1 Midterm Exam

Directions: Answer all questions - Point totals for each are given in parentheses.

1. (20) Consider two representative agent, exchange economies in which one period bonds and equity are traded. Equity entitles the owner to the future stream of output (as in the Lucas tree model) while bonds cost \( p_t \) units of consumption in period \( t \) and return 1 unit of consumption upon maturity in period \( t + 1 \). In Economy I, the level of the endowment, \( x_t \), is random while in Economy II, the growth rate of the endowment, \( \lambda_t \), is stochastic. Both random variables are independently and identically distributed over time. Assume that agents instantaneous utility is logarithmic. Within this environment, answer the following:

   a. Write down the equations which determine the equilibrium prices of bonds and equity (i.e. it is not necessary to write down the associated dynamic programming problem - you can work straight from the appropriate necessary conditions).

   b. Define equilibrium prices in both economies.

   c. Compare the volatility of asset prices in both economies.

   d. Prove that the equity premium is time varying in Economy I but constant in Economy II.

2. (10) Analyze the following quote: “In the Lucas tree model, the dividend associated with equity is consumption, therefore it must be the case that the equity premium is positive.”

3. (10) Describe the intuition for Hall’s test of the permanent income hypothesis. Based on his analysis, do you think that the permanent income hypothesis is a reasonable assumption to make in representative agent models?
4.(25) Consider the following stochastic growth model. Assume the economy is populated by identical agents with preferences given by:

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \ln c_t \right] \]

while technology for each agent is given by

\[ y_t = z_t k_t \]

The depreciation rate of capital is 100\%. The technology shock is assumed to be independently distributed over time. In addition to capital, agents trade one and two period bonds that cost one unit of consumption at time \( t \) and return \((1 + r_{1_t})\) and \((1 + r_{2_t})^2\) units of consumption, respectively, upon maturity.

a. Setup the problem as a dynamic programming problem, derive and interpret the associated necessary conditions. (Assume that agents sell their two period bonds after one period.)

b. Define a recursive competitive equilibrium in this economy.

c. Define the term premium as:

\[ TP_t = E \left( \frac{(1 + r_{2_t})^2}{(1 + r_{1_{t+1}})} \right) - (1 + r_{1_t}) \]

Prove that the term premium in this economy is zero. Explain.