Final Exam

Directions: Answer all questions; the questions are equally weighted.

1. The typical real business cycle model is solved by first solving a corresponding social planner problem. But in this social planner environment there are no prices so how is it possible to talk about the cyclical behavior of factor prices?

2. Consider a simple RBC model in which agents have log utility and the production function is $y_t = z_t k_t^\alpha$ where $y$ is output, $z$ is an i.i.d. shock, and $k_t$ is beginning of period capital. Depreciation is 100%. Answer the following:
   i. Prove that consumption is a constant fraction of output.
   ii. Suppose one-period bonds were introduced into this economy. What is the correlation of interest rates and the marginal productivity of capital?

3. Suppose one and two period bonds are traded in a Lucas tree model economy in which the growth rate of the endowment is random and agents have isoelastic preferences. Define the term premium as:

   $$ tp_t = E \left[ \frac{(1 + r_{2_t})^2}{(1 + r_{1_{t+1}})} \right] - (1 + r_{1_t}). $$

   That is, the term premium is the difference between the expected return from liquidating a two-period bond after one-period and the current certain one-period yield. Prove that for the term premium to be positive, the endowment growth rate must exhibit negative autocorrelation. Interpret this result in terms of the consumption-based capital asset pricing model. (It is not necessary to set up the full maximization problem - work directly from the appropriate equilibrium conditions.)

4. Much of the evolution of RBC models has taken place in the depiction of the labor market. Describe the modeling changes that have been implemented and the motivations behind them.
5. Consider a RBC model with habit persistence in consumption and adjustment costs to capital. That is, preferences are:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t - hc_{t-1}) \right\}$$

and the economy's resource constraint is:

$$z_t^\alpha k_t^\lambda h_t^{1 - \alpha} = c_t + i_t + \frac{q}{2}(k_{t+1} - k_t)^2.$$ 

The parameter $h > 0$ captures habit persistence and the parameter $q$ measures the resource cost associated with changing the capital stock. Consumption and investment are denoted $c$ and $i$ respectively. The law of motion for capital is:

$$k_{t+1} = k_t(1 - \delta) + i_t.$$ 

a. Set up the problem as a dynamic programming problem. Identify the state and control variables.

b. Derive the necessary conditions associated with the maximization problem.

c. How would the parameters $h$ and $q$ affect the equilibrium characteristics of the economy?