1 Answers to Final Exam, ECN 200E, Spring 2004

1. A good answer would include the following elements: The equity premium puzzle demonstrated that with standard (i.e. time separable and constant relative risk aversion) preferences, the consumption based capital asset pricing model is not consistent with the differences in average return between stocks and bonds. The reason was that the covariance between agents’ marginal utility of consumption and the return on equity was too low. Since the return on equity is fairly volatile, the fact for the low covariance is that consumption is very smooth and, therefore, with standard preferences and moderate levels of risk aversion, marginal utility is relatively smooth. One can increase the volatility of $MU$ by increasing risk aversion - but then one runs into the risk free rate puzzle. Increasing the risk aversion parameter causes the risk free rate to increase. This can be seen from the intertemporal equation for bonds:

\[
(1 + r_t)^{-1} = \beta E (c_{t+1}^{-\gamma})
\]

where I have assumed $c_t$ denotes one-period consumption growth and, for convenience, I have assumed that this is i.i.d. It is clear that, since consumption growth is on average greater than one, increasing relative risk aversion (i.e. $\gamma$) will result in a greater risk free rate.

The reason these puzzles are of interest to macroeconomists is that it shows dramatically that our models of intertemporal choice are missing something when it comes to capturing risk preferences. This is important since it casts doubt on Lucas’s exercise about the cost of business cycles. He found that agents would pay a very small amount to perfectly insure their consumption against the volatility implied by business cycles. Lucas used per-capita consumption which obviously ignores the differential impact of business cycles but he did use the same kinds of preferences examined in the original equity premium puzzle paper by Mehra and Prescott. Since that paper demonstrated that these preferences severely underestimate the way investors perceive the risk associated with equity, it is not surprising that Lucas found business cycle risk (of consumption) to be quite small. Something along these lines would be sufficient.

2. The definition of equilibrium need not be as complete (note that the consistency condition between individual and aggregate policy functions is the assumption of rational expectations) but it should have the functions noted and make a distinction between the individual and aggregate state variables. Also, the social planner problem should be stated - the equilibrium behavior should be described. Here are the details:
For the individual, the maximization problem is:

\[
v(z_{t-1}, \zeta_t) = \max_{(c_t, h_t, z_t)} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + A(1-h_t) + \beta E[v(z_{t+1}, \zeta_{t+1})] \right\} \tag{2}
\]

subject to:

\[
w_t h_t + z_{t-1}(\pi_t + q_t) = c_t + q_t z_t
\]

where \(z_t\) denotes equity holdings in period \(t\) and \(\pi_t = \zeta_t h_t^{\alpha} - w_t h_t\) denotes profits. Note the value function, \(v(z_{t-1}, \zeta_t)\), is a function of the individual’s beginning of period equity holdings and the current technology shock. Prices are also part of the state vector, but since these will be functions of the aggregate state (\(\zeta_t\)), I suppress these in the state vector.

The associated necessary conditions are (using \(\lambda_t\) to denote the Lagrange multiplier for the budget constraint):

\[
c_t : \quad c_t^{1-\gamma} - \lambda_t = 0
\]
\[
h_t : \quad -A + \lambda_t w_t = 0
\]
\[
z_t : \quad \beta E \left[ \frac{\partial V(z_{t+1}, \zeta_{t+1})}{\partial z_t} \right] - \lambda_t q_t = 0
\]

Or, combining terms and using the envelope theorem:

\[
Ac_t^{\gamma} = w_t
\]
\[
c_t^{-\gamma} q_t = \beta E \left[ c_t^{-\gamma}(\pi_{t+1} + q_{t+1}) \right] \tag{3}
\]

These have the standard interpretations.

The firms maximization problem is static and is simply:

\[
\max_{h_t} (\zeta_t h_t^\alpha - w_t h_t)
\]

which leads to the necessary condition:

\[
\alpha \zeta_t h_t^{\alpha-1} = w_t \tag{4}
\]

b. Definition of a recursive competitive equilibrium

A recursive competitive equilibrium can be defined by a set of functions: a value function that defines the household’s problem (2), a wage function, \(w(\zeta_t)\), an equity price function, \(q(\zeta_t)\), a set of decision rules for households, \(c(z_{t-1}, \zeta_t), h(z_{t-1}, \zeta_t)\), and a corresponding set of aggregate per capita decision rules, \(C(\zeta_t), H(\zeta_t)\). These functions must satisfy:

a. The household’s problem (as noted above for the value function.)

b. The necessary condition for households: i.e. \(q_t = q(\zeta_t)\) in eq. (3) when evaluated at market clearing quantities.

