Midterm Examination

Directions: Answer all questions - I am most interested in the intuition behind the answers so please provide sufficient discussion with your results.

1. Consider a simple production economy in which agents have log utility and the production function is \( y_t = z_t k_t^a \) where \( y \) is output, \( z \) is an i.i.d. technology shock, and \( k_t \) is beginning of period capital. Depreciation is 100%. Answer the following
   i. Prove that consumption is a constant fraction of output.
   ii. Suppose one-period bonds were introduced into this economy. What is the correlation of interest rates and the marginal productivity of capital?
   iii. Capital in this economy is a risky asset. Prove that the risk premium associated with capital is positive.

2. Again consider a stochastic growth model. Preferences exhibit habit persistence (represented by the parameter \( h > 0 \)) and are given by:

\[
E_0 \sum_{t=0}^{\infty} \beta^t U (c_t - hc_{t-1})
\]

Capital is subject to adjustment costs so that the economy wide resource constraint is:

\[
z_t k_t^a = c_t + i_t + \frac{q}{2} (k_{t+1} - k_t)^2
\]

where \( i \) represents investment and \( q \) is the adjustment cost parameter. The law of motion for the capital stock is:

\[
k_{t+1} = k_t (1 - \delta) + i_t
\]

where \( \delta \) represents the depreciation rate. Given this environment, do the following:
   i. Set up the social planner problem as a dynamic programming problem. Identify the state and control variables.
   ii. Derive and interpret the necessary conditions.
   iii. How would the parameters \( h \) and \( q \) effect the equilibrium characteristics of the economy?
3. Empirically, the return from selling a 6-month bond after holding it for 3 months is on average greater than that from purchasing a 3-month bond. That is, there is a positive term premium in the data. Describe in detail how you would apply Mehra and Prescott’s calibration methodology to study this term premium within the context of the consumption based capital asset pricing model.

4. What is the intuition behind Hall’s test of the permanent income hypothesis? Do you agree (as Romer points out in a footnote) that Hall “must have been on drugs” when he wrote the paper?