Midterm Examination

Directions: Answer all questions (the questions are weighted equally). I am most interested in the intuition behind the answers so please provide sufficient discussion with your results.

1. Consider the following stochastic growth model. Assume the economy is populated by identical agents with preferences given by:

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \ln c_t \right] \]

while technology for each agent is given by

\[ y_t = z_t k_t \]

The depreciation rate of capital is 100%. The technology shock is assumed to be independently distributed over time. In addition to capital, agents trade one and two period bonds that cost, respectively, \( p_1 t \) and \( p_2 t \) units of consumption at time \( t \) and return 1 unit of consumption, upon maturity.

(a) Setup the problem as a dynamic programming problem, derive and interpret the associated necessary conditions. (Assume that agents sell their two-period bonds after one period.) Use the necessary conditions for bonds to derive the relevant intertemporal efficiency condition between periods \( t \) and \( t+2 \) for the two-period bond; i.e. the condition that would hold if agents held the bond to maturity.

(b) Define a recursive competitive equilibrium in this economy. Determine the equilibrium policy functions. Characterize the behavior of equilibrium interest rates.

(c) Define the term premium at time \( t \) as the difference between the return from holding a two-period bond until maturity and the expected return from purchasing a sequence of one period bonds. Using the intuition of the CCAPM, what is the sign of this term premium. (Note: this term premium, defined as the rolling premium, is the negative of a risk premium.)
2. Write a brief essay on the relevance of asset pricing to macroeconomics. Include in your discussion the importance of the “risk-free rate puzzle” and the “equity premium puzzle”.

3. Consider an exchange economy populated by identical agents that trade equity shares, $z_t$, defined as title to the endowment process. (That is, this is the same asset priced in the Lucas tree model.) Denote the price of equity as $q_t$. Agents also trade one-period bonds which cost $p_t$ units of consumption in period $t$ and return 1 unit of consumption in the following period. In addition to these assets, a one-period forward contract on bonds is traded. In this contract, agents agree at time $t$ to pay $\phi_t$ units of consumption in period $t + 1$ for the promise of one unit of consumption to be received in period $t + 2$. The endowment, $x_t$, is stochastic and varies over the interval $(\underline{x}, \overline{x})$; furthermore, $x_t$ is assumed to be independently and identically distributed. Given this environment, agents choose a sequence of consumption and assets in order to maximize

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t) \right]$$

(a) Formulate the agent’s problem as a dynamic programming problem. Be explicit in identifying the state and control variables.

(b) Derive and interpret the necessary conditions which characterize the solution to this maximization problem.

(c) Define a recursive equilibrium in this economy. How are rational expectations imposed in this context?

(d) Prove that equilibrium bond and equity prices are positively correlated with the endowment while the price of the forward contract is constant. Explain these results.

4. When Robert Hall first presented his paper on the permanent income hypothesis, a prominent macroeconomist claimed that he must have been on drugs when he wrote the paper. Why would the prominent macroeconomist make this charge? Do you agree with him?