Calibration and the Volatility of Labor: A Cautionary Note

Abstract
A key parameter in real business cycle models is the weight on the utility of leisure. Typically this parameter is chosen so that the steady-state level of work activity matches the corresponding measure in the data, i.e. the amount of time workers spend in market activity. While the calibration of this parameter is often highlighted in business cycle research, this paper demonstrates that this parameter has no influence on the cyclical properties of the Hansen (1985) indivisible labor model, when solved using traditional methods. Hence, the functional form of utility rather than the parameterization of utility is the critical factor.

- **JEL Classification:** E32, C68
- **Keywords:** calibration, real business cycles

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*I am indebted to Parantap Basu and Kevin Hoover for insightful comments and suggestions.
1 Introduction

The empirical methodology of quantitative macroeconomics is now firmly established: construct a general equilibrium model with explicit functional forms for preferences and technology, calibrate the parameters of these functions so that features of the model (e.g. characteristics of steady-state equilibrium) match key characteristics of the data, and then use numerical approximation methods to construct the equilibrium of the model. By generating artificial data from the solution, one can study the equilibrium characteristics of the model and compare these to the data equivalents.

While this approach has improved the profession’s understanding of the economy, it is also the case that, because one must use numerical methods to solve the model, the analysis often comes at the expense of intuition. With the intent of improving the intuition behind these models, this note illustrates that the standard procedure for calibrating and solving a standard real business cycle model implies that the importance of leisure in agent’s utility function has no implications for the cyclical behavior of labor.

This point is established using the standard indivisible labor model due to Hansen (1985). This model, in which utility at the aggregate level is linear in leisure, was a critical development in the real business cycle approach since it improved the model’s ability to match the cyclical volatility of labor. That is, standard preferences with diminishing marginal utility of leisure produced too little volatility; the assumption of linear preferences produced behavior more in line with observation.\footnote{For a rigorous and thorough analysis of the indivisible labor model, see Mulligan (1999).} For this reason, this model, while not the canonical model
of RBC analysis, has become a common workhorse for business cycle analysis. An often highlighted step in making this model operational is the calibration of the parameters and, in particular, determining the weight on the utility from leisure. This paper demonstrates that such attention on this parameter is unwarranted; i.e. the value of this parameter is irrelevant for the cyclical behavior of the economy under the proviso that the model’s remaining parameters are calibrated in the usual fashion and the model is solved using typical approximation methods. The implication is that the functional form and not the parameterization of preferences is the critical feature in determining the cyclical properties of labor.

It is well known that the choice of approximation method is not innocuous because of the associated approximation errors. The example presented here represents another point of caution in using numerical methods to solve business cycle models. Namely, the choice of one solution method, say a linear-quadratic approximation approach, may result in a greater loss of intuition than an alternative method, e.g. linearizing the conditions that define an equilibrium. Hence, one should employ an approximation method only after the structure of the model has been fully examined.

2 The Model

To show this, I start with the standard RBC model with indivisible labor written as a social planner problem:

$$\max_{\{c_t, h_t, k_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\ln c_t + A (1 - h_t)] \right\}$$  \hspace{1cm} (1)
subject to:
\[ z_t k_t^\alpha h_t^{1-\alpha} + (1 - \delta) k_t = c_t + k_{t+1} \]  

(2)

Current consumption is \( c_t \), \( h_t \) is labor, and \( k_t \) is beginning of period capital stock. Output in each period is denoted \( y_t = z_t k_t^\alpha h_t^{1-\alpha} \). The two critical preference parameters are agents’ discount factor, \( \beta \), and the weight on leisure in the utility function, \( A \). The technology parameters are the depreciation rate, \( \delta \), and the elasticity of output with respect to capital, \( \alpha \). The technology shock, \( z_t \), is assumed to follow an autoregressive process with autoregressive parameter \( \rho \) and innovations such that \( E(z_t) = 1 \).

This maximization problem produces two necessary conditions representing the labor-leisure trade-off and the consumption-investment decision:
\[ c_t = \frac{(1 - \alpha)}{A} z_t k_t^\alpha h_t^{1-\alpha} \]  

(3)

\[ c_t^{-1} = \beta E_t \left[ c_{t+1}^{-1} (\alpha z_{t+1} k_{t+1}^\alpha h_{t+1}^{1-\alpha} + 1 - \delta) \right] \]  

(4)

The solution to this economy is represented by functions for capital (i.e. investment), consumption, and labor that satisfy the two necessary conditions (eqs. (3),(4)), the economy-wide resource constraint, eq. (2) and are consistent with expectations implied by the Markov process for the technology shock; the arguments of these functions are the state variables \( k_t \) and \( z_t \). A typical method to find this solution is to “linearize” these three equations by taking a first-order Taylor series expansion of the expressions around the steady-state values. The resulting set of linear expectational difference equations implies that the solution functions will also be linear.
The first step in solving the model involves choosing values for the parameters describing tastes and technology - that is, the model is calibrated. As is well know, this is typically done so that the steady-state of the model replicates some of the stylized facts of growth. For the point that is being made in this paper, it is important to note that this is usually done in a sequential manner. That is, labor’s share, equal to \((1 - \alpha)\), is used to pin down \(\alpha\). Then the discount factor is chosen so that the implied steady state interest rate, equal to \(\beta^{-1} - 1\), is consistent with the data. Next, note that in steady-state (in which \(z_t = 1, k_t = \bar{k}, \) and \(y_t = \bar{y} \forall t\)), eq. (4) implies

\[
\frac{\beta^{-1} - 1 + \delta}{\alpha} = \bar{k}^{-1} \bar{h} \beta^{1-\alpha} = \frac{\bar{y}}{\bar{k}}
\]

Therefore, the observed capital-output ratio in the economy is used to determine \(\delta\).

