Economics 200E: Midterm Examination

Directions: Answer all questions. While the details are important, I am most interested in the intuition.

1. Consider a stochastic growth model in which output is a linear function of the beginning of period capital stock and a random technology shock. That is \( y_t = z_t k_t \). The shock follows a two state Markov process; the transition probability matrix is symmetric with diagonal elements \( \pi = 1/2 \). The social planner chooses sequences for consumption and capital in order to maximize:

\[
E_0 \left[ \sum_{t=1}^{\infty} \beta^{t-1} \frac{c_t^{1-\gamma} - 1}{1-\gamma} \right]
\]

Given this environment, answer the following questions:

a. Express the maximization problem as a dynamic programming problem. Be explicit in identifying the state and policy variables.

b. Derive and interpret the associated necessary conditions.

c. Make an educated guess for the form of the policy function describing optimal consumption. Show that this conjecture is verified by the model. (Hint: Note that the shock is positively autocorrelated. Also, there is no closed form solution for this model but it is possible to derive the expressions which define the policy function.)

d. Using the expressions derived in part c., characterize the behavior of consumption as a function of agents’ relative risk aversion. (It is not necessary to provide formal proofs for this, instead focus on the intuition.)

2. Consider a representative agent exchange economy in which the endowment takes on two states, \( x_1 < x_2 \). The transition probability matrix is symmetric with diagonal elements \( \pi = 1/2 \). Agents make consumption and portfolio decisions in order to maximize lifetime utility

\[
E_0 \left[ \sum_{t=1}^{\infty} \beta^{t-1} U(c_t) \right]
\]

Agents trade two assets: one period bonds that cost 1 unit of consumption in period \( t \) and yield \( R1_t \) in period \( t+1 \) and two period bonds that sell for 1 unit of consumption in period \( t \) and yield \( R2_t \) in period \( t+2 \). (Note that both returns represent gross returns and are expressed as one period rates of return.)

a. Setup the maximization problem as a dynamic programming problem and derive the necessary conditions associated with optimal bond holdings.

b. Prove that equilibrium interest rates (both long and short term) are negatively related to the endowment. What is the intuition?

c. The term premium can be defined as: \( TP_t = E \left[ \frac{R2_t}{R1_{t+1}} \right] - R1_t \). That is, the term premium is the difference between the expected return from liquidating a two period bond after one period and the certain return on a one period bond. What is the sign of the term premium in this economy? (It is not necessary to derive this explicitly – use the consumption based capital asset pricing model’s characterization of risk to determine the sign.)

3. What is the intuition behind Hall’s test of the permanent income hypothesis? Do you agree with the prominent economist’s assessment that Hall “… must have been on drugs when he wrote the paper?”