Midterm Exam

Directions: Answer all questions; the questions are weighted equally.

1. An individual is endowed at birth with a given amount of cake, \( \Psi_0 \). In each period, the individual decides how much cake to eat; that which is left over is available for consumption in future periods. Preferences are given by:

\[
\sum_{t=0}^{\infty} \beta^t \ln c_t
\]

Set up the maximization as a dynamic programming problem and solve for the optimal cake eating rule.

2. Consider a stochastic growth model with habit persistence and capital adjustment costs. That is, preferences are given by

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t U (c_t - h c_{t-1})
\]

while the resource constraint is:

\[
z_t k_t^\alpha = c_t + x_t + \frac{q}{2} (k_{t+1} - k_t)^2
\]

and the law of motion of capital is:

\[
k_{t+1} = k_t (1 - \delta) + x_t
\]

Note that \( h \) denotes the degree of habit persistence, \( \delta \) is the depreciation rate \((0 < \delta < 1)\), and \( q \) denotes the cost of capital adjustment. Given this environment, answer the following:

(a) Set up the dynamic programming problem and use investment \( x_t \) as a control variable. Derive the associated necessary conditions.

(b) How would the parameters \( h \) and \( q \) affect equilibrium behavior?

3. The costs of business cycles as measured by Robert Lucas: Suppose household’s current consumption path is growing at the constant rate of \( \mu \). If uncertainty was introduced into this path, how much would household’s have to be compensated to be indifferent between the random and non-random consumption streams. Assuming CRRA utility, this formalizes to finding the value of \( \lambda \) that solves:

\[
E \left\{ \sum_{t=0}^{\infty} \beta^t [(1 + \lambda) c_t]^{1-\gamma} \right\} = \sum_{t=0}^{\infty} \beta^t \left[ Ae^{\mu t} \right]^{1-\gamma} \quad (1)
\]
Lucas assumes that, in the stochastic case, consumption is described by the following process:

$$c_t = Ae^{ut} e^{-\frac{\sigma^2}{2} t} e_t$$

where the innovation is assumed to be lognormally distributed with mean 0 and variance, $\sigma^2$. Lucas makes use of the result that, if $\ln z_t \sim N(\mu_z, \sigma_z^2)$, then

$$E(z_t) = \exp \left( \mu_z + \frac{1}{2} \sigma_z^2 \right).$$

Use this to demonstrate that the costs of business cycles ($\lambda$) is approximately proportional to the variance of business cycles ($\sigma^2$) with the factor of proportionality determined by agents' risk aversion ($\gamma$).

(After using the lognormal assumption to simplify eq.(1), take logs of the resulting expression.)

4. Consider a two-state version of the Lucas tree model; that is, the endowment is assumed to follow a two-state Markov process with possible realizations $x_1 < x_2$ and symmetric transition probability matrix with diagonal elements $\pi = 1/2$. Suppose agents in this economy trade one- and two period bonds that cost $p_1$ and $p_2$ units of consumption, respectively, and return one unit of consumption upon maturity. Define the term premium as the difference between the expected return from selling a two-period bond after holding it for one-period and the certain return from a one-period bond. What is the sign of this term premium? Provide an explanation for your answer.