The final parameter, \(A\), is determined by exploiting the steady-state relationship implied by eq. (3):

\[
\bar{h} = \frac{(1 - \alpha) \bar{y}}{A \bar{c}}
\]

Since all terms on the right-hand side other than \(A\) have been determined, the choice of this parameter pins down the amount of time spent in work activity in the steady-state.

Authors that use the linearization approximation to solve real business cycle models have calibrated their models along these lines and, in particular, have stressed the importance of calibrating the weight of leisure in the utility function, i.e. the parameter \(A\). I now demonstrate that, for this particular model (see footnote 2 below) the parameter \(A\) has absolutely no influence on the cyclical equilibrium behavior of this model.
3 Equilibrium

This can be seen by rewriting the equations defining equilibrium as follows:

\[ h_t = \frac{(1 - \alpha) y_t}{A c_t} \quad \text{(7)} \]

\[ c_t^{-1} = \beta E_t \left[ c_{t+1}^{-1} \left( \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right] \quad \text{(8)} \]

\[ k_{t+1} = y_t + k_t (1 - \delta) - c_t \quad \text{(9)} \]

\[ \rho z_t = E_t (z_{t+1}) \quad \text{(10)} \]

Changes in the parameter \( A \) will influence equilibrium behavior through two possible channels - directly through its presence in an equation (as in eq. (7)) or indirectly through its impact on the steady-state values around which the equations are being linearized. But linearizing the equations and expressing the variables as percentage deviations from steady-state values is akin to taking log approximations so the direct effect of parameter \( A \) in eq. (7) will drop out. As seen in eqs. (7) and (8), the effect on steady-state values is mitigated since the capital-output ratio and output-consumption ratio are both independent of \( A \). However, the steady-state values of \((y_t, c_t, k_t)\) are all affected by \( A \) so that the impact through the second channel is still possible.

To see that this channel is not present in the linearized model, replace \( h_t \) in eq. (7) using the production function so that this equilibrium condition is now written in terms of \( y_t \):

\[ y_t = \left( \frac{1 - \alpha}{A} \right)^{1/\alpha} z_t^{1/\alpha} k_t c_t^{-\left(\frac{1-\alpha}{\alpha}\right)} \quad \text{(11)} \]

Then linearizing the system of equations (11), (8), (9), and (10) around the steady-state values yields the following system of equations: (Note all variables are expressed as
percentage deviations from steady-state, \( \hat{x}_t \equiv (x_t - \bar{x}) / \bar{x} \).

\[
\hat{y}_t = -\frac{1 - \alpha}{\alpha} \hat{c}_t + \hat{k}_t + \frac{1}{\alpha} \hat{z}_t \tag{12}
\]

\[
\hat{c}_t = E_t [\hat{c}_{t+1}] + \alpha \beta \frac{\bar{y}}{k} \hat{k}_{t+1} - \alpha \beta \frac{\bar{y}}{k} E_t [\hat{y}_{t+1}] \tag{13}
\]

\[
-\frac{\bar{c}}{k} \hat{c}_t + (1 - \delta) \hat{k}_t + \frac{\bar{y}}{k} \hat{y}_t = \hat{k}_{t+1} \tag{14}
\]

\[
\rho \hat{z}_t = E_t [\hat{z}_{t+1}] \tag{15}
\]

As is clear, all coefficients in this system of equations are independent of the parameter \( A \). Consequently, the implied solutions, i.e. the functions describing the equilibrium policy functions for consumption, capital (or investment) and output will also be independent of \( A \). Furthermore, due to eq. (7), this implies that the policy function for labor will also be independent of this parameter.

Hence, the exercise of choosing this parameter so that the steady-state features of the model match the data equivalent is a meaningless step with regard to the cyclical characteristics of the model. Rather than the parameterization of the utility function, it is the functional form that drives equilibrium business cycle behavior.

It is curious why this irrelevance has not been noted in previous work; however, it is possible that this is due to the numerical approximation method employed by some RBC analysts. Namely, the approach used in the early literature was to take a second-order approximation of the utility function around the steady-state. The resulting quadratic objective function implies linear necessary conditions; note that, the linear conditions analyzed

\[2 \text{ It is important to point out that this result does not generalize to all preferences; in particular, if the utility of leisure is not linear, then the weighting parameter will influence cyclical behavior. The critical feature in this model is that eq.}(7) \text{ is linear in labor.} \]
here, however, would imply, up to a constant, the same objective function. But working with the quadratic objective function may make these relationships, and, in particular, the irrelevance of the parameter $A$, less transparent.$^3$

As stated earlier, the use of numerical methods to solve economic models is highly beneficial since it greatly expands the environments which can be studied. However, it is important to analytically investigate the structure of these models so that the cost in terms of intuition is minimized.

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$^3$ Another possible reason for why this result was not noted in the early literature is that the policy functions were often defined in terms of the levels of the variables rather than percentage deviations from steady-state. Consequently, the coefficients on the equilibrium policy function, in particular, the constant term, would change as the value of $A$ changed. But since the end analysis typically involves percentage deviations from trend (defined by the H-P filter), this difference is eliminated.
References
