1 Final Examination

Directions: Answer all questions. Point totals for each question are in parentheses.

1. Consider a representative agent exchange economy in which agents trade ownership shares in the endowment process. That is, equity purchased in period $t - 1$ (denoted $0 < z_t < 1$) yields dividends in period $t$ equal to the corresponding fraction of the current endowment (i.e. this is identical to the Lucas tree model). These proceeds, along with capital gains, are used to purchase current consumption and new shares of equity. The endowment, $x_t$, is growing stochastically over time. That is, $x_{t+1} = \lambda_{t+1} x_t$ where $\lambda_t$ is a random variable assumed to be independently and identically distributed. Agents each period choose $(c_t, z_t)$ in order to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^*)$$

where the utility function exhibits constant relative risk aversion (denoted $\gamma$) and $c_t^* = c_t - \alpha c_{t-1}$ where $0 < \alpha < 1$. That is, preferences exhibit habit persistence. Assume that $\gamma > 1$. Given this environment, answer the following:

Express the agent’s maximization problem as a dynamic programming problem and derive the associated necessary conditions.

a. Define a stationary rational expectations equilibrium in this economy.

b. Solve for the equilibrium equity price. Demonstrate that habit persistence increases the volatility of equity prices. (Note - use the price-dividend for your analysis and analyze the behavior of this ratio when $\alpha = 0$ compared to $\alpha > 0$.)

2. Modern macroeconomic models focus on impulse, amplification, and propagation mechanisms. Within the context of real business cycle theory, identify these mechanisms and discuss their strengths and weaknesses.
3. Consider a cash-in-advance model identical to the Stockman model in which only consumption is subject to the cash-in-advance constraint - the only modification is that one period nominal bonds are introduced into this economy. These bonds cost $1 in period \( t \) and return $\( (1 + n_t) \) in the following period. The agents’ budget and cash-in-advance constraints (expressed in nominal terms) are:

\[
P_t f (K_t) + M_t + B_{t-1} (1 + n_{t-1}) + T_t = P_t (c_t + i_t) + M_t + B_t
\]

\[
M_{t-1} + T_t \geq P_t c_t
\]

Note that \( T_t \) denotes the lump-sum monetary transfer received from the government. This is due to new money introduced into the economy each period - the money stock is assumed to be growing at the constant rate of \( g > 0 \) each period. The law of motion of the capital stock is the standard

\[
K_{t+1} = K_t (1 - \delta) + i_t
\]

where \( i_t \) denotes investment. In this economy, do the following:

a. Setup the problem as a dynamic programming problem, derive and interpret the associated necessary conditions.

b. Prove that, in steady-state, money growth does not affect the capital stock.

c. In a money-in-the-utility function model, it was demonstrated that money demand could be expressed in the form

\[
\frac{U_{m,t}}{U_{c,t}} = \frac{n_t}{1 + n_t}
\]

Where \( U_m \) and \( U_c \) represent the marginal utility of real balances and consumption respectively. Prove that a similar expression holds in this economy - and prove that nominal interest rates are zero if and only if the cash-in-advance constraint is not binding.
4. Geometric growth in an RBC model can be introduced by expressing the social planner problem as:

\[
\max E \left\{ \sum_{t=0}^{\infty} \beta^t [\ln c_t + A (1 - h_t)] \right\}
\]

subject to:

\[
\begin{align*}
c_t + x_t &= z_t k_t^{\alpha} h_t^{1-\alpha} \\
k_{t+1} &= k_t (1 - \delta) + x_t \\
z_t &= \theta^t \lambda_t \\
\lambda_t &= \lambda_{t-1} \varepsilon_t
\end{align*}
\]

The steady-state of this economy (i.e. setting \( \lambda_t = 1 \forall t \)) is a balanced growth path in which consumption, capital, and output all grow at the same rate, \( \nu \). (Note \( \nu \neq \theta \).) Labor, on the other hand is constant. Assume that \( \beta = 0.99 \) and \( \nu = 1.05 \). Determine the values of the other parameters so that along the balanced growth path:

a. The time spent in work activity is 25%.

b. Gross investment, \( x_t \), is 7% of beginning-of-period capital, \( k_t \).

c. The capital-output ratio is 4.

d. What is the implied growth rate of the technology shock, i.e. \( \theta \).