1 Final Exam

Directions: Answer all questions - they are equally weighted. Good luck and enjoy your summer — after the prelims.

1. Consider a two-state version of the Lucas tree model; that is, the endowment is assumed to follow a two-state Markov process with possible realizations \( x_1 < x_2 \) and symmetric transition probability matrix with diagonal elements \( \pi = 1/2 \). Suppose agents in this economy trade one- and two period bonds that cost \( p_1t \) and \( p_2t \) units of consumption, respectively, and return one unit of consumption upon maturity. Define the term premium as the difference between the expected return from selling a two-period bond after holding it for one-period and the certain return from a one-period bond. What is the sign of this term premium? Provide an explanation for your answer.

2. Consider the following version of the Stockman CIA model: The economy is populated by identical agents that have preferences given by:

\[
\sum_{t=0}^{\infty} \beta^t [u(c_t) + W(1 - h_t)]
\]

where \( c_t \) is consumption and \( h_t \) is labor. Each period, agents produce output via a concave production function with labor as the only input (there is no capital): \( y_t = f(h_t) \). The revenue from the sale of this output is combined with money carried over from the previous period, the monetary transfer, and returns from both one-period nominal and real bonds purchased in period \( t \) in order to acquire new money holdings and new bonds. The monetary transfer from the government is proportional to their initial money holdings; i.e. they receive an interest payment on their money holdings. This interest payment is equal to the (constant) monetary growth rate, \( \mu \). To be precise, if their money choice at period \( t \) is \( M_t \) then they receive a transfer of \( \mu M_t \) at the beginning of period \( t + 1 \). Consumption purchases in period \( t \) must be financed by cash acquired in period \( t - 1 \) and augmented by the monetary transfer.

(a) Set up the problem as a dynamic programming problem, derive and interpret the first-order conditions.

(b) Solve for the steady-state in this economy. Does the monetary growth rate influence the level of steady-state output? Explain.

(c) Does the Fisher relation hold in this economy?

(d) Is the equilibrium in this economy Pareto optimal?

3. Consider a simple stochastic production economy in which agents have log utility and the production function is \( y_t = z_t k_t^\alpha \) where \( y \) is output, \( z \) is an i.i.d. technology shock, and \( k_t \) is beginning of period capital. Depreciation is 100%. Answer the following
(a) Solve for the policy functions characterizing consumption and capital choices.

(b) Suppose one-period bonds were introduced into this economy. What is the correlation of interest rates and the marginal productivity of capital?

(c) Capital in this economy is a risky asset. Prove that the risk premium associated with capital is positive.

(d) Suppose an asset was introduced this economy which gave title to the output process. That is, purchase of this asset at time $t$ entitles the owner to the dividend process $\{y_{t+i}\}_{i=1}^{\infty}$. Determine the equilibrium price of this asset. What is the economic intuition of this price?

4. Over the last fifteen years, much research in macroeconomics has focused on three “puzzles”: the equity premium puzzle, the risk-free rate puzzle, and the small costs of business cycles (as demonstrated by R. Lucas, Jr.). Describe these puzzles and discuss to what extent they are related. In your discussion, be sure to develop the implications that these puzzles have for macroeconomists.