Answers to Final

1. This question had several elements:
   1. Habit persistence implies lagged consumption is a state variable.
   2. The fact that the endowment is growing means that equity prices will be a function of both \( x_t, \lambda_t \). The assumption of CRRA (even with habit persistence) implies that the function will be h.d. 1 in the level of the endowment (as in Mehra-Prescott).
   3. The definition of equilibrium is critical - refer to Lucas 1978. It is defined in terms of two functions: a value function and a price function.

To illustrate these, first set up the dynamic programming problem:

\[
\begin{align*}
\text{max} & \quad \left( c_t - \alpha c_{t-1} \right)^{-\gamma} + \beta \mathbb{E} \left[ v(z_{t+1}, x_{t+1}, \lambda_{t+1}) \right] \\
& + \mu_t (q_t + x_t) - c_t - q_t z_t
\end{align*}
\]

Using the envelope theorem yields the following:

(1) \[ q_t \mu_t = \beta \mathbb{E} [q_{t+1} + x_{t+1}] \]

with:

(2) \[ \mu_t = \left( c_t - \alpha c_{t-1} \right)^{-\gamma} - \alpha \beta \mathbb{E} \left[ (c_{t+1} - \alpha c_t)^{-\gamma} \right] \]

In equilibrium, \( z_t = 1, c_t = x_t \).

Here's what Bob has to say about the definition of equilibrium:

**Definition:** An equilibrium is a continuous function \( p : E^r \times E^r \rightarrow \mathbb{R} \) such that...

(c) To solve for equilibrium, first examine the Lagrange multiplier. Note that it can be written as:

\[ \mu_t = x_t^{-\gamma} (\lambda_t - \alpha)^{-\gamma} - \alpha \beta \mathbb{E} \left[ x_{t+1}^{-\gamma} (\lambda_{t+1} - \alpha)^{-\gamma} \right] = x_t^{-\gamma} \left[ \lambda_t^\gamma (\lambda_t - \alpha)^{-\gamma} - K \right] \]

where \( K = \alpha \beta \mathbb{E} [ (\lambda_{t+1} - \alpha)^{-\gamma} ] \) is a constant due to the i.i.d. assumption.

Use this in the necessary condition for equity. The equilibrium price of equity is \( q_t = q(x_t, \lambda_t) \).

Given that we are using CRRA utility, make the conjecture that the price is homogeneous of degree one in the level of the endowment. That is, \( q(x_t, \lambda_t) = x_t \theta (\lambda_t) = x_t \theta (\lambda_t) \).

If the conjecture is true, using the conjecture in eq. (1) will result in an expression that does not contain \( x_t \).

Making the substitution and manipulating the expression yields the following:
\[
(3) \quad \left[ \lambda_t^\gamma (\lambda_t - \alpha)^\gamma - K \right] \theta_t = \beta E \left\{ \lambda_{t+1} \left[ \lambda_t^\gamma (\lambda_t - \alpha)^\gamma - K \right] \theta_{t+1} + 1 \right\}
\]

this verifies the conjecture. Note the right hand side is a constant again due to the i.i.d. assumption. Also, if \( \alpha = 0 \) (i.e., no habit persistence) then the price-dividend ratio \( (\theta_t) \) is constant. Hence, habit persistence will increase the volatility of stock prices. More can be said but that is enough for this question.

2. This question was designed to see how well you understood the relationships implied by the necessary conditions. HOWEVER, there was a mistake in one of the given values. Gross investment is 7\% of beginning of period capital rather than 20\% as stated. Using the 20\% yields a nonsensical answer for capital's share. Consequently, in grading the question, I simply wanted to see how you proceeded with your analysis.

The equations defining a balanced growth equilibrium (with \( \lambda_i = 1 \)) are:

1. intratemporal efficiency: \( c_t = \frac{(1 - \alpha)}{A} k_t^{\alpha} h_t^{-\alpha} \).
2. intertemporal efficiency: \( c_t^{-1} = \beta \left\{ \frac{1}{\bar{h}_{t+1}} \right\} \alpha_0 h_t^{\alpha - 1} + 1 - \delta \} \)\( .
3. law of motion of capital: \( k_{t+1} = k_t (1 - \delta) + x_t \).
4. resource constraint: \( k_t^{\alpha} h_t^{-\alpha} = y_t = c_t + x_t \).

We are given \( \beta = 0.99, c_t, k_t, y_t = \) 1.05. 

First, using eq. (3) we have:

\[
\frac{k_{t+1}}{k_t} = (1 - \delta) + \frac{x_t}{k_t} \quad \text{Given that the LHS = 1.05 and under the assumption that} \quad \frac{x_t}{k_t} = 0.07, \text{this implies} \quad \delta = 0.02. 
\]

Eq. (2) can be written as: \( \left( \frac{c_{t+1}}{c_t} \right) \beta^{-1} = \alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \). This can be solved for \( \alpha = 0.967 \).

(Pretty high - but this is a simple numerical example.)

From (1), we have: \( \frac{\bar{h}}{A} = \frac{(1 - \alpha)}{c_t} y_t \). Using the resource constraint and the assumption that \( \bar{h} = 0.25 \), this implies that \( A = 0.825 \).

Finally, the growth rate of the technology shock is determined by:

\[
\frac{y_{t+1}}{y_t} = \nu z_t k_t^{\alpha} \bar{h}_t^{1-\alpha} = \theta \nu^\alpha \quad \text{which implies} \quad \theta = 1.0016. 
\]

3. The main point about calibration as used in RBC models is that the parameters of the model are pinned down by features of the data which do not vary over the cycle. That is, the capital output ratio, time spent in work, the long run interest rate are used to pin down the key parameters. Then these are used in the model to generate cyclical output which are compared to actual business cycle characteristics. This constitutes a test of the model since the data which determines the parameters (long run or acyclical) is not the same as the data which the model is compared to (cyclical).
4. In the CCAPM, agents attempt to smooth consumption - the risk of an asset is determined by how the asset’s payoff affects investors’ MU. Specifically by the covariance between the payout and MU. We would expect the equity premium to be positive because the states in which the payout is high are also states in which consumption is high. Hence, the aforementioned covariance is negative implying a positive risk premium on equity.