Answers to Final Exam

I sketch out some answers below. Not enough intuition was provided in the answers.

1. The typical real business cycle model is solved by first solving a corresponding social planner problem. But in this social planner environment there are no prices so how is it possible to talk about the cyclical behavior of factor prices?

I was looking for some discussion of the social planner problem and the equivalence to a competitive equilibrium via the second welfare theorem. Review “Theory ahead of measurement.”

2. Consider a simple RBC model in which agents have log utility and the production function is

\[ y_t = z_t k_t^\alpha \]

where \( y \) is output, \( z \) is an i.i.d. shock, and \( k_t \) is beginning of period capital. Depreciation is 100%. Answer the following:

i. Prove that consumption is a constant fraction of output.

ii. Suppose one-period bonds were introduced into this economy. What is the correlation of interest rates and the marginal productivity of capital?

Part (i) was seen repeatedly in class. For part (ii), think of the intuition. Consumption is proportional to the technology shock. This implies, via the equilibrium condition for one period interest rates, that a positive technology shock will result in lower interest rates. Since the technology shock determines the productivity of capital, the correlation will be negative. You can prove this by using the functional forms (it is easiest to take logs of everything – this makes the expressions linear). If you grind through all the expressions (using the answer to (i)), then you can obtain:

\[ \ln\ln\ln(1 + r_t) = A' + (\alpha - 1)\ln z_t + \alpha(\alpha - 1)\ln k_t \]

where \( A' \) is a constant, and

\[ \ln MPK_t = \ln \alpha + \ln z_t + (\alpha - 1)\ln k_t. \]

The fact that the coefficient on the technology shock is negative in the interest rate expression while positive in the MPK expression proves the result. The intuition, however, is what I was looking for.

3. Suppose one and two period bonds are traded in a Lucas tree model economy in which the growth rate of the endowment is random and agents have isoelastic preferences. Define the term premium as:

\[ tp_t = E\left[ \frac{(1 + r_{2,t})^2}{(1 + r_{1,t+1})} \right] - (1 + r_{1,t}). \]

That is, the term premium is the difference between the expected return from liquidating a two-period bond after one-period and the current certain one-period yield. Prove that for the term premium to be positive, the endowment growth rate must exhibit negative autocorrelation. Interpret this result in terms of the consumption-based capital asset pricing model. (It is not necessary to set up the full maximization problem - work directly from the appropriate equilibrium conditions.)
Again, intuition is useful. The first term in the term premium is the expected return from liquidating a two-period bond after one period. As shown, this yield depends inversely on the one period rate that prevails next period. Suppose the growth rate is positively autocorrelated and, to make life easy, suppose there are only two states, high and low. Then it is easy to show (as we did in class) that the one period rate will be lower in the low growth rate state. Now suppose that in period t+1 the growth rate is high => this implies that MU is low but also it implies that the yield from liquidating a two period bond is low since the one period rate will be high. If the growth rate is low next period, all relationships are reversed. Hence, the two period bond is smoothing agents MU – the risk premium (which is identical to the term premium as it is defined) will be negative. Again, you can prove this by working from the expressions. Let

$$S_{t,t+1} = \frac{U'_t}{U'_{t+1}},$$

then the term premium can be written as:

$$tp_t = \frac{\beta E_t \left( S_{t,t+1,t+2} \right)}{\beta^2 E_t \left( S_{t,t+1,t+1,t+2} \right)} - \frac{1}{\beta E_t \left( S_{t,t+1} \right)}. \text{ Using the definition of the covariance generates the result.}$$

4. Much of the evolution of RBC models has taken place in the depiction of the labor market. Describe the modeling changes that have been implemented and the motivations behind them.

*See the Hansen and Wright article on the labor market in RBC models.*
5. Consider a RBC model with habit persistence in consumption and adjustment costs to capital. That is, preferences are:

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t - hc_{t-1}) \right\} \]

and the economy’s resource constraint is:

\[ z_t k_t^\alpha h_t^{1-\alpha} = c_t + i_t + \frac{q}{2} (k_{t+1} - k_t)^2. \]

The parameter \( h > 0 \) captures habit persistence and the parameter \( q \) measures the resource cost associated with changing the capital stock. Consumption and investment are denoted \( c \) and \( i \) respectively. The law of motion for capital is:

\[ k_{t+1} = k_t (1 - \delta) + i_t. \]

a. Set up the problem as a dynamic programming problem. Identify the state and control variables.
b. Derive the necessary conditions associated with the maximization problem.
c. How would the parameters \( h \) and \( q \) affect the equilibrium characteristics of the economy?

I screwed up when writing this question and forgot to put labor in the utility function – this caused some confusion. As written, labor would just be equal to 1 (or whatever is the endowment) since there is no disutility. The necessary conditions for consumption and investment are straightforward if you keep the state variables organized and use the envelope theorem appropriately. The greater “\( h \)” the smoother consumption will be, the greater “\( q \)”, the more volatile consumption will be.