1 Midterm Exam

Directions: Answer all questions - point totals for each are provided. Remember, the most important part of your answer is the intuition behind the result - so be sure to explain your answers.

1.(25) Consider an exchange economy populated by identical agents that trade equity shares, $z_t$, defined as title to the endowment process. (That is, this is the same asset priced in the Lucas tree model.) Denote the price of equity as $q_t$. Agents also trade one-period bonds which cost $p_t$ units of consumption in period $t$ and return 1 unit of consumption in the following period. In addition to these assets, a one-period forward contract on bonds is traded. In this contract, agents agree at time $t$ to pay $\phi_t$ units of consumption in period $t+1$ for the promise of one unit of consumption to be received in period $t+2$. The endowment, $x_t$, is stochastic and varies over the interval $(\bar{x}, \overline{\bar{x}})$; furthermore, $x_t$ is assumed to be independently and identically distributed. Given this environment, agents choose a sequence of consumption and assets in order to maximize

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t) \right]$$

a. Formulate the agent’s problem as a dynamic programming problem. Be explicit in identifying the state and control variables.

b. Derive and interpret the necessary conditions which characterize the solution to this maximization problem.

c. Define a recursive equilibrium in this economy. How are rational expectations imposed in this context?

d. Prove that equilibrium bond and equity prices are positively correlated with the endowment while the price of the forward contract is constant. Explain these results.

2.(25) Consider the following optimal growth model. Agents’ preferences are given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \ln c_t \right]$$

while output is a concave function of beginning of period capital and a technology shock. That is,

$$y_t = z_t k_t^\alpha$$

It is assumed that $z_t$ is i.i.d. The law of motion for the capital stock is

$$k_{t+1} = k_t^{1-\delta} + \delta k_t$$
Note that the parameter $\delta$ incorporates both depreciation and costs of adjustment that affect investment, $i_t$. Solve for the policy functions describing optimal consumption and investment. Show that if $\delta = 1$, the solution is identical to that studied in class. (Note: When manipulating the associated necessary conditions, it is useful to express the variables as ratios when possible. For instance, the $MPK = \alpha z_t k_t^{\alpha-1} = \frac{y_t}{k_t}$. And the resource constraint is $\frac{y_t}{c_t} = 1 + \frac{i_t}{c_t}$.)

3. (15) Empirically, the average return from selling a 6-month bond after holding it for 3 months is greater than that on a 3-month bond over the same period. That is, there exists a positive term premium on 6-month bonds. Describe carefully the steps one would take to apply the empirical methodology used by Mehra and Prescott in their study of the equity premium to analyze this term premium within the context of the CCAPM.

4. (10) The equity premium puzzle is often associated with the “risk-free rate puzzle.” What does this refer to - and what is its relevance to the equity premium puzzle?