1 Final Exam

Directions: Answer all questions - they are equally weighted. Good luck and enjoy your summer — after the prelims.

1. Consider an exchange economy populated by identical agents that trade equity shares, $z_t$, defined as title to the endowment process. (That is, this is the same asset priced in the Lucas tree model.) Denote the price of equity as $q_t$. Agents also trade one-period bonds which cost $p_t$ units of consumption in period $t$ and return 1 unit of consumption in the following period. In addition to these assets, a one-period forward contract on bonds is traded. In this contract, agents agree at time $t$ to pay $\phi_t$ units of consumption in period $t + 1$ for the promise of one unit of consumption to be received in period $t + 2$. The endowment, $x_t$, is stochastic and varies over the interval $(x, \bar{x})$; furthermore, $x_t$ is assumed to be independently and identically distributed. Given this environment, agents choose a sequence of consumption and assets in order to maximize

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t) \right]$$

(a) Formulate the agent’s problem as a dynamic programming problem. Be explicit in identifying the state and control variables.

(b) Derive and interpret the necessary conditions which characterize the solution to this maximization problem.

(c) Define a recursive equilibrium in this economy. How are rational expectations imposed in this context?

(d) Prove that equilibrium bond and equity prices are positively correlated with the endowment while the price of the forward contract is constant. Explain these results.

2. Consider a competitive economy with an infinite number of identical firms and households. Each period, firms hire labor at the wage rate, $w_t$, in order to maximize profit. The production function for a representative firm is

$$y_t = z_t h_t^\alpha$$

where $y_t$ denotes output, $h_t$ is labor, and $z_t$ is an i.i.d. technology shock with $E(z_t) = 1$. It is assumed that $0 < \alpha < 1$.

Income in each period is determined by labor income and the proceeds from equity purchased in the previous period. This consists of current profits and the price of equity. (Initially, each household is endowed with one unity of
equity denoted \(z_t\). Household income is then allocated between consumption and purchases of equity. These choices are made in order to maximize the discounted stream of expected utility given by:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\gamma}}{1-\gamma} + A(1-h_t) \right] \right\}
\]

The parameters \((\beta, \alpha, A)\) are all positive with \((\beta, \alpha) \in (0, 1)\). Given this environment, do the following:

a. Define the representative firm’s and household’s maximization problems. Derive and interpret the associated necessary conditions.

b. Define a recursive competitive equilibrium in this economy.

c. Rather than solve directly for the competitive environment, one can instead solve an associated social planner problem. For this environment, what is the relevant social planner problem. Show that the necessary conditions associated with this problem do indeed correspond to those in (a).

d. Characterize the equilibrium behavior of labor, the wage rate, and the price of equity.

3. Consider the following optimal growth model. Agents’ preferences are given by

\[
E_0 \left[ \sum_{t=0}^{\infty} \beta^t \ln c_t \right]
\]

while output is a concave function of beginning of period capital and a technology shock. That is,

\[y_t = z_t k_t^\alpha\]

It is assumed that \(z_t\) is i.i.d. The law of motion for the capital stock is

\[k_{t+1} = k_t^{1-\delta} \frac{1}{i_t}\]

Note that the parameter \(\delta\) incorporates both depreciation and costs of adjustment that affect investment, \(i_t\). Solve for the policy functions describing optimal consumption and investment. Show that if \(\delta = 1\), the solution is identical to that studied in class. (Note: When manipulating the associated necessary conditions, it is useful to express the variables as ratios when possible. For instance, the \(MPK = \alpha z_t k_t^{\alpha-1} = \frac{y_t}{k_t}\) . And the resource constraint is \(\frac{\mu}{c_t} = 1 + \frac{i_t}{c_t}\).)
4. Consider the following variation of the Sidrauski model. Assume that preferences are given by:

\[ \sum_{t=0}^{\infty} \beta^t U \left( c_t, \frac{M_{t-1}}{P_t} \right) \]

Note that utility from real balances depends upon money chosen in the previous period. Output is produced using the technology \( y_t = f(k_t) \) where \( f' > 0, f'' < 0 \). In addition to money and capital, agents also trade one-period nominal bonds that cost $1 at time \( t \) and return $(1 + n_t)$ in period \( t + 1 \). The money supply is assumed to grow at the rate \( \mu \) per period so that the law of motion of the money stock is \( \bar{M}_t = (1 + \mu) \bar{M}_{t-1} \). New money is introduced via lump-sum transfers.

a. Express the maximization problem as a dynamic programming problem; derive and interpret the associated first-order conditions.

b. Define a steady-state equilibrium in this economy.

c. Using steady-state analysis, calculate and interpret: \( \frac{dk}{d\mu}, \frac{dm}{d\mu}, \frac{dn}{d\mu} \). (Where \( m \) is steady-state level of real balances.)

5. Out of the many intriguing theories and techniques discussed in class, what did you find most interesting?