Monetary Policy in a Credit-in-Advance Economy*

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**Abstract**

It is well known that, in modern economies, most money has its origin in credit that nonbank economic agents obtain from the banking industry. In this paper, we develop a dynamic general equilibrium model where all money corresponds to credit that the households obtain from commercial banks. Technically, the credit structure of our economy is obtained by using a specific initial condition on the typical household’s optimization problem. This initial condition transforms a cash-in-advance economy into a credit-in-advance economy. We then look at the impact of monetary policy shocks on the behaviour of real and nominal variables in the model. We find that monetary policy has only very small real effects.

Keywords: Dynamic General Equilibrium Models, Cash-in-Advance, Monetary Policy.

JEL Classification: E13, E17, E41, E52.
1 Introduction

It is a fact of modern economies that most money has its origin in loans from commercial banks to households, firms and the government. Hence, a calibrated general equilibrium model should include the demand for credit by the households as an important dimension. In this paper, we take some steps into modelling borrowing by the households in a dynamic general equilibrium framework. We think that to make some progress in this direction is important for various reasons. On the one hand, in the real world households do actually borrow significant amounts from commercial banks (e.g. loans to buy a home). On the other hand, the behaviour of the demand for credit by households is likely to have specificities of its own implying that the response of the households’ demand for credit to increases in interest rates could differ from the firms’ response and could also differ among different types of households because the income and substitution effects may be different across households (for example, households with different wealth levels will behave differently).

When we start thinking about modelling household borrowing, a key issue that one has to deal with is the household’s initial debt position. If period 0 is the period where our analysis of the economy is starting (the present date, for example) and the household has been living for some periods before arriving at the beginning of period 0, what debt should we assume that it carries from the previous period? In this article, we propose using a specific initial condition. In the Appendix we show that this is the initial condition which naturally arises when we think the economy back to its initial moment (note that period 0 is not the period where the household’s life starts but rather the period where we catch her in time and try to model her behaviour). In any case, other initial conditions can and should be tried. What we cannot do is avoid modelling the initial debt position of the household.

The model we build is a model of monetary creation and destruction with a completely defined asset structure and where a complete description of monetary flows can be presented. We then use the model to look at the impact of technological shocks on the behaviour of nominal variables and at the impact of monetary policy (temporary increases in the rate of growth of the money supply) on the behaviour of both real and nominal variables.

We construct a model where there are only households, firms and banks in operation and we then
compute the decentralized competitive equilibrium. At the beginning of each period, commercial banks obtain reserves from the central bank which allow them to make loans to households. In making loans to households at the beginning of each period, commercial banks create money which did not exist before. Households pay the amount borrowed at the end of the same period thereby destroying the money which had been created at the beginning of the period. In the real world, money is constantly being created and destroyed. In our model, the cycles of monetary creation and destruction correspond to the periods. At the beginning of each period there is no money in the economy. Then, in the first moments of the period, money is created which will be destroyed at the end of the same period.

In this model, firms and banks are owned by the households. As a consequence, both the firms’ profits and the banks’ profits are distributed to households (the shareholders) at the end of each period.

In our model, the source of all money is loans from commercial banks to households. Commercial banks make loans to households and households then use the money obtained in this way to buy consumption goods from the firms.

In order to model the demand for credit by the households, we introduce the following initial condition on the typical household’s optimization problem: at the beginning of the period where our analysis of the economy is starting (period 0), the typical household owes the banks an amount which equals the sum of the wage earnings and dividend earnings that she received at the end of the previous period. After paying the debt to the banks, the household is left with nothing (the household also owns shares but, because households are all alike, shares are not traded in equilibrium) and must therefore borrow again from the banks to finance her consumption expenditure during the period that is just beginning (period 0). The market clearing conditions of the model together with the definitions of profits imply that the amount the household borrows at the beginning of period 0 is such that when she arrives at the end of period 0 she will again be left with nothing (after paying the debt to the banks). The consequence is that the household will need to borrow again from the banks at the beginning of period 1 in order to finance her consumption expenditure during this period. And this story is repeated every period into the future. All this is achieved by the introduction of the initial condition that we have just mentioned.
The complete description of monetary flows among economic agents is as follows.

At the beginning of each period, the typical household borrows from the banks the amount that she needs in order to be able to buy consumption goods from the firms during the period that is beginning. Loans obtained from a bank take the form of checkable deposits. During the period, the typical household spends these checkable deposits buying consumption goods from the firms. At the end of the period, the household receives back from the firms these checkable deposits (as wage payments and dividend payments). Then, the household pays the banks interest on the amount borrowed at the beginning of the period (the amount of interest due is paid by reducing the amount of checkable deposits that the household owns at the banks). However, since banks are owned by the households and households are all alike, the household immediately receives back the amount of interest paid to the banks (in the form of bank dividends). Afterwards, the household pays the banks the principal of the debt contracted at the beginning of the period. The structure of the model is such that, after all these payments, the household is left with nothing and must therefore borrow again from the banks at the start of the new period.

Since all transactions in our economy are financed by bank loans, it can be labelled a pure credit economy (as defined by Wicksell).

In order to be able to study the dynamic properties of our model, we have log-linearized the model around the steady-state values of its variables. The model was then calibrated using Postwar U.S. data.

We performed two types of experiments: impulse response experiments and stochastic simulation experiments.

The results obtained when only technological shocks are considered were as follows. In our monetary economy, the response of the real variables to technological shocks is exactly the same as would be obtained with a zero growth version of the RBC model without money presented in King, Plosser and Rebelo (1988). On the other hand, in a stochastic simulation experiment where only technological shocks were considered, we obtained a value for the correlation between the price level and real output which is very close to the correlation value reported by empirical researchers.

The response of the model to temporary money shocks (temporary increases in the rate of growth of the money supply) can be summarized as follows. Real variables are almost unaffected.
Inflation increases temporarily. The nominal interest rate responds positively to the increase in the rate of growth of the money supply. In other words, the expected inflation effect dominates the liquidity effect.

An important issue is the relation between our model and the literature on monetary economics.

The banking system is very simple. It consists of banks that have no costs and which are simply endowed with numbers that specify the maximum amount of credit that they can supply in each period. This amount of credit can be interpreted as being set by a legal constraint such as a required reserve ratio together with a fixed amount of reserves made available to the banking system (alternatively, we can think of the maximum amount of credit that each bank can supply as corresponding to “credit ceilings” set by the central bank). The interest earned by the banks is then distributed to the banks’ shareholders.

The structure of the firms’ optimization problem is completely standard: firms use “labour effort” and physical capital to produce physical output.

The best way to describe the formal structure of the consumer’s optimization problem is to say that it adds a specific initial condition to a cash-in-advance format. The initial condition we add is such that, in equilibrium, it makes the cash-in-advance structure become a credit-in-advance constraint in the first period (the first period being the period where our analysis of the economy is starting); since it creates a situation where the household starts the first period with zero net wealth, the initial condition makes the household’s need to obtain “cash” in order to buy consumption goods become synonymous with a need to borrow from a bank. With Rational Expectations the credit-in-advance constraint is propagated to all future periods.

The structure of the article is as follows. In section 2, we describe the economic environment: preferences, technology, resource constraints and market structure. In section 3, we describe the typical bank’s behaviour. In section 4, we describe the typical firm’s behaviour. In section 5, we describe the typical household’s behaviour. In section 6, we write down the market clearing conditions. In section 7, we write the set of equations that describes the competitive general market equilibrium. In section 8, we describe the calibration of the model. In section 9, we look at the response of the model to technological shocks and to monetary policy shocks. In section 10, we make an overview and conclusion.
2 The Economic Environment

This is a closed economy model with no government. There are H homogeneous households, F homogeneous firms and L homogeneous banks. There is only one (homogeneous) physical good produced in this economy which we denote physical output. There are two possible uses for this output: it can either be consumed or used for investment (i.e., used to increase the level of the stock of capital). We next examine the typical household’s preferences, the technology available in the economy (production function and capital accumulation equation), the resource constraints that exist in a given period and the market structure.

