1 Homework #3 - Due 5/10 (in class)

This homework set involves some numerical computation. Hence, it would be a good idea to use the computer to help you—I recommend using Matlab (or Mathematica). We will be using Matlab later and the sooner you become familiar with the syntax, the better.

1. Suppose an economy is populated by infinitely lived identical agents that receive a stochastic endowment, \( x_t \), every period. The motion of the endowment is given by the two-state Markov process:

\[
x_t \begin{cases} 
  x1 = x - \delta \\
  x2 = x + \delta 
\end{cases}
\]

The transition probability matrix is symmetric with diagonal elements \( \pi \). Agents buy and sell one-period bonds which cost one unit of consumption in period \( t \) and return \((1 + r_t)\) units of consumption next period. Consumption and bond purchases are made in order to maximize

\[
E_0 \left[ \sum_{t=0}^{\infty} \delta^t U(\hat{c}_t) \right]
\]

where \( \hat{c}_t = c_t - \alpha c_{t-1} \). This specification of preferences incorporates a simple form of habit persistence in which the utility of consumption depends on the relative magnitude of current and past levels of consumption.

(a) Formulate the individual’s problem as a dynamic programming problem and derive the necessary conditions associated with optimal bond purchases

(b) Equilibrium real interest rates will be determined by the time series of \( U'(\hat{x}_t) \) where \( \hat{x}_t \) is the equilibrium level of consumption services. Derive the Markov process for \( \hat{x}_t \) and determine the associated unconditional probability distribution.

(c) Assume that \( U(\cdot) = \ln \hat{c} ; \beta = 0.98, \pi = 0.60, x = 10, \delta = 0.2 \).

   i. Assume that \( \alpha = 0 \)
      A. Determine equilibrium interest rates.
      B. Compute the unconditional mean and variance of interest rates.
      C. Compute the first-order serial correlation of the marginal utility of consumption.

   ii. Assume that \( \alpha = 0.25 \). Repeat (a)-(c).

2. Consider an actual economy in which the (gross) growth rate of consumption follows the \( AR(1) \) process

\[
\lambda_t = 0.408 + 0.6\lambda_{t-1} + \varepsilon_t
\]

where \( \varepsilon_t \) is a white noise disturbance term with \( \sigma^2 = (0.02)^2 \). Use this process to calibrate and solve the Mehra-Prescott version of the Lucas asset pricing model in which the growth rate of consumption is assumed to follow a two-state Markov process. That is, \( c_{t+1} = c_t \lambda_{t+1} \) where

\[
\lambda_t = \begin{cases} 
  \lambda_1 = \lambda - \delta \\
  \lambda_2 = \lambda + \delta 
\end{cases}
\]

and the one-period transition probability matrix is symmetric with diagonal elements of \( \pi \). Assume that agents have constant relative risk aversion utility with the risk aversion parameter \( = 2 \); moreover, agents’ discount factor, \( \beta = 0.5 \). Calibrate the parameters of the Markov process using the estimated \( AR(1) \) for consumption growth and compute the following (note: the equity premium in this economy is quite small so you need to present your calculations out to 5 decimal places):

(a) Determine the conditional expected return on equity.
(b) Determine the conditional interest rates. Interpret your results.
(c) Determine the unconditional equity premium. Interpret the sign of the equity premium.
3. The Rietz crash state model. Using the basic Mehra-Prescott economy (as studied in their paper on the equity premium), assume that the growth rate of the endowment follows a 3-state Markov process:

\[
\lambda_t = \begin{cases} 
    \lambda_1 = 1 + m + v \\
    \lambda_2 = 1 + m - v \\
    \lambda_3 = k (1 + m) 
\end{cases}
\]

with transition probability matrix of

\[
\begin{pmatrix}
    \pi & 1 - \pi - \delta & \delta \\
    1 - \pi - \delta & \pi & \delta \\
    1/2 & 1/2 & 0
\end{pmatrix}
\]

Note that states one and two are normal growth rate states while state 3 is crash state. Also note that there is no persistence to the crash state - the economy immediately moves to either state 1 or 2. Calibrate this model to the same set of moments used in Mehra and Prescott: \( E(\lambda_t) = 1.018, \text{Sd}(\lambda_t) = 0.036, \text{Corr}(\lambda_t, \lambda_{t-1}) = -0.14 \). Study two different economies: In Economy 1 \( \delta = 0.01 \) and \( k = 0.75 \) while in Economy 2, \( \delta = 0.002 \) and \( k = 0.5 \). Assume in both economies that \( \beta = 0.98 \).

(a) Determine the unconditional probabilities.
(b) Determine the values of \((\pi, m, v)\).
(c) Calculate the average equity premium in both economies when RRA = 6.
(d) Calculate the average risk free rate in both economies. Do these economies explain the equity premium puzzle? Discuss your results.