1 Final Exam

Directions: Answer all questions - they are equally weighted. Good luck and enjoy your summer — after the prelims.

1. Real business cycle models can be characterized by their impulse, amplification, and propagation mechanisms. Identify these mechanisms and discuss how and why they have evolved over time.

2. Consider the Stockman cash-in-advance (CIA) model. Prove the following:

   (a) Show that money is not superneutral when the CIA constraint applies to both investment and consumption.

   (b) Show that the model exhibits superneutrality when the CIA constraint applies only to consumption.

What is the reason for these results?

3. Consider the following optimal growth model. Agents’ preferences are given by

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t A_t \ln c_t \right] \]

The term \( A_t \) denotes a preference shock which is assumed to follow a two-state Markov process with symmetric transition probability matrix with diagonal elements \( \frac{1}{2} \) and realizations \( A_1 < A_2 \). Output is a concave function of beginning of period capital and a technology shock. That is,

\[ y_t = z_t k_t^\alpha \]

It is assumed that \( z_t \) is i.i.d. both over time and with respect to \( A_t \). The depreciation rate of capital is 100%. Within this environment, do the following

   (a) Set up the social planner problem as a dynamic programming problem and derive the necessary conditions.

   (b) Conjecture a solution to the policy functions for consumption and savings. Derive the equations that determine these optimal policy functions and intuitively characterize their qualitative behavior. (Hint: The solution requires solving a set of nonlinear equations - derive these expressions and use them intuitively to motivate equilibrium behavior.)
4. The basic real business cycle studied in class was expressed as a social planner problem with household preferences given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t [\ln c_t + A (1 - h_t)] \right]$$

and technology represented by:

$$y_t = z_t k_t^\alpha h_t^{1-\alpha}$$

with $z_t$ a stochastic technology shock whose evolution is described by the process

$$z_t = z_{t-1} \varepsilon_t$$

where $\varepsilon_t \sim i.i.d.$ The solution to this problem was approximated by a set of linear policy functions. Using standard parameter values, the policy function for capital, consumption and labor are given below for two values of $\rho$:

For $\rho = 0.60$:

$$c_t = 0.532 k_t + 0.189 z_t$$
$$h_t = -0.477 k_t + 2.25 z_t$$
$$k_{t+1} = 0.942 k_t + 0.224 z_t$$

For $\rho = 0.99$:

$$c_t = 0.532 k_t + 0.654 z_t$$
$$h_t = -0.477 k_t + 1.78 z_t$$
$$k_{t+1} = 0.942 k_t + 0.110 z_t$$

What explains the differences in the policy functions?