Let us suppose that we are at the beginning of period 0 and that households, firms and banks are considering decisions for periods \( t \) with \( t = 0, 1, 2, 3, \ldots \).

Let us start by describing the preferences of the typical household. The typical household maximizes the discounted sum of utilities from now till the end of time. Utility in period \( t \) is given by \( u(c_t, \ell_t) \) where \( c_t \) is the flow amount of consumption and \( \ell_t \) is the amount of leisure enjoyed in that period. The function \( u(\cdot, \cdot) \) has the usual properties. At the beginning of period 0, the household maximizes \( U_0 = E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t) \right] \) where \( \beta \) is a discount factor \((0 < \beta < 1)\) that reflects a preference for current over future consumption-leisure bundles. Application of the operator \( E_0[\cdot] \) yields the mathematical expectation, conditional on complete information pertaining to the beginning of period 0 and earlier, of the indicated argument.

Let us now describe the technology available in the economy: production function and capital accumulation equation. Each firm’s production function is described by \( y_t = A_t F(k_t, n_t^d) \) where \( y_t \) is the physical output of the firm, \( A_t \) is a technological parameter, \( k_t \) is the firm’s stock of capital and \( n_t^d \) is the firm’s labour demand in period \( t \). Capital accumulation is described by \( k_{t+1} = (1 - \delta)k_t + i_t \) where \( i_t \) is the flow of investment in period \( t \) and \( \delta \) is the per-period rate of depreciation of the stock of capital which is assumed to be constant and belonging to the closed interval \([0,1]\).

The resource constraints are as follows. Each firm starts period \( t \) with a stock of capital \( k_t \) which is pre-determined \([ \text{which was determined at the beginning of period} \ (t - 1) \] ). In other words, the stock of capital that will enter the production function in period \( t \) cannot be changed.
by decisions taken during period $t$. Each household has an endowment of time per-period which is normalized to be equal to one by an appropriate choice of units. This amount of time can be used to work or to rest. Therefore, we can write $n_t^s + \ell_t = 1$ where $n_t^s$ is the household’s supply of labour during period $t$.

There is a legal constraint which implies that the total amount of credit that each commercial bank can offer cannot exceed a certain amount. This may be seen as a restriction set by the existence of a required reserve ratio together with a fixed amount of reserves made available to each bank. Therefore, the maximum amount of credit that each bank can lend is denoted by $\frac{1}{r^{req}}RES_t$ where $RES_t$ is the total reserve endowment of each bank and $r^{req}$ is the required reserve ratio. In this model, reserves pay no interest.

Let us now describe the market structure. There are five markets: the goods market, the labour market, the bank loans’ market, the market for firms’ shares and the market for banks’ shares. We assume that each household behaves as a price-taker, each firm behaves as a price-taker and each bank also behaves as a price-taker. Prices are perfectly flexible and adjust so as to clear all markets in every period.

### 3 The Typical Bank’s Behaviour

The aim of this paper is to model the demand for bank credit by the households. Hence, the supply of credit is modelled in the simplest possible way. In this model, each commercial bank receives an endowment of reserves from the central bank at the beginning of period $t$ which is denoted $RES_t$. We assume that the commercial bank pays no interest on this amount of reserves. It is also assumed that commercial banks have no costs of operation. On the other hand, commercial banks operate under a fractional reserve system (with the required reserve ratio denoted $r^{req}$).

With the present assumptions (no costs of obtaining reserves and no costs of operation), as long as the lending rate is always strictly positive, banks will always want to supply the maximum amount of credit that they can. Therefore, the supply of credit by each bank in period $t$ is just the fixed amount $\frac{1}{r^{req}}RES_t$. To see this mathematically, we first note that, under the present assumptions, the profits of bank $l$ in period $t$ are given by $\Pi_{t}^{bank,l} = R_t B_t^l$ where $R_t$ is the nominal interest rate (lending rate) between the beginning of period $t$ and the beginning of period $(t + 1)$.
and $B_t^*$ is the nominal amount of credit supplied by the bank. The profits earned by each bank during period $t$ are distributed to households at the end of the period in the form of dividends.

Each bank maximizes the Value of its Assets (VA), i.e., the expected discounted value of its stream of present and future dividends. Therefore, when we are at the beginning of period 0, the typical bank’s optimization problem is

$$\max_{B_t^*} VA = E_0 \left[ \sum_{t=0}^{\infty} \frac{1}{1+R_{0,t+1}} R_t B_t^* \right]$$

s.t. $B_t^* \leq \frac{1}{\rho_{req}} RES_t$ for $t = 0, 1, 2, ...$

Given the economic environment we are working with, we think it is appropriate to assume that

$$(1 + R_{0,t+1}) = (1 + R_0)(1 + R_1)(1 + R_2)...(1 + R_t)$$

for $t = 0, 1, 2, ...$

Note that we are at the beginning of period 0. Therefore, because dividends are only distributed at the end of the period, we discount period 0 dividends by multiplying them by $1/(1 + R_0)$, we discount period 1 dividends by multiplying them by $1/[(1 + R_0)(1 + R_1)]$,...

As long as the nominal interest rate is always strictly positive, the solution to the bank’s optimization problem will obviously be

$$B_t^* = \frac{1}{\rho_{req}} RES_t$$ (1)

for $t = 0, 1, 2, ...$

The profits of bank $l$ in period $t$ can then be written as

$$\Pi_t^{bank,l} = \frac{1}{\rho_{req}} RES_t R_t$$ (2)

4 The Typical Firm’s Behaviour

As already mentioned, the focus of the article is on modelling the demand for bank credit by households. Hence, the behaviour of firms is modelled in a standard way. In nominal terms, the profits of firm $f$ in period $t$ are given by income from the sale of output minus the wage bill minus investment expenditure
where \( P_t \) is the nominal price of goods in period \( t \) and \( W_t \) is the nominal wage in period \( t \). The firm pays wages to households at the end of the period. The profits earned by each firm during period \( t \) are distributed to households at the end of the period in the form of dividends. Each firm \( f (f = 1, 2, ..., F) \) maximizes the Value of its Assets (VA), i.e., the expected discounted value of the stream of present and future dividends. Therefore, when we are at the beginning of period 0, the typical firm’s optimization problem is

\[
\max_{n_t^d, k_{t+1}} VA = E_0 \left[ \sum_{t=0}^{\infty} \frac{1}{1 + R_{0, t+1}} \Pi_t^f \right]
\]

where

\[
\Pi_t^f = P_t A_t F(k_t, n_t^d) - W_t n_t^d - P_t[k_{t+1} - (1 - \delta)k_t]
\]

for \( t = 0, 1, 2, ... \).

There is also an initial condition for the capital stock, the standard transversality condition for the capital stock and non-negativity constraints.

### 5 The Typical Household’s Behaviour

In this section we present the typical household’s problem written in a cash-in-advance form which was learned from Lucas (1982), i.e., using a portfolio allocation constraint and a cash-in-advance constraint.

The way loans work in this model is as follows. We have mentioned that \( R_t \) denotes the nominal interest rate between the beginning of period \( t \) and the beginning of period \( (t+1) \). At the beginning of period \( t \), the household borrows from banks the amount \( \frac{B_{t+1}}{1 + R_t} \). This means that the household receives \( \frac{B_{t+1}}{1 + R_t} \) monetary units at the beginning period \( t \) and that she will have to pay \( \frac{B_{t+1}}{1 + R_t} (1 + R_t) = B_{t+1} \) monetary units at the end of period \( t \) [beginning of period \( (t + 1) \)].

