costs have little effect on the short-term dynamics of output.

IV. Employment Lags and Labor Adjustment Costs

Baseline RBC models also abstract from employment lags and labor adjustment costs, and this section examines two models that incorporate these features. The first is a difference-stationary version of the labor-hoarding model of Burnside et al. (1993). They assume that firms must choose the size of the labor force before observing the current state of the economy but can vary the intensity of work effort after observing the current state. Although employment adjustment costs are not explicitly modeled, one can interpret this as the reduced form of a model in which it is infinitely costly to make current-quarter adjustments on the extensive margin (e.g., by hiring, layoffs, or overtime). Thus firms choose to make all their current-quarter adjustments on the intensive margin (i.e., by varying effort). Since the ability to vary work effort only partially compensates for the inability to make current-quarter employment adjustments, these assumptions introduce a one-period lag in adjusting labor input, and this helps to propagate shocks over time.

We also consider a dynamic labor-demand model which assumes that the marginal cost of adjusting employment is a linear function of its rate of change. This model extends the previous section's q-theoretic model to include quadratic costs of adjusting labor input. In particular, the production function now becomes

\[
\ln(y_t) = \ln\left[f(k_t, a, n_t)\right] - \frac{\alpha_k}{2}\left[\frac{\Delta k_t}{k_{t-1}}\right]^2 - \frac{\alpha_n}{2}\left[\frac{\Delta n_t}{n_{t-1}}\right]^2
\]

while the other elements of the model remain the same. For \(\alpha_k\), we use the previous section's benchmark value of 2.2, but the results do not depend on the value of this parameter.

The labor-adjustment parameter, \(\alpha_n\), is calibrated using Shapiro's estimates. He distinguishes between adjustment costs for production and nonproduction workers and finds that the latter are substantial but that the former are negligible. In particular, the marginal cost of a 1-percent change in the number of nonproduction workers is roughly 0.36 percent of a quarter's output, while the marginal cost of a 1-percent change in the number of production workers cannot be distinguished from zero.

Since our model has only one kind of worker, it is a bit difficult to translate Shapiro's (1986) estimates directly into a value of \(\alpha_n\). If employment of production and nonproduction workers varied in the same proportion, one could simply set \(\alpha_n\) equal to 0.36. However, employment of production workers appears to be more cyclically sensitive than employment of nonproduction workers. If employment adjustments occur primarily on the production-worker margin, \(\alpha_n\) should be less than 0.36. We take \(\alpha_n = 0.36\) as our benchmark case, but we recognize that this probably overstates the size of aggregate labor adjustment costs. We do sensitivity analysis to deal with the resulting ambiguity. Fortunately, the results are robust to the choice of \(\alpha_n\).
Figure 6 summarizes the output dynamics in the labor-hoarding and adjustment-cost models, and Table 4 reports test statistics. The upper left panel of Figure 6 shows the sample- and model-generated autocorrelation functions. In contrast to the other models studied in this paper, the labor-hoarding and adjustment-cost models endogenously generate positive autocorrelation in output growth. In the Burnside et al. (1993) model, output growth is positively autocorrelated at lag 1 and has modest negative autocorrelation at higher-order lags, and the model easily passes the autocorrelation test (see the first row of Table 4, column 1). In the adjustment-cost model, output growth is well approximated by an AR(1) representation, with positive autocorrelation at lag 1 and monotonic decay at higher-order lags. Although the adjustment-cost model generates rather modest serial correlation, it still passes the autocorrelation test (see the second row of Table 4). This result is robust to changes in the value of $\alpha_n$; for example, the model passes even when $\alpha_n = 0.09$ (see the third row of Table 4).

The propagation mechanism in the Burnside et al. (1993) model derives from the assumption that employment is predetermined. Although firms can vary work effort, this is relatively costly, both because wages are higher (firms have to pay a premium in order to compensate workers for supplying greater effort) and because the marginal product is lower (there are sharply diminishing marginal returns to greater ef-
Table 4—Employment Lags and Labor Adjustment Costs

<table>
<thead>
<tr>
<th>Model</th>
<th>$Q_{act}$</th>
<th>$y_p$</th>
<th>$y_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burnside et al. (1993)</td>
<td>6.7</td>
<td>31.5</td>
<td>72.7</td>
</tr>
<tr>
<td></td>
<td>(0.469)</td>
<td>(0.035)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Adjustment costs in labor and capital:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_n = 0.36$, $\alpha_k = 2.2$</td>
<td>9.2</td>
<td>34.6</td>
<td>76.0</td>
</tr>
<tr>
<td></td>
<td>(0.326)</td>
<td>(0.031)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Sensitivity analysis:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_n = 0.36/4$, $\alpha_k = 2.2$</td>
<td>12.9</td>
<td>37.0</td>
<td>193.7</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(0.031)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$\alpha_n = 0.36/2$, $\alpha_k = 2.2$</td>
<td>9.8</td>
<td>39.2</td>
<td>123.1</td>
</tr>
<tr>
<td></td>
<td>(0.224)</td>
<td>(0.024)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>$\alpha_n = 0.36/2$, $\alpha_k = 2.2$</td>
<td>9.0</td>
<td>37.1</td>
<td>52.4</td>
</tr>
<tr>
<td></td>
<td>(0.339)</td>
<td>(0.024)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>$\alpha_n = 0.36/4$, $\alpha_k = 2.2$</td>
<td>10.1</td>
<td>48.7</td>
<td>54.8</td>
</tr>
<tr>
<td></td>
<td>(0.257)</td>
<td>(0.016)</td>
<td>(0.018)</td>
</tr>
</tbody>
</table>

