Final Examination

Directions: Answer all questions. Point totals for each question are in parentheses.

1. (30) Consider a representative agent exchange economy in which agents trade ownership shares in the endowment process. That is, equity purchased in period \( t-1 \) \((0 < z_{t-1} < 1)\) yields dividends in period \( t \) equal to the corresponding fraction of the current endowment. These proceeds are used to purchase current consumption and new shares of equity. The endowment, \( x_t \), is growing stochastically over time. That is, 
\[
 x_{t+1} = \lambda_{t+1} x_t 
\]
where \( \lambda_t \) is a random variable assumed to be independently and identically distributed over time.

Agents each period choose \((c_t, z_t)\) in order to maximize:

\[
 E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\} 
\]

The utility function exhibits habit persistence and has the following functional form:

\[
 U(c_t, c_{t-1}) = \frac{(c_t - \alpha c_{t-1})^{1-\gamma}}{1-\gamma} 
\]

with \( \alpha \in (0,1) \) and \( \gamma > 1 \). Given this environment, answer the following:

a. Express the agent’s maximization problem as dynamic programming problem and derive the associated necessary conditions.

b. Define a rational expectations equilibrium in this economy.

c. Solve for the equilibrium equity prices. Demonstrate that habit persistence increases the volatility of equity prices. Why is this?

2. (25) Geometric growth in an RBC model can be introduced by expressing the social planner problem as:

\[
 \max E \left\{ \sum_{t=1}^{\infty} \beta^{t-1} [\ln c_t + A(t - h_t)] \right\} 
\]

subject to:

\[
 c_t + x_t = z_t k_t^{\alpha} h_t^{1-\alpha} \\
 k_{t+1} = k_t (1 - \delta) + x_t \\
 z_t = \theta' \lambda_t \\
 \lambda_t = \theta^e e_t 
\]

The steady-state of this economy (i.e. setting \( \lambda_t = 1 \forall t \)) is a balanced growth path in which consumption, capital, and output all grow at the same rate, \( \nu \). (Note, \( \nu \neq \theta \).) Labor, on the other hand, is constant. Assume that \( \beta = 0.99 \) and \( \nu = 1.05 \). Determine the values of the other parameters, \((A, \delta, \alpha)\), so that along the balanced growth path:

a. The time spent in work activities is 25%.

b. Gross investment, \( x_t \), is 20% of beginning-of-period capital, \( k_t \).

c. The capital-output ratio is 12.

What is the growth rate of the technology shock, \( \theta \)?

3. (20) Typically, real business cycle models are not evaluated through standard econometric procedures but instead use calibration. Describe what this means and how it is implemented. In what sense does this procedure allow one to test the predictions of the model?

4. (15) How is risk defined in the consumption-based capital asset pricing model? Why does this model predict that the equity premium will be positive?