1. Consider a simple production economy in which agents have log utility and the production function is $y_t = z_t k_t^\alpha$ where $y_t$ is output, $z_t$ is an i.i.d. technology shock, and $k_t$ is beginning of period capital. Depreciation is 100%. Answer the following
i. Prove that consumption is a constant fraction of output.
ii. Suppose one-period bonds were introduced into this economy. What is the correlation of interest rates and the marginal productivity of capital?
iii. Capital in this economy is a risky asset. Prove that the risk premium associated with capital is positive.

2. Again consider a stochastic growth model. Preferences exhibit habit persistence (represented by the parameter $h > 0$) and are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U (c_t - hc_{t-1})$$

Capital is subject to adjustment costs so that the economy wide resource constraint is:

$$z_t k_t^\alpha = c_t + i_t + \frac{q}{2} (k_{t+1} - k_t)^2$$

where $i$ represents investment and $q$ is the adjustment cost parameter. The law of motion for the capital stock is:

$$k_{t+1} = k_t (1 - \delta) + i_t$$

where $\delta$ represents the depreciation rate. Given this environment, do the following:

i. Set up the social planner problem as a dynamic programming problem. Identify the state and control variables.
ii. Derive and interpret the necessary conditions.
iii. How would the parameters $h$ and $q$ effect the equilibrium characteristics of the economy?
3. Consider a growing economy identical to that studied in Mehra and Prescott’s analysis of the equity premium puzzle. Assume agents have log preferences and that the growth rate of the economy can take on two values $\lambda_1 < \lambda_2$. The transition probability matrix is symmetric with diagonal elements $\pi > 1/2$. Suppose one- and two-period bonds are traded in this economy. Prove that the term premium (defined as the difference between the expected return from selling a two-period bond after one period and the return on a one-period bond) is negative. Why? (You do not need to set up the dynamic programming problem - you can work directly from the equilibrium pricing equations.)

4. In the economy described in Question #1, agents follow the permanent income hypothesis as described in the article by Robert Hall. Yet, agents consume a constant fraction of their income regardless of the serial correlation properties of the shock. That is, consumption is the same whether the shock is perceived to be long-lasting (i.e. exhibits positive serial correlation) or transitory (the shock is i.i.d.) - this is similar to a simple Keynesian consumption function. Do you see this as a contradiction?