Notes: This table reports test statistics for the autocorrelation and impulse-response functions. The statistics $Q_{act}$ and $Q_{ur}$ are defined by equations (6)–(8) and (9)–(11), respectively. The variable $y_p$ refers to the permanent component of output, and $y_T$ refers to the transitory component. The autocorrelation and impulse-response functions are truncated at lag 8, and probability values are in parentheses. The parameters $\alpha_p$ and $\alpha_t$ govern the marginal cost of a 1-percent change in labor and capital, respectively [see equation (15) and the discussion in Section IV].

output to decline from its peak. This generates a hump-shaped response of output to technology shocks (see the upper right panel of Fig. 6). If we take first differences, we find that output growth has positive low-order autocorrelation and weak negative high-order autocorrelation.\(^{15}\)

The propagation mechanism in the cost-of-adjustment model operates in a similar fashion, except that the cost of a contemporaneous change in employment is finite and the cost of a future change is positive. Thus, the impact effect of a technology shock is somewhat larger than in the Burnside et al. model, and the lagged effects are somewhat smaller. This generates weaker serial correlation in output growth.

Although the labor-hoarding and cost-of-adjustment models account for serial correlation in output growth, they are less successful at replicating the impulse-response functions (see the right-hand panels of Fig. 6 and the second and third columns of Table 4). Both models overstate the short-term response of output to technology shocks and underestimate its response to transitory shocks. In particular, conditional on the baseline parameterization of the transitory shock, neither model generates an important trend-reverting component in output. On this dimension, the models are rejected at the 1–3-percent level. Although the models generate the right qualitative response to transitory shocks, it is strongly damped and much too small in magnitude. In order to match the magnitude of the transitory impulse response, the innovation variance of government-spending shocks would have to be considerably larger than the value found in the data. Thus, the labor-hoarding and adjustment-cost models

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\(^{15}\)If the constraint on current-quarter employment adjustments were relaxed, firms would immediately raise employment and would not increase work effort. In this case, the Burnside et al. model would become observationally equivalent to the Christiano-Eichenbaum model and thus would not generate serial correlation in output growth. Therefore, the constraint on contemporaneous employment adjustments is critical for generating serial correlation in this model.
must be joined with large transitory shocks in order to generate an important trend-reverting component in output.

V. Conclusion

The time-series literature on aggregate output dynamics documents two stylized facts about U.S. GNP. The first is that GNP growth is positively autocorrelated in the short run and weakly negatively autocorrelated over longer horizons. The second is that GNP appears to have an important trend-reverting component that has a hump-shaped moving-average representation. This paper considers whether various RBC models can replicate these stylized facts.

We find that existing RBC models must rely heavily on exogenous factors to replicate both stylized facts. Many RBC models have weak internal propagation mechanisms and do not generate interesting dynamics via their internal structure. In particular, in models that rely on intertemporal substitution, capital accumulation, and costs of adjusting the capital stock, output dynamics are nearly the same as impulse dynamics. Hence, these models must rely on impulse dynamics to replicate the dynamics found in U.S. data. Models that rely on lags or costs of adjusting labor input are partially successful. Although they endogenously generate the right pattern of autocorrelation in output growth and a small hump in the transitory impulse-response function, they must rely on implausibly large transitory shocks to match the large transitory impulse response found in the data.

From the perspective of the literature of the 1970’s and early 1980’s, it is perhaps surprising that RBC propagation mechanisms do not generate business-cycle dynamics in output. For example, in response to James Tobin’s (1977) criticism that equilibrium monetary business-cycle models fail to deliver serially correlated movements in output, Robert E. Lucas and Thomas J. Sargent (1981) noted that capital accumulation and costs of adjustment could turn serially uncorrelated shocks into serially correlated movements in output. Although RBC theorists have explored this idea in great detail, the propagation mechanisms embodied in current models do not generate the right kind of output dynamics. Our results suggest that RBC theorists ought to devote further attention to modeling internal sources of propagation.

Data Appendix

Output is defined as real gross national product, and population is measured by the civilian noninstitutional population aged 16 or older. Labor input is measured by hours worked by all workers in all industries, and consumption is defined as real personal consumption expenditures. Government spending is measured by real purchases of goods and services by federal, state, and local governments. The capital stock measure that was used to compute Solow residuals was constructed by summing net private investment in equipment and structures, which was measured by subtracting the capital consumption allowance and the change in business inventories from gross private domestic investment. The initial value for the quarterly capital stock series was taken from the annual series on the stock of private residential and nonresidential capital. The data are available on the Federal Reserve System’s FAME Economic Database and on CITIBASE.

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