c. The necessary condition for firms: i.e. \(w_t = w(\zeta_t)\) in eq.(4).

d. The consistency of individual and aggregate decisions:

\(c(1, \zeta_t) = C(\zeta_t), h(1, \zeta_t) = H(\zeta_t)\). (that is, market clearing requires \(z_t = 1 \forall t\).
e. The aggregate resource constraint: $C(\zeta_t) = \zeta_t [H(\zeta_t)]^\alpha$

c. Social planner problem

Since there is no capital, the social planner problem is simply to maximize utility each period given by:

$$\max_{c_t, h_t} \left[ \frac{c_t^{1-\gamma}}{1-\gamma} + A (1-h_t) \right]$$

subject to $c_t = \zeta_t h_t^\alpha$

The necessary condition is:

$$Ac_t^{\gamma} = \alpha \zeta_t h_t^{\alpha-1}$$

This is clearly the same as in the competitive economy when the household’s and firm’s necessary conditions are combined. Note that equity has no relevance for the social planner problem since it does not influence equilibrium allocations.

3. a. I was looking for a discussion about the use of the Second Welfare Theorem so that the competitive equilibrium prices are certainly present in the behavior of RBC models. Hence, Summers’ critique is misplaced.

b. Many students discussed the behavior of the labor market in RBC models and the inconsistency with the data - this was fine but did not answer the question. The typical RBC model does not have strong propagation mechanisms so the dynamic properties are primarily inherited from the exogenous
technology shock. See the article by Cogley & Nason, the article by Stadler, and the discussion in Romer’s text.

4. This problem was clearly too hard: nobody set up the budget constraint correctly in this problem. Here is what the problem stated:

In the asset market, agents receive the returns from capital (i.e. the revenue from the sale of output in last period’s goods market and the sale of undepreciated capital), the returns from bonds, money not spent in last period’s goods market, and the lump-sum monetary transfer and use this to buy new capital, bonds and money. Next agents visit the goods market where money is used to finance consumption (i.e. consumption is subject to the cash-in-advance constraint).

This corresponds to the following budget and CIA constraints:

\[ P_{t-1}f(k_{t-1}) + P_t k_t (1 - \delta) + B_{t-1} N_{t-1} + P_t b_{t-1} R_{t-1} + (M_{t-1} - P_{t-1} c_{t-1}) + (\delta) \]

\[ M_t \geq P_t c_t \tag{7} \]

The dynamic programming problem is

\[ V(s_t) = \max \left\{ +\lambda_t \left( \frac{P_{t-1} f(k_{t-1})}{P_{t+1}} k_t (1 - \delta) + \frac{B_{t-1} N_{t-1}}{P_{t+1}} + \frac{b_{t-1} R_{t-1}}{P_{t+1}} + \frac{M_{t-1} - P_{t-1} c_{t-1}}{P_{t+1}} \right) \right\} \]

\[ \lambda_t \left( \frac{M_t}{P_{t+1}} - c_t \right) \tag{8} \]

where the state vector is \( s_t = (M_{t-1}, c_{t-1}, k_{t-1}, B_{t-1}, b_{t-1}) \). Note that consumption last period affects how much money is brought into period \( t \) hence that is why it is a state variable, along with \( M_{t-1} \). Taking derivatives and applying the envelope theorem yields the following first order conditions:

\[ c_t : U_t'' = \gamma_t + \beta \lambda_{t+1} + P_t \]

\[ k_t : \lambda_t = \beta \lambda_{t+1} + \frac{P_t}{P_{t+1}} f'(k_t) + 1 - \delta \] \tag{9}

\[ M_t : \lambda_t = \frac{\gamma_t}{P_t} + \beta \lambda_{t+1} + P_t \]

\[ B_t : \lambda_t = \beta \lambda_{t+1} N_t \]

\[ b_t : \lambda_t = \beta \lambda_{t+1} R_t \] \tag{11}

First combining eqs. (11) and (9) yields:

\[ \lambda_t = U_t'' \]

This is the answer to (b) - the reason is that wealth of any kind can be converted into money in the asset market which precedes the goods market.
Using this in eq. (10) and solving for steady-state (imposing \( \frac{P_t}{P_{t+1}} = \frac{1}{1+\mu} \)) yields:

\[
\beta^{-1} = \frac{f'(\bar{k})}{1+\mu} + 1 - \delta
\]

This is the answer to part (c) - money is not superneutral in this economy since the inflation tax affects the revenues from capital. (Why is this different from Stockman’s model?)

Finally, manipulating the last two necessary conditions yields the Fisher relationship:

\[
R_t = N_t \frac{P_t}{P_{t+1}}
\]