The way shares work in this model is as follows. \( Q_t^f \) is the nominal price that the household would have to pay to buy 100% of firm \( f \) at the beginning of period \( t \). \( z_t^f \) is the percentage of firm \( f \) [i.e. the share of firm \( f \)] that the household bought at the beginning of period \( (t-1) \) and sells at the beginning of period \( t \). \( z_{t+1}^f \) is the percentage of firm \( f \) that the household buys at the beginning...
of period \( t \). These percentages are measured as a number belonging to the closed interval \([0,1]\).

Therefore, \( (z_{t+1}^f - z_t^f) \) is the net percentage of firm \( f \) that the household buys at the beginning of period \( t \). \( Q_t^f (z_{t+1}^f - z_t^f) \) is the total (nominal) amount that the household spends on shares of firm \( f \) at the beginning of period \( t \). To make this clear, suppose that \( z_t^f = 0 \) and that the household wants to buy the whole of firm \( f \) at the beginning of period \( t \) so that \( z_{t+1}^f = 1 \); then clearly it would have to pay \( Q_t^f (1 - 0) = Q_t^f \) monetary units which fits our definition of \( Q_t^f \).

The shares of banks work in the same way: \( Q_{t}^{bank,l} \) is the nominal price that the household would have to pay to buy 100% of bank \( l \) at the beginning of period \( t \). \( \pi_{t}^{bank,l} \) is the percentage of bank \( l \) [i.e. the share of bank \( l \)] that the household bought at the beginning of period \( (t-1) \) and sells at the beginning of period \( t \). \( \pi_{t+1}^{bank,l} \) is the percentage of bank \( l \) that the household buys at the beginning of period \( t \).

Let \( CD_t \) denote the amount of checkable deposits that the household decides to hold at the beginning of period \( t \).

Let us now write the portfolio allocation constraint and the cash-in-advance constraint that capture the structure of this model’s consumer problem. At the beginning of period \( t \), the household faces the following portfolio allocation constraint

\[
(CD_{t-1} - P_{t-1}c_{t-1}) + W_{t-1}n_{t-1} + \sum_{f=1}^{F} z_t^f \Pi_{t-1}^f + \sum_{f=1}^{F} Q_t^f z_t^f + \sum_{l=1}^{L} z_t^{bank,l} \Pi_{t-1}^{bank,l} + \sum_{l=1}^{L} Q_t^{bank,l} z_t^{bank,l} - B_t + \frac{B_{t+1}}{1 + R_t} =
\]

\[
= \sum_{f=1}^{F} Q_t^f z_{t+1}^f + \sum_{l=1}^{L} Q_t^{bank,l} z_{t+1}^{bank,l} + CD_t \quad (4)
\]

The first term on the left-hand side of the equation denotes checkable deposits not spent during the previous period. The second term denotes the household’s wage earnings received at the end of period \( (t-1) \) [beginning of period \( t \)] in return for work effort supplied during period \( (t-1) \). These wage earnings are received in the form of checkable deposits (transferred from the firms’ accounts into the household’s account). The third term denotes the amount of dividends received from the \( F \) firms at the end of period \( (t-1) \) corresponding to shares of firms bought by the household.
at the beginning of period (t-1). These dividends are received in the form of checkable deposits (transferred from the firms’ accounts into the household’s account). The fourth term corresponds to the amount that the household receives at the beginning of period t from selling the shares of firms she had bought at the beginning of period (t-1). This amount is received in the form of checkable deposits. The fifth term denotes the amount of dividends received from the L banks at the end of period (t-1) corresponding to shares of banks bought by the household at the beginning of period (t-1). These dividends are received in the form of checkable deposits. The sixth term corresponds to the amount that the household receives at the beginning of period t from selling the shares of banks she had bought at the beginning of period (t-1). This amount is received in the form of checkable deposits. The seventh term subtracts the amount that the household uses to pay the debt contracted from commercial banks at the beginning of period (t-1). This payment is made by destroying part of the checkable deposits that the household owns in its current account. The eighth term adds the amount received from the new loan that the household obtains from the banks at the beginning of period t. This amount is received in the form of new checkable deposits created by the banks.

In short, the left-hand side of the equation gives the total amount of checkable deposits that the household owns at the beginning of period t. At the beginning of period t, the household uses the whole of this amount in the following way: she buys shares of firms and of banks and she keeps the rest as checkable deposits (the terms on the right-hand side of the equation).

In deciding the amount that she will keep as checkable deposits, $CD_t$, the household must be aware that to buy consumption goods during period t, it can only use checkable deposits (the amount spent buying shares cannot be used to buy consumption goods during the period). This is what the cash-in-advance constraint (which follows) states in a very clear way. This cash-in-advance constraint is

$$P_t c_t = CD_t$$

Note that the cash-in-advance constraint is not written as an inequality because checkable deposits are dominated in terms of return by other assets (shares, in this case). This being so, it wouldn’t be optimal for the household to hold an amount of checkable deposits greater than the
amount she needs to buy consumption goods during the period.

Let us now show that the portfolio allocation constraint (equation 4) and the cash-in-advance constraint (equation 5) together imply a budget constraint similar to the budget constraints we can find in RBC models. Since the cash-in-advance constraint (equation 5) is always binding, it must have been binding in period \( (t-1) \): Therefore, we have \( P_{t-1}c_{t-1} = CD_{t-1} \). Using this equality and equation 5 in the portfolio allocation constraint (equation 4), and then rearranging the equation, we obtain

\[
W_{t-1}n_{t-1} + \sum_{f=1}^{F} x_f^t \Pi_{t-1}^f + \sum_{l=1}^{L} z_l^{ban k,l} \Pi_{t-1}^{ban k,l} + \frac{B_{t+1}}{1 + R_t} =
\]

\[
= B_t + P_t c_t + \sum_{f=1}^{F} Q_t^f (z_{t+1}^f - z_t^f) + \sum_{l=1}^{L} Q_t^{ban k,l} (z_{t+1}^{ban k,l} - z_t^{ban k,l})
\]

(6)

This equation is the consumer’s budget constraint. This equation simply states that the total amount of money the household obtains at the beginning of period t (wage earnings, dividend earnings from firms, dividend earnings from banks and the amount it borrows from the banks at the beginning of period t) must be equal to what the household spends at the beginning or during period t [payment of the debt contracted from banks at the beginning of period (t-1), consumption expenditure, net purchase of shares of firms and net purchase of shares of banks].

We can summarize by saying that the portfolio allocation constraint and the cash-in-advance constraint together imply the consumer’s budget constraint.

Let us now normalize the household’s budget constraint (the purpose of the normalization is to write the model in terms of variables that are constant in the steady-state). We can do that by dividing both sides of the constraint by \( RES_{t-1} \). After doing this, we rearrange the equation and then define the following new variables:

\[
\mu_t = \frac{RES_t}{RES_{t-1}} - 1, \quad p_t = \frac{P_t}{RES_{t-1}}, \quad w_t = \frac{W_t}{RES_{t-1}}, \quad \bar{q}_t = \frac{Q_t}{RES_{t-1}};
\]

\[
q_t^{ban k,l} = \frac{Q_t^{ban k,l}}{RES_{t-1}}, \quad b_{t+1} = \frac{B_{t+1}}{RES_{t-1}}, \quad \pi_t^f = \frac{\Pi_t^f}{RES_{t-1}}, \quad \pi_t^{ban k,l} = \frac{\Pi_t^{ban k,l}}{RES_{t-1}}
\]

After all these steps, we arrive at

\[
\frac{w_{t-1}}{1 + \mu_{t-1}} n_{t-1} + \sum_{f=1}^{F} x_f^t \frac{\pi_t^{f-1}}{1 + \mu_{t-1}} + \sum_{l=1}^{L} z_l^{ban k,l} \frac{\pi_t^{ban k,l}}{1 + \mu_{t-1}} + \frac{b_{t+1}}{1 + R_t} =
\]
We suggest using the following initial condition concerning the household’s debt position at the beginning of period 0

\[ B_0 = W_{-1} n_{-1}^s + \sum_{f=1}^{F} z_f^0 \sum_{l=1}^{L} z_{bank;l}^0 \]

