1 Intuition behind superneutrality in the Stockman model when the CIA constraint applies only to consumption

Stockman motivates the result that
\[ \frac{dk}{dg} = 0 \]

when the CIA constraint takes the form
\[ M_{t-1} + T_t \geq P_t c_t \]

by constructing a MC=MB argument. First, we start with the case when the CIA constraint applies to both consumption and investment. That is, the constraint is:
\[ M_{t-1} + T_t \geq P_t (c_t + K_{t+1} - (1 - \delta) K_t) \]

Suppose, at time \( t \), agents decrease consumption by \( dC \) and use this to buy capital. Since consumption and capital are both subject to the CIA constraint, \( dC = dK \) (in absolute value terms).

The loss in utility is:
\[ U_0' dC \]

The gain in utility is due to the stream of increased consumption that the unit of capital will provide. Recall in this model that the nominal proceeds from sales of output can not be used until the future period so the first increase in consumption will occur at time \( t+2 \). After that, the unit of capital depreciates and so output declines. Hence, the gain in utility is:
\[ \beta^2 \left( \frac{1}{1+g} \right) f' (\bar{K}) dK_t U'' (\bar{c}) \left[ 1 + \beta (1 - \delta) + \beta^2 (1 - \delta)^2 + ... \right] \]

Or, imposing steady-state, this becomes:
\[ \beta^2 \left( \frac{1}{1+g} \right) f' (\bar{K}) \frac{f' (\bar{K})}{1 - \beta (1 - \delta)} dK_t U'' (\bar{c}) \]

Setting the loss in utility equal to the gain in utility results in:
\[ \beta^2 \left( \frac{1}{1+g} \right) \frac{f' (\bar{K})}{1 - \beta (1 - \delta)} = 1 \]
Which is the same as eq. (7) in Stockman and establishes that $\frac{d\bar{K}}{dt} < 0$.

If the CIA constraint applies only to consumption, then the dollars that were going to used for purchases of capital at time $t$ can now be used to acquire capital at time $t-1$. That is, now $P_t dC_t$ fewer dollars are needed at time $t$ so that $\frac{P_t}{P_{t-1}} dC_t$ units of capital can be purchased at time $t-1$. That is:

$$dK_t = \frac{P_t}{P_{t-1}} dC_t$$

Replacing $dK_{t+1}$ in eq. (1) with this expression and noting that, since the capital is purchased at time $t-1$, the first increase in consumption will now occur in period $t+1$, the expression in eq. (1) becomes:

$$\beta \frac{P_t}{P_{t+1}} f'(K_t) \frac{P_t}{P_{t-1}} dC_t U_{t+1}' + \beta^2 \frac{P_{t+1}}{P_{t+2}} f'(K_t) (1 - \delta) \frac{P_t}{P_{t-1}} dC_t U_{t+2}' \quad (2)$$

$$+ \beta^3 \frac{P_{t+2}}{P_{t+3}} f'(K_t) (1 - \delta)^2 \frac{P_t}{P_{t-1}} dC_t U_{t+3}' + ...$$

Note that the inflation rates in each term will cancel out in steady-state - this is precisely why money growth does not affect the capital stock in steady-state. In steady-state the expression becomes

$$\frac{\beta f'(\bar{K})}{1 - \beta (1 - \delta)} dC_t U'(\bar{c})$$

Setting this equal to the loss in utility results in:

$$\frac{\beta f'(\bar{K})}{1 - \beta (1 - \delta)} = 1$$

This is the same condition as in Sidrauski and in the basic optimal growth model.