1 Midterm Answer Key

1. Economy I - endowment is stationary in levels (as in original Lucas model and in LeRoy & LaCivita). Economy II - endowment is stationary in growth rates (as in Mehra & Prescott).
   a. In both economies, the necessary conditions evaluated at the market clearing quantities \( c_t = x_t \) determines the price of the assets.

   \[
   \frac{q_t}{x_t} = \beta E \left( \frac{(q_{t+1} + x_{t+1})}{x_{t+1}} \right) \quad (1)
   \]

   \[
   \frac{p_t}{x_t} = \beta E \left( \frac{1}{x_{t+1}} \right) \quad (2)
   \]

   Note the absence of a time subscript on the expectations operator since in both economies the exogenous random variable is i.i.d.

   b. In equilibrium, the prices of assets will be functions of the state variables. Hence in Economy I: \( q_t^I = q_t^I (x_t) \); \( p_t^I = p_t^I (x_t) \). In Economy II, the state variables are both the level of the endowment and the growth rate: \( q_t^{II} = q_t^{II} (x_t, \lambda_t) \).

   For bond prices, since these define a rate of return (rather than a level of price), it is reasonable to think that bond prices will be a function of the growth rate only (bond prices will be stationary, equity prices will not be, although equity rates of return will be). \( p_t^{II} = p_t^{II} (\lambda_t) \)

   c. To determine the equilibrium behavior of asset prices in both economies, we exploit the i.i.d. assumption.

   **Economy I**

   Rewrite eq.(1) as

   \[
   \frac{q_t^I (x_t)}{x_t} = \beta E \left( \frac{q_{t+1}^I (x_{t+1})}{x_{t+1}} + x_{t+1} \right) \]

   The right-hand side is a constant since all terms are functions of next period’s endowment - this implies that the price-dividend ratio is constant. Hence, stock prices are as volatile as dividends.

   From eq.(2), it is clear that the bond price will behave identically. So in this economy equity and bond prices are equally volatile.

   Let the price-dividend ratio be denoted as \( \omega_t^I \). We have

   \[
   \omega_t^I = \beta E \left[ \omega_t^I + 1 \right]
   \]

   Solving yields:

   \[
   \omega_t^I = \frac{\beta}{1 - \beta} \quad (3)
   \]
For completeness, note

\[ p'(x_t) = x_t \beta E \left( \frac{1}{x_{t+1}} \right) \]

**Economy II**

Given the assumption of log preferences, it makes sense to conjecture that the function for the price of equity will be homogeneous of degree one in the endowment. As stated earlier, since the bond price determines a rate of return, the function for this price will be a stationary function of \( \lambda_t \) only.

\[ q^{II}(x_t, \lambda_t) = x_t \omega^{II}(\lambda_t) \tag{4} \]

Using these in the equilibrium conditions, we have

\[ \omega^{II}(\lambda_t) = \beta E [\omega^{II}(\lambda_{t+1}) + 1] \]

which implies

\[ \omega^{II} = \frac{\beta}{1 - \beta} = \omega^I \]

For bond prices, we have:

\[ p^{II}(\lambda_t) = \beta E \left( \frac{1}{\lambda_{t+1}} \right) \]

The implication is that equity prices will be as volatile as dividends while bond prices will be constant in the growing economy. (Note the difference: in the growing economy, the expected growth rate of the endowment is constant, while in Economy I, the expected growth rate is time varying and determined by the current level of the endowment.)

d. The equity premium is defined as:

\[ ep_t = E \left[ \frac{q_{t+1} + x_{t+1}}{q_t} \right] - \frac{1}{p_t} \tag{5} \]

**Economy I**

Rewrite eq.(5) as

\[ ep'_I = E \left( x_{t+1} \right) \left( \frac{\omega' + 1}{\omega} \right) - \frac{1}{x_t \beta E \left( \frac{1}{x_{t+1}} \right)} = \frac{1}{\beta x_t} \left( E \left( x_{t+1} \right) - \frac{1}{E \left( \frac{1}{x_{t+1}} \right)} \right) \]

Clearly this will be time varying.

**Economy II**

Rewrite eq.(5) as

\[ ep''_I = E \left( \lambda_{t+1} \right) \left( \frac{\omega'' + 1}{\omega} \right) - \frac{1}{\beta E \left( \frac{1}{\lambda_{t+1}} \right)} = \frac{1}{\beta} \left( E \left( \lambda_{t+1} \right) - \frac{1}{E \left( \frac{1}{\lambda_{t+1}} \right)} \right) \]
Clearly this will be constant.

2. The sign of the (conditional) equity premium is the negative of the sign of:

\[\text{Cov}_t \left[ \frac{q(x_{t+1}) + x_{t+1}}{q(x_t)}, U'(x_{t+1}) \right]\]

Since the dividend, \( x_t \), is consumption, this component of equity return will indeed imply a positive equity premium. And if \( q(x_t) \) is monotonically increasing in the endowment, then this will reinforce that result. However, if agents are highly risk averse and the endowment is autocorrelated, it is possible for the demand for equity to fall with an increase in the endowment - this, in turn, raises the possibility that stock prices fall. If they fall by a greater percentage than the dividend increases, the covariance can turn positive implying a negative equity premium.

3. See Romer’s discussion of the Hall analysis. Since departures from the permanent income hypothesis are often motivated by liquidity constrained agents or imperfect capital markets, these criticisms are less applicable to representative agent economies since they imply a complete set of asset markets.

4. The necessary conditions are (Define \( R1_t = (1 + r1_t) \), \( R2_t^2 = (1 + r2_t)^2 \))

\[
\frac{1}{c_t} = \beta E \left[ \frac{1}{c_{t+1}} \right] \tag{6}
\]

\[
\frac{1}{c_t} = R1_t \beta E \left[ \frac{1}{c_{t+1}} \right] \tag{7}
\]

\[
\frac{1}{c_t} = R2_t^2 \beta E \left[ \frac{1}{c_{t+1} R1_{t+1}} \right] \tag{8}
\]

b. A recursive competitive equilibrium is defined by five functions: a value function (defined by the agent’s maximization problem), two functions for the one- and two-period interest rates: \( R1(k_t, z_t) \), \( R2^2(k_t, z_t) \), and two policy functions: \( c(k_t, z_t) \), \( k(k_t, z_t) \) describing agents’ optimal choices. These latter four functions are determined by the necessary conditions and the aggregate resource constraint: \( z_t k_t = c(k_t, z_t) + k(k_t, z_t) \). Agents expectations are rational in that the price functions they use in their optimizing problem are the same as implied by equilibrium.

c. Before analyzing the term premium, first determine the behavior of consumption. Since we have log preferences, it is reasonable to conjecture that:

\[c_t = y_t \theta\]

where \( y_t = z_t k_t \). That is, agents consume (and, therefore, save) a constant fraction of their income. Using this in eq.(6) yields:

\[c_t = (1 - \beta) z_t k_t \tag{9} \]
Now use this in the equation for the one period interest rate:

\[
\frac{1}{\beta N_z t} = R_1 \beta E \left[ \frac{1}{(1 - \beta) z_{t+1} k_{t+1}} \right]
\]

Since \( k_{t+1} = \beta z_t k_t \), this simplifies to:

\[
R_1^{-1} = E \left[ \frac{1}{z_{t+1}} \right]
\]

Since \( z_t \) is i.i.d, this implies one-period interest rates are constant. But from eq.(8), this implies that two period interest rates are also constant. Therefore the term premium must be zero since returns are constant.