This initial condition simply states that the household starts period 0 (period 0 being the period where our analysis of the economy is starting) with a debt which equals the sum of the wage earnings, dividend earnings from firms and dividend earnings from banks that she receives there (at the beginning of period 0) because of the hours she worked during period (−1) and because of the shares of firms and of banks it bought at the beginning of period (−1). Note that period 0 is not the period where the household’s life starts but rather the period where our analysis of the economy begins (the household has been living for some periods and we catch her in period 0). In the Appendix, we show that this is the initial condition which naturally arises when we think a closed economy without government and where firms don’t borrow back to its initial moment. As will be shown in section 7, this initial condition implies that the household will borrow from the banks at the beginning of each period an amount equal to the amount she needs to buy consumption goods from the firms. This initial condition can also be normalized by dividing both sides of the equation by \( RES_{-1} \) giving

\[ \frac{b_0}{1 + \mu_{-1}} = \frac{w_{-1}}{1 + \mu_{-1}} n_{-1}^s + \sum_{f=1}^{F} z_f^0 \sum_{l=1}^{L} z_{bank;l}^0 \]

Consequently, at the beginning of period 0 the household is looking into the future and acts in a way that can be described as follows

Max

\[ c_t, \ell_t, n_{t+1}^s, b_{t+1}, z_{f,t+1}, z_{bank;l,t+1} \]

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t) \right] \]

s.t.

\[ \frac{w_{t-1}}{1 + \mu_{t-1}} n_{t-1}^s + \sum_{f=1}^{F} z_f^{t-1} \sum_{l=1}^{L} z_{bank;l}^{t-1} \frac{1}{1 + \mu_{t-1}} + \frac{b_{t+1}}{1 + R_t} = \]
\[ b_t = \frac{b_0}{1 + \mu_{t-1}} + pc_t + \sum_{f=1}^{F} q_{t}^f (z_{t+1}^f - z_{t}^f) + \sum_{l=1}^{L} q_{t}^{bank,l} (z_{t+1}^{bank,l} - z_{t}^{bank,l}) \]

\[ n_t^s + \ell_t = 1 \]

for \( t = 0,1,2,3,... \)

\[ \frac{b_0}{1 + \mu_{-1}} = \frac{w_{-1}}{1 + \mu_{-1}} n_{-1}^s + \sum_{f=1}^{F} z_0^f \frac{n_{-1}^s}{1 + \mu_{-1}} + \sum_{l=1}^{L} z_0^{bank,l} \frac{n_{-1}^{bank,l}}{1 + \mu_{-1}} \]

There are also initial conditions on holdings of shares [assuming market clearing in the shares market in period \((-1)\), these initial conditions will be \( z_0^f = \frac{1}{H} \) and \( z_0^{bank,l} = \frac{1}{H} \)], a standard transversality condition on the pattern of borrowing and non-negativity constraints.

We can summarize by saying that the way we have set the household’s problem means that we are adding a specific initial condition to a cash-in-advance format.

### 6 The Market Clearing Conditions

With \( H \) homogeneous households, \( F \) homogeneous firms and \( L \) homogeneous banks, the market clearing conditions for period 0 are as follows. In the goods market, the condition is

\[ Hc_0 + Fi_0 = Fy_0 \iff c_0 + \frac{F}{H} [k_1 - (1 - \delta)k_0] = \frac{F}{H} A_0 F(k_0, n_0^d) \quad (8) \]

In the labour market, the condition is

\[ Hn_0^s = Fn_0^d \iff n_0^s = \frac{F}{H} n_0^d \quad (9) \]

In the bank loans market, the condition is

\[ H \frac{B_1}{1 + R_0} = L \star B_0^s \iff \]

\[ \iff H \frac{(B_1/RES_{-1})}{1 + R_0} = L \star \frac{B_0^s}{RES_{-1}} \]
Defining a new variable \( b^*_t = \frac{B^*_t}{RES_{t-1}} \) this last equation can be rewritten as

\[
\frac{b_1}{1 + R_0} = \frac{L}{H} * b^*_0
\]

(11)

The market clearing condition in the shares market is that each firm and each bank should be completely held by the households (the only owners of shares in this model). Since households are all alike, each holds an equal share of each firm and an equal share of each bank. Therefore, the market clearing conditions in the shares’ market are

\[
z^f_1 = \frac{1}{H}
\]

(12)

and

\[
z^{bank, l}_1 = \frac{1}{H}
\]

(13)

7 The Competitive General Market Equilibrium assuming H homogeneous households, F homogeneous firms and L homogeneous banks plus Rational Expectations

To obtain the system that describes the competitive general market equilibrium, we put together in a system the typical household’s first order conditions (that give, in an implicit way, the household’s demand and supply functions), the typical firm’s first order conditions (that give, in an implicit way, the firm’s demand and supply functions), the typical bank’s supply of credit equation and the market clearing conditions.

The next step in solving this model is to normalize the typical bank’s supply of credit equation (equation 1) by dividing both sides of the equation by \( RES_{t-1} \). Then, we normalize the typical firm’s first-order conditions by dividing both sides of the equations by \( RES_{t-1} \). We then assume
Rational Expectations and use a Certainty Equivalence argument. After all these steps and if we also assume that the production function is homogeneous of degree one and define the following new variables

\[
\begin{align*}
\bar{k}_t &= \frac{F}{H} k_t, \quad \bar{n}_t^f = \frac{F}{H} n_t^f, \quad \bar{\pi}_t^f = \frac{F}{H} \pi_t^f, \quad \bar{q}_t^f = \frac{F}{H} q_t^f, \\
\bar{\pi}_{t}^{bank,l} &= \frac{F}{H} \pi_{t}^{bank,l}, \quad \bar{q}_{t}^{bank,l} = \frac{F}{H} q_{t}^{bank,l}
\end{align*}
\]

we can write the system describing the Competitive General Market Equilibrium assuming \( H \) homogeneous households, \( F \) homogeneous firms and \( L \) homogeneous banks plus Rational Expectations as

\[
\begin{align*}
u_1(c_t, 1 - n_t^s) &= \lambda_t p_t \quad (14) \\
u_2(c_t, 1 - n_t^s) &= \beta \frac{w_t}{1 + \mu_t} E_t[\lambda_{t+1}] \quad (15) \\
\lambda_t &= \beta E_t[\lambda_{t+1}] \left( \frac{1 + R_t}{1 + \mu_t} \right) \quad (16) \\
\lambda_t \bar{\pi}_t^f &= \beta E_t[\lambda_{t+1}] \left( \frac{\pi_t^f}{1 + \mu_t} + E_t[\bar{q}_{t+1}^{f}] \right) \quad (17) \\
\lambda_t \bar{\pi}_{t}^{bank,l} &= \beta E_t[\lambda_{t+1}] \left( \frac{\pi_{t}^{bank,l}}{1 + \mu_t} + E_t[\bar{q}_{t+1}^{bank,l}] \right) \quad (18)
\end{align*}
\]
\[
\frac{b_{t+1}}{1 + R_t} = p_t c_t
\]  
\hspace{1cm} (19) \\

\[
p_t A_t F_2(\overline{k}_t, \overline{n}^d_t) = w_t
\]  
\hspace{1cm} (20) \\

\[
E_t [A_{t+1}] F_1(\overline{k}_{t+1}, E_t [\overline{n}^d_{t+1}]) + (1 - \delta) = (1 + E_t [R_{t+1}]) \frac{p_t}{E_t [p_{t+1}]} \frac{1}{(1 + \mu_t)}
\]  
\hspace{1cm} (21) \\

\[
b^*_t = \frac{1}{\nu_{eq}} (1 + \mu_t)
\]  
\hspace{1cm} (22) \\

\[
c_t + [\overline{k}_{t+1} - (1 - \delta) \overline{k}_t] = A_t F(\overline{k}_t, \overline{n}^d_t)
\]  
\hspace{1cm} (23) \\

\[
n^*_t = \overline{n}^d_t
\]  
\hspace{1cm} (24) \\

\[
\frac{b_{t+1}}{1 + R_t} = \frac{F}{H} \phi^*_t
\]  
\hspace{1cm} (25) \\

\[
z^I_{t+1} = \frac{1}{H}
\]  
\hspace{1cm} (26) \\

\[
\frac{\phi_{bank}^I}{z_{t+1}} = \frac{1}{H}
\]  
\hspace{1cm} (27)
\[
\pi_t^f = p_t A_t F(k_t, \pi_t^d) - w_t \pi_t^d - p_t \left[ k_{t+1} - (1 - \delta)k_t \right]
\]

