Midterm Examination

Answer all questions – points for each question are in parentheses. Provide explanations and intuition for your answers.

1.(10) What is the characterization of risk in the Lucas asset pricing model?

2.(10) Describe the intuition for Hall’s test of the permanent income hypothesis.

3.(20) Consider the economy studied by Mehra and Prescott: a representative agent exchange economy in which the endowment grows at the random rate $\lambda_t$. Assume that this growth rate is independently and identically distributed over time. Agents have isoelastic preferences and trade equity which entitles the owner to the future dividend stream, $\{x_{t+1}\}_{t=1}^\infty$, and one period bonds which cost $p_t$ units of consumption in period $t$ and return 1 unit of consumption in the following period.

   a. Set up the agent’s optimization problem as a dynamic programming problem. Identify the state variables.
   b. Derive and interpret the necessary conditions associated with the maximization problem.
   c. Solve for equilibrium equity and bond prices. (Are there any restrictions on parameter values that must be made to ensure an equilibrium exists?)
   d. Suppose utility is logarithmic. Prove the conditional equity premia is constant and positive.

4.(20) Consider a representative agent, exchange economy (a Lucas tree model) in which it is assumed that the endowment is independently and identically distributed and can take on any value in the interval $(x', x'')$ where $x' > 0$. In this economy, agents trade one and two-period pure discount bonds - i.e. these bonds sell for prices $p_{1t}$ and $p_{2t}$, respectively, and yield one unit of consumption at maturity. Agents choose bonds and consumption in every period in order to maximize:

$$E \sum_{t=0}^\infty \beta^t U(c_t)$$

where $E$ denotes expectations with respect to the distribution for $x_t$.

   a. Formulate the individual's maximization problem as a dynamic programming problem. (Hint: To simplify things, assume that agents always sell their two period bonds after one period.)
   b. Prove that in equilibrium, even though bond prices are random, they are proportional to each other.
   c. What determines the volatility of bond prices? Why?