1 Final Exam

Directions: Answer all questions - Point totals for each are given in parentheses. Good luck and enjoy your summer — after the prelims.

1. (20) Modern macroeconomic models can be distinguished by their impulse, amplification, and propagation mechanisms. Within the context of real business cycle models, identify those mechanisms and discuss their merits and weaknesses.

2. (25) Geometric growth in an RBC model can be introduced by expressing the social planner problem as

\[ \max_E \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \left[ \ln c_t + A (1 - h_t) \right] \right\} \]

subject to:
\[ c_t + x_t = z_t k_t^\alpha h_t^{1-\alpha} \]
\[ k_{t+1} = k_t (1 - \delta) + x_t \]
\[ z_t = \theta^t \lambda_t \]
\[ \lambda_t = \lambda_{t-1} \eta_t \]

The steady-state of this economy (i.e. setting \( \lambda_t = 1^{\forall t} \)) is a balanced growth path in which consumption, capital, and output all grow at the same rate, \( \nu \). (Note, \( \nu \neq \theta \)) Labor, on the other hand, is constant. Assume that \( \beta = 0.99 \) and \( \nu = 1.05 \). Determine the value of the other parameters \( (A, \delta, \alpha) \) so that, along the balanced growth path:

a. The time spent in work activities is 25%.

b. Gross investment, \( x_t \), is 7% of beginning-of-period capital, \( k_t \).

c. The capital-output ratio is 3.5.

3. (25) Consider a simple production economy in which agents have log utility and the production function is \( y_t = z_t k_t^\alpha \) where \( y \) is output, \( z \) is an i.i.d. technology shock, and \( k_t \) is beginning of period capital. Depreciation is 100%. Answer the following

a. Prove that consumption is a constant fraction of output.

b. Suppose one-period bonds were introduced into this economy. What is the correlation of interest rates and the marginal productivity of capital?

c. Capital in this economy is a risky asset. Prove that the risk premium associated with capital is positive.
Consider a cash-in-advance model identical to the Stockman model in which only consumption is subject to the cash-in-advance constraint - the only modification is that one period nominal bonds are introduced into this economy. These bonds cost $1 in period $t$ and return $(1 + n_t)$ in the following period. The agents’ budget and cash-in-advance constraints (expressed in nominal terms) are:

$$P_t f(K_t) + M_t + B_{t-1} (1 + n_{t-1}) + T_t = P_t (c_t + i_t) + M_t + B_t$$

$$M_{t-1} + T_t \geq P_t c_t$$

Note that $T_t$ denotes the lump-sum monetary transfer received from the government. This is due to new money introduced into the economy each period - the money stock is assumed to be growing at the constant rate of $g > 0$ each period. The law of motion of the capital stock is the standard

$$K_{t+1} = K_t (1 - \delta) + i_t$$

where $i_t$ denotes investment. In this economy, do the following:

a. Setup the problem as a dynamic programming problem, derive and interpret the associated necessary conditions.

b. Prove that, in steady-state, money growth does not affect the capital stock.

c. In a money-in-the-utility function model, it was demonstrated that money demand could be expressed in the form

$$\frac{U_{m,t}}{U_{c,t}} = \frac{n_t}{1 + n_t}$$

Where $U_m$ and $U_c$ represent the marginal utility of real balances and consumption respectively. Prove that a similar expression holds in this economy - and prove that nominal interest rates are zero if and only if the cash-in-advance constraint is not binding.

5.(10) Out of the many intriguing theories and techniques discussed in class, what did you find most interesting?