(28)

\[
\pi_t^{ban,k,f} = \frac{L_t}{H_t} \frac{1}{\nu^e_q} (1 + \mu_t) R_t
\]

(29)

for \( t = 0, 1, 2, 3, \ldots \)

Equations 14-19 have their origin in the typical household’s first order conditions.

Equation 19 is the credit-in-advance constraint which results from combining the household’s budget constraint with the initial condition and then using the market clearing conditions from the shares’ market. Let us be more precise about this. We start by showing that if we add our specific initial condition to the household’s period 0 budget constraint, in equilibrium we obtain a credit-in-advance condition for period 0. In order to prove this, we start by writing the household’s budget constraint (equation 6) for period \( t = 0 \)

\[
W_{-1} n_{z1} + \sum_{f=1}^{F} z_0^f \Pi_{-1}^f + \sum_{l=1}^{L} z_0^{ban,k,l} \Pi_{-1}^{ban,k,l} + \frac{B_1}{1 + R_0} =
\]

\[
= B_0 + P_0 c_0 + \sum_{f=1}^{F} Q_0^f (z_1^f - z_0^f) + \sum_{l=1}^{L} Q_0^{ban,k,l} (z_{1}^{ban,k,l} - z_{0}^{ban,k,l})
\]

Using the initial condition before normalization (equation 7) in this last equation yields

\[
\frac{B_1}{1 + R_0} = P_0 c_0 + \sum_{f=1}^{F} Q_0^f (z_1^f - z_0^f) + \sum_{l=1}^{L} Q_0^{ban,k,l} (z_{1}^{ban,k,l} - z_{0}^{ban,k,l})
\]

Using the market clearing conditions in the firms’ shares market and in the banks’ shares market in periods \((-1)\) and \(0\) (which are \( z_0^f = \frac{1}{H}, z_1^f = \frac{1}{H}, z_0^{ban,k,l} = \frac{1}{H} \) and \( z_1^{ban,k,l} = \frac{1}{H} \)) this last equation becomes

\[
\frac{B_1}{1 + R_0} = P_0 c_0
\]

(30)
which means that, in equilibrium, the household will have to borrow at the beginning of period 0 an amount equal to the amount it wants to spend buying consumption goods during period 0 (i.e., in equilibrium, there is a credit-in-advance constraint for period 0). Dividing both sides of this equation by \( RES_{-1} \), we obtain equation 19 for period \( t = 0 \). If we assume Rational Expectations, this credit-in-advance constraint is propagated to all future periods (periods 1, 2, 3,...). Let us show why this happens.

By assuming Rational Expectations we introduce all future budget constraints of the household and all future market clearing conditions into the structure of the mathematical representation of this economy. Hence, we can reason starting from period 0 and going successively into every future period in the following way. We first write the following tautology

\[
B_1 = \frac{B_1}{1 + R_0} (1 + R_0)
\]

This is equivalent to writing

\[
B_1 = \frac{B_1}{1 + R_0} + \frac{B_1}{1 + R_0} R_0
\]

Using the credit-in-advance constraint for period 0 which we have just derived (equation 30), this last equation can be written as

\[
B_1 = P_0 c_0 + \frac{B_1}{1 + R_0} R_0
\]

Using the market clearing condition in the goods market in period 0 (equation 8), we obtain

\[
B_1 = \frac{F}{H} \left[ P_0 A_0 F(k_0, n^d_0) - P_0 [k_1 - (1 - \delta) k_0] \right] + \frac{B_1}{1 + R_0} R_0
\]

Using the definition of profits of firm \( f \) (equation 3) in period 0, this becomes

\[
B_1 = \frac{F}{H} \left[ \Pi^f_0 + W_0 n^d_0 \right] + \frac{B_1}{1 + R_0} R_0 \Leftrightarrow
\]

\[
\Leftrightarrow B_1 = W_0 \frac{F}{H} n^d_0 + \frac{F}{H} \Pi^f_0 + \frac{B_1}{1 + R_0} R_0
\]

With the market clearing condition in the labour market (equation 9), we obtain

\[
B_1 = W_0 n^*_0 + \frac{F}{H} \Pi^f_0 + \frac{B_1}{1 + R_0} R_0
\]
Using the market clearing condition from the bank loans market (equation 10), we obtain

\[ B_1 = W_0 n_0^* + \frac{F}{H} \Pi_0^f + \frac{L}{H} B_0^s R_0 \]

Using equation 1, this last equation becomes

\[ B_1 = W_0 n_0^* + \frac{F}{H} \Pi_0^f + \frac{L}{H} \Pi_0^{bank,l} \]

Using the definition of profits of bank l in period 0 (equation 2), we obtain

\[ B_1 = W_0 n_0^* + \frac{F}{H} \Pi_0^f + \frac{L}{H} \Pi_0^{bank,l} \]

Finally, using the market clearing conditions in the shares market, this can be written as

\[ B_1 = W_0 n_0^* + \sum_{f=1}^{f=F} z_1^f \Pi_0^f + \sum_{l=1}^{l=L} z_1^{bank,f} \Pi_0^{bank,l} \]  \hspace{1cm} (31)

Note that this equation is identical in form to the initial condition we propose using for period 0 but written one period ahead. In other words, it is an initial condition for period \( t = 1 \). This is an interesting property of the initial condition we propose using: if we assume it holds at the beginning of period 0, then the structure of the model will reproduce it automatically into the following periods. Combining equation 31 with the household’s budget constraint for period \( t = 1 \), we obtain

\[ \frac{B_2}{1 + R_1} = P_1 c_1 \]

which is a credit-in-advance constraint identical in form to the credit-in-advance constraint for period 0 which we have already obtained (equation 30) but written one period ahead. In other words, it is a credit-in-advance constraint for period 1. Dividing both sides of this equation by \( RES_0 \), we obtain equation 19 for period \( t = 1 \).

If we repeat the whole reasoning we will also obtain a credit-in-advance constraint for period \( t = 2 \). And if we successively repeat the whole reasoning, we will obtain a credit-in-advance constraint for all future periods.

The intuition behind this propagation of the credit-in-advance constraint is as follows. The initial condition for the beginning of period 0 forces the household to borrow from the banks at the
beginning of period 0 the amount she wants to spend buying consumption goods from the firms during period 0. In equilibrium, consumption is equal to output per household minus investment per household (this follows from the market clearing condition in the goods market). Using the definition of firm’s profits and the market clearing conditions in the labour market and in the firm shares market, it is easy to show that output per household minus investment per household is equal to the sum of wage earnings and firm dividend earnings of each household. Hence, what the household wants to borrow at the beginning of period 0, is an amount which is equal to the sum of her wage earnings and firm dividend earnings. Therefore, at the end of the period 0 (beginning of period 1) the household’s debt will be this amount plus interest on it. However, since the interest paid to banks is equal to the dividends the banks will pay the household, the total debt of the household at the end of the period 0 (beginning of period 1) can be expressed as the sum of wage earnings and firm dividend earnings and bank dividend earnings of each household. But this household’s debt at the beginning of period 1 is just what the initial condition had forced the household’s debt to be at the beginning of period 0.

Equations 20 and 21 have their origin in the typical firm’s first order conditions. Equation 22 has its origin in the typical bank’s supply of credit equation. Equations 23-27 are the market clearing conditions. Equation 28 results from multiplying the typical firm’s normalized profit function by $(F/H)$. Equation 29 results from multiplying the typical bank’s normalized profit function by $(L/H)$.

We have 2 exogenous variables ($A_t$ and $\mu_t$) and 16 endogenous variables. The specific utility and production functions used were $u(c_t, \ell_t) = \ln c_t + \phi \ln \ell_t$ and $A_t F(k_t, n^d_t) = A_t (k_t)^{1-\alpha} \left( n^d_t \right)^\alpha$.

8 Calibration

In order to study the dynamic properties of the model, we first log-linearize each of the equations in the system 14-29 around the steady-state values of the variables. The log-linearized system was then calibrated. To calibrate the log-linearized system we used the following parameters. With the specific utility function we are using we obtain
Elasticity of the MU of consumption with respect to consumption -1
Elasticity of the MU of consumption with respect to leisure 0
Elasticity of the MU of leisure with respect to consumption 0
Elasticity of the MU of leisure with respect to leisure -1

where MU denotes “Marginal Utility”. From the U.S. data, we obtain

<table>
<thead>
<tr>
<th></th>
<th>value</th>
<th>source</th>
</tr>
</thead>
<tbody>
<tr>
<td>investment share of output in</td>
<td>0.167</td>
<td>Barro (1993)</td>
</tr>
<tr>
<td>the s.s. (i/y)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>labour’s share of output (α)</td>
<td>0.58</td>
<td>King et al. (1988)</td>
</tr>
<tr>
<td>labour supply in the steady-state (n*)</td>
<td>0.2</td>
<td>King et al. (1988)</td>
</tr>
<tr>
<td>Nominal interest rate in the</td>
<td>0.016383</td>
<td>FRED</td>
</tr>
<tr>
<td>steady-state (R)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate of growth of the money</td>
<td>0.009258</td>
<td>Barro (1993)</td>
</tr>
<tr>
<td>supply in the s.s. (μ)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As usual we take the Postwar average as representing the steady-state value. The last two values in the table are per-quarter values. The nominal interest rate used was the Bank Prime Loan Rate. We have used data from the Federal Reserve Economic Data (FRED) to computed the average quarterly value of this rate for the period 1949-1986.

The value used to calibrate the steady-state rate of growth of the money supply (0.009258) was the average quarterly inflation rate for the period 1949-1986. The argument for using this value is as follows. Combining equations 19, 22 and 25 above, which corresponds to equating the supply and the demand for money, we obtain

$$\frac{L}{H r^{req}} (1 + μ_t) = p_t c_t \Leftrightarrow \frac{L}{H r^{req}} R E S_t = \frac{P_t}{R E S_{t-1}} c_t \Leftrightarrow$$

$$\Leftrightarrow L \cdot \frac{1}{r^{req}} * R E S_t = P_t \cdot H * c_t$$

(32)

Since $c_t$ is constant in the steady-state, this equation implies that in the steady-state the rate of growth of the money supply [the rate of growth of ($L \cdot \frac{1}{r^{req}} \cdot R E S_t$)] is equal to the rate of growth of $P_t$. In other words, since velocity is constant in the model (it is equal to one and this is because each unit of money is only used once during the period) and consumption is constant in the steady-state, we have a steady-state where the rate of growth of the money supply is equal
to the rate of growth of the price level. We can therefore use the average inflation rate from the data to calibrate the steady-state rate of growth of the money supply in our model. The advantage of calibrating the steady-state rate of growth of the money supply in this way can be stated as follows. When we set a value for $\mu$ in this model, we are also setting a value for the model’s steady-state inflation rate (as we have just seen). Hence, if we want to calibrate the steady-state nominal interest rate in this model using the average value of the nominal interest rate obtained from the U.S. data, the only way to obtain a value for the steady-state real interest rate in the model equal to the average value it takes in the U.S. data is to force the steady-state inflation rate in the model to be equal to the average value it takes in the data. This requires making the steady-state rate of growth of the money supply in the model ($\mu$) equal the average inflation rate obtained from the data.

The values in the preceding table imply

<table>
<thead>
<tr>
<th>Consumption share of output ($c/y$)</th>
<th>0.833</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per-quarter rate of depreciation of the capital stock ($\delta$)</td>
<td>0.0047</td>
</tr>
<tr>
<td>Household’s discount factor ($\beta$)</td>
<td>0.993</td>
</tr>
</tbody>
</table>

With this calibration, the model in this chapter has exactly the same steady-state values of physical output, consumption, investment, labour effort, real wage and real interest rate as would be obtained in a zero growth version of the RBC model presented in King, Plosser and Rebelo (1988) calibrated with our parameters. This means that we have built an economy with money which has exactly the same steady-state values of the key real variables that we would obtain in a benchmark economy without money.

9 The Dynamic Properties of the Model

The response of the log-linearized model to shocks in the exogenous variables ($A_t$ and $\mu_t$) can be obtained using the King, Plosser and Rebelo (1988) method. We first look at the impact of shocks in the firms’ technological parameter. Afterwards, we look at the impact of shocks in the rate of growth of the money supply.
9.1 The impact of technological shocks

We next examine the results of two experiments which use shocks in the firms’ technological parameter: impulse response experiment and stochastic simulation. In order to perform these two experiments, we assumed that the technological parameter evolves according to

\[ \dot{A}_t = 0.9 \dot{A}_{t-1} + \varepsilon_t \]  

(33)

where \( \varepsilon_t \) is a white noise.

9.1.1 Impulse Response

The first experiment we carried out was a standard impulse response experiment: a 1\% shock in the typical firm’s technological parameter. The results are plotted in figures 1 to 9. The first important thing to notice is that, in spite of the fact that ours is a monetary economy, it is capable of reproducing the key results that the King, Plosser and Rebelo (1988) paper is able to reproduce. First: consumption, investment and “hours of work” are procyclical. Second: consumption is less volatile than output and investment is more volatile than output. These are very well documented stylized facts about the United States economy [references on this include Kydland and Prescott (1990) and Backus and Kehoe (1992)].

We have checked the actual numbers and concluded that the impulse response results for the real variables \( \dot{y}_t, \dot{c}_t, \dot{h}_t, \dot{n}^*_t \), the real wage and the capital stock are exactly the same as would be obtained in a zero growth version of the model presented in King, Plosser and Rebelo (1988) calibrated with our parameters. We emphasize that this is only true with the log-linear utility function and in the absence of monetary shocks.

As already mentioned, the model in this chapter has exactly the same steady-state values of physical output, consumption, investment, labour effort, real wage and real interest rate as the zero growth version of the model presented in King, Plosser and Rebelo (1988). This means that in this paper we have a model with money which not only has the same steady-state values of...
the key real variables as the zero growth version of the model without money presented in King, Plosser and Rebelo (1988) but also reacts in exactly the same way to an exogenous technological shock.

The results in figures 1-9 were obtained with $\mu = 0.009258$. It is important to point out that changes in the steady-state rate of growth of the money supply used in the calibration (changes in $\mu$) do not affect the impulse response results obtained with technological shocks as long as we adjust the steady-state nominal interest rate so as to make the steady-state real interest rate remain the same in our calibration (note that changing $\mu$ implies changing the steady-state rate of inflation). In other words, when money is supplied to the economy at a constant rate, the actual rate at which it is supplied has no influence on the results we obtain with technological shocks.

Let us try to explain why the results obtained so far (results obtained with a constant rate of growth of the money supply) seem to be the same as the results we would obtain with a zero growth version of the non-monetary economy presented in King, Plosser and Rebelo (1988) calibrated with our parameters. We can summarize our description of the monetary flows among economic agents, done in the introduction to this article, as follows. At the beginning of each period, commercial banks tell the nonbank economic agents: “Take this money, use it for your transactions and, at the end of the period, give it back to us”. We can say this because, although households actually have to pay interest on the amount that they borrow from the banks, they then receive the interest paid back in the form of bank dividends (this happens because banks are owned by the households and households are all alike). This means that the fact that households have to borrow to finance their expenditure does not involve an income effect. It seems that, with the specific utility function we are using and in the absence of monetary shocks, introducing money in the model in the way we did does not involve a substitution effect either. With the specific utility function we are using (log-linear utility function) and in the absence of monetary shocks, the ex-ante real interest rate is given by the same expression in the model of this paper and in the zero growth version of the model presented in King, Plosser and Rebelo. Also important to explain the fact that we obtain the same results with this model as we obtain with the zero growth version of the model presented in King, Plosser and Rebelo is the fact that in this model commercial banks provide money to the economy without spending resources (banks have no costs of operation).
Figure 5 shows that the normalized price of physical output, \( p_t = \frac{P_t}{RES_t} \), reacts negatively to the positive technological shock. Since physical output reacts positively to the positive technological shock, we can conclude that if we only consider technological shocks then the price of physical output will display countercyclical behaviour in our model.

Working with the log-linearized version of system 14-29, and using our specific utility function (log-linear utility function), it is possible to show that the nominal interest rate percentage deviations from its steady-state are given by

\[
\hat{R}_t = \frac{\mu (1 + R)}{(1 + \mu) R} E_t [\hat{p}_{t+1}]
\]

where \( \hat{R}_t \) and \( \hat{\mu}_t \) denote percentage deviations from the steady-state values and \( R \) and \( \mu \) denote the steady-state values. This equation simply means that \( R_t \) will only deviate from its steady-state if the level of reserves is expected to grow more (or less) than it grows at the steady-state (i.e., \( R_t \) will only deviate from its steady-state if there are monetary shocks). Since we are only considering a technological shock, the nominal interest rate does not deviate from its steady-state (figure 6).

9.1.2 Stochastic Simulation

The second experiment we carried out was a standard stochastic simulation exercise where only technological shocks were considered. The results are shown in table 1.

In our model, the correlation between “detrended prices” and “detrended output” is represented by the correlation between the “deviations of the normalized price of physical output from its steady-state” and the “deviations of physical output from its steady-state”. As can be seen in table 1B, the value we obtained for this correlation in the stochastic simulation experiment was \(-0.59\).

There are a number of relatively recent empirical studies concerning the relationship between prices and output. Since we have calibrated our model with measures computed using U.S. Postwar data, we shall concentrate only on empirical studies about the U.S. and only in that period.

Cooley and Hansen (1989) use quarterly data for the U.S. (the period considered is 1955:3-1984:1). All time series were seasonally adjusted, logged and detrended. The correlation of the “implicit GNP Deflator” with real GNP that they obtain is \(-0.53\). The correlation of the “Consumer Price Index” with real GNP that they obtain is \(-0.48\). Using deviations from trend and
U.S. quarterly data for the period 1954-1989, Kydland and Prescott (1990) report a correlation coefficient of the cyclical deviations of the “implicit GNP Deflator” with the cyclical deviations of real GNP equal to \(-0.55\). They also report a correlation coefficient of the cyclical deviations of the “Consumer Price Index” with the cyclical deviations of real GNP equal to \(-0.57\). In the study by Cooley and Ohanian (1991), the simple contemporaneous correlation between detrended output and detrended prices is \(-0.67\) for the period 1948:2-1987:2; it is \(-0.69\) for the period 1954:1-1973:1 (a sample period beginning immediately after the Korean War and terminating before the first oil shock); and it is \(-0.87\) for the period 1966:1-1987:2 (a period of high average inflation rate). Using Hodrick-Prescott filtered data the results are \(-0.57\), \(-0.36\) and \(-0.68\), respectively. Other studies that quote similar results include Backus and Kehoe (1992), Smith (1992), Chada and Prasad (1994) and King and Watson (1996).

These studies seem to indicate that the price level is countercyclical. This corresponds to the results from our impulse response experiment and stochastic simulation experiment. It is easy to understand why our model produces these simulation results. Combining equations 19, 22 and 25 above, which corresponds to equating the supply and the demand for money, we obtain

\[ p_t c_t = \frac{L}{H} \frac{1}{\rho \log (1 + \mu_t)} \]

Since we are not considering monetary shocks (\(\mu_t = \mu\)), when \(c_t\) moves upward (as a consequence of the positive technological shock), \(p_t\) must move in the opposite direction.

### 9.2 Monetary Policy

We next examine the effect of a temporary increase in the rate of growth of the money supply on the behaviour of both nominal and real variables (in the model). In order to perform this experiment, we assumed that money shocks follow the following process

\[ \hat{\mu}_t = 0.481 \times \hat{\mu}_{t-1} + \frac{\hat{\xi}_t}{0.01538} \]  \hspace{1cm} (35)
9.2.1 Impulse Response

Let us first look at the response of the model to a 1% shock in the rate of growth of the money supply. The main results are plotted in figures 10 to 18. The results in figures 10 to 13 show that a shock in the rate of growth of the money supply has an effect on the behaviour of real variables but that this effect is very small. We can therefore conclude that, in our model, shocks to the rate of growth of the money supply have a “small but significant” impact on the behaviour of real variables.

In spite of the fact that the impact is very small, the fact remains that the model does not give a purely nominal answer to a purely nominal shock. The reason why the increase in the rate of growth of the money supply triggers some (small) real effects is as follows.

Because (nominal) wages are only paid at the end of the period, the increase in the rate of growth of the money supply decreases the expected marginal benefit of devoting time to work (i.e., the right hand side of equation 15 increases). This makes sense because more inflation reduces the real value of the wage received at the end of the period. Hence, households reduce their supply of “hours of work” (figure 13).

This reduction in the supply of labour puts upward pressure on the real wage (figure 16).

Since \( \overline{k_t} \) is pre-determined, we can see from equation 20 that the increase in the real wage will lead firms to reduce their labour demand. Because \( \overline{k_t} \) is pre-determined, this decrease in the firms’ labour demand implies an decrease in output (figure 10).

Let us now look at the response of the nominal interest rate to the temporary increase in the rate of growth of the money supply. Figure 15 shows a positive response of the nominal interest rate to the positive shock in the rate of growth of the money supply. As already mentioned (equation 34 above), it is possible to show that in this model the nominal interest rate percentage deviations from its steady-state are given by

\[
\hat{R}_t = \frac{\mu(1 + R)}{(1 + \mu)R} E_t [\hat{\mu}_{t+1}]
\]

In other words, if the level of reserves is expected to grow more in a given period than it grows
on average (at the “steady-state”), then the nominal interest rate for that period will be above its average value. This is contrary to the beliefs of most central bankers. They believe that more liquidity makes short-term nominal interest rates go down (the so-called “liquidity effect”). In fact, this liquidity effect is often viewed as a channel through which monetary policy might affect real variables: more liquidity would make interest rates fall and this would stimulate consumption, investment and real output.

If the price level is constant and assuming that no real shocks occur, then it is likely that an increase in liquidity will make interest rates fall (because banks have more to lend and they have to induce households and firms to borrow more). However, if the price level is fully flexible and expectations are rational (as in our model), an increase in the rate of growth of the money supply will cause an increase in the expected rate of inflation. The increase in the expected rate of inflation pushes nominal interest rates up. This “expected inflation effect” is seen by central bankers as an effect that will only appear in the long-run. Hence their assumption that, in the short-run, an increase in liquidity will make nominal interest rates fall.

There are a number of empirical studies that have tried to evaluate the impact of an increase in liquidity on the behaviour of nominal interest rates. While the evidence is mixed, recent studies suggest that the liquidity effect dominates the expected inflation effect [see Reichenstein (1987), Gordon and Leeper (1992), Eichenbaum (1992), Christiano and Eichenbaum (1992a), Strongin (1995), Bernanke and Blinder (1992), Bernanke and Mihov (1995), Pagan and Robertson (1995), Hamilton (1997), Bernanke and Mihov (1998)].

9.2.2 Stochastic Simulation

Let us now see what happens when we use an erratic money supply process to generate the rate of growth of the money supply in each period. Table 2 shows the stochastic simulation results obtained using the process given by 35 to generate the percentage deviations of the rate of growth of the money supply from the steady-state rate of growth of the money supply of that same process (this stochastic simulation experiment combines technological shocks generated using equation 33 with monetary shocks generated using equation 35). Note that if $\xi_t \sim N(0, \sigma^2)$, then $\frac{\xi_t}{0.01538} \sim N(0, \left(\frac{\sigma}{0.01538}\right)^2)$. For $\sigma$ we used the value obtained by Cooley and Hansen (1989) in their estimation (0.0086). The average nominal interest rate used in the calibration ($R$) was adjusted so
that, given the new steady-state value used for $\mu$, the steady-state real interest rate would remain the same as the one we had in the previous stochastic simulation exercise.

We can see in table 2 that in terms of the real variables $\hat{y}_t$, $\hat{c}_t$, $\hat{n}_t$, $\hat{w}_t$ and $\hat{p}_t$, the results (obtained with an erratic rate of growth of the money supply) are similar to the results we obtained in the stochastic simulation exercise reported in table 1 (where the rate of growth of the money supply was constant and we only had technological shocks). Hence, we can conclude that in our model money shocks have a very small impact on the behaviour of real variables. These results are not surprising. Other authors who have worked with flexible price models have also found that monetary policy has only a very small impact on the behaviour of real variables [e.g., Cooley and Hansen (1989), Kydland (1989)]. The literature suggests that for monetary policy to have important real effects we need some kind of rigidity in the model (for example, some stickiness in the price level or some kind of friction in financial markets).

10 Conclusion

We have built a model of monetary creation and destruction which has exactly the same steady-state values of physical output, consumption, investment and “hours of work” as a zero growth version of the model without money presented in King, Plosser and Rebelo (1988).

The technological shock impulse response experiment performed using a constant rate of growth of the money supply gave us exactly the same results in terms of the real variables as would be obtained with the zero growth version of the King, Plosser and Rebelo model calibrated with our parameters.

Monetary policy shocks have only a very small impact on the behaviour of real variables. Most of the impact goes to the price level. This also explains why the nominal interest rate reacts positively to a temporary increase in the rate of growth of the money supply.

Many of the results we obtained with our model are probably linked with the assumptions of fully flexible prices, permanent market clearing and Rational Expectations (for example, the effect of money shocks on the price level and on the behaviour of real variables; or the fact that the “expected inflation effect” determines the behaviour of the nominal interest rate). As is well known, these assumptions tend to make adjustments be very quick. It would be interesting to
examine the impact of introducing different types of rigidities in our model. These could include some form of price stickiness (as in sticky price models) or some kind of friction in financial markets (as in limited participation models).

It would also be interesting to examine what would happen if we used the same model but with different initial conditions on the household’s optimization problem (in this article, we have used the initial condition which naturally arises when we think a closed economy without government and where firms don’t borrow back to its initial moment). In fact, we have tried another initial condition: an initial condition stating that the household starts period 0 with a debt which equals the profits she receives at the end of the previous period, i.e., $B_0 = \sum_{f=1}^{F} z_f \Pi^f_{t-1} + \sum_{l=1}^{L} z_{0}^{bank,l} \Pi^{bank,l}_{t-1}$. This is the initial condition which naturally arises when we think a closed economy without government and where firms borrow the wages at the beginning of the period back to its initial moment. This initial condition implies that the amount the household wants to borrow at the beginning of each period will be given by $\frac{B_t}{1+R_t} = P_t c_t - W_t n_t$. This makes sense because the fact that the household does not need to use the wage earnings to pay the debt from the previous period, means that she can use those wage earnings to buy consumption goods during the current period. Hence, the amount she will want to borrow equals the amount she wants to spend buying consumption goods minus the amount she already has (wage earnings). The results we obtained with this initial condition were not very different.
In this Appendix we show that the initial condition we used in the main text is the initial condition which naturally arises when we think a closed economy without government and where firms don’t borrow back to its initial moment. We show that when we start our analysis of the economy at a certain point in time which is in the middle of History (let us denote this point by period $0$), the specific initial condition (concerning the household’s debt position at the beginning of that period) that we ought to use is the one we have used in this article.

Let us consider a closed economy where there are only households, firms and banks and where firms don’t borrow. Since there is no government there are no monetary transfers from the government to the households. Let us imagine that we are at the moment in time where serious economic activity is going to begin and denote this “Beginning of the economy” period by $t_B$. Since production is only going to take place during period $t_B$, firms will only receive income during the period and hence will only pay wages and dividends at the end of the period. However, to be able to buy goods from the firms during the period households must use money. The only possibility they have to obtain this money is to go to the bank loans market and obtain a loan from a bank. Hence, when the household is at the beginning of period $t_B$ and considering choices for $t_B$ and future periods it faces the following set of budget constraints. For period $t_B$ the budget constraint is

$$\frac{B_{t_B+1}}{1+R_{t_B}} = P_{t_B} c_{t_B} + \sum_{f=1}^{f=F} Q_{t_B}^f (z_{t_B+1}^f - z_{t_B}^f) + \sum_{l=1}^{l=L} Q_{t_B}^{bank,l} (z_{t_B+1}^{bank,l} - z_{t_B}^{bank,l})$$ \hspace{1cm} (36)$$

Since the household is looking into the future, when she is at the beginning of period $t_B$ she also takes into account the constraints for periods $(t_B + 1), (t_B + 2), (t_B + 3), \ldots$ which are given by

$$W_{t_B-1+i} = B_{t_B-i} + \frac{B_{t_B+i+1}}{1+R_{t_B+i}} + \sum_{f=1}^{f=F} z_{t_B+i}^f \Pi_{t_B+i}^f + \sum_{l=1}^{l=L} z_{t_B+i}^{bank,l} \Pi_{t_B+i}^{bank,l}$$

$$= P_{t_B+i} c_{t_B+i} + B_{t_B+i} + \sum_{f=1}^{f=F} Q_{t_B+i}^f (z_{t_B+i+1}^f - z_{t_B+i}^f) + \sum_{l=1}^{l=L} Q_{t_B+i}^{bank,l} (z_{t_B+i+1}^{bank,l} - z_{t_B+i}^{bank,l})$$ \hspace{1cm} (37)$$

for $i = 1, 2, \ldots$
Note that when the household is at the beginning of period \( t_B \), the budget constraint for \( t_B \) is different from the budget constraint for following periods (this happens because at \( t_B \) there is no previous period). All that the budget constraint for period \( t_B \) (equation 36) is saying is that in the initial period, to finance her purchases of consumption and of shares, the household must obtain credit. The analysis of the budget constraints for following periods [periods \( (t_B + i) \) with \( i = 1, 2, \ldots \)], which are given by equation 36, is identical to the analysis we did in the main text for the period \( t \) budget constraint.

The important thing that follows from this structure is that, whatever the optimal decisions of economic agents are, in equilibrium there is an initial condition for period \((t_B + 1)\) which can be computed. In other words, we can use the household’s budget constraint in period \( t_B \), the market clearing conditions for period \( t_B \) and the definitions of profits of firms and of profits of banks to compute an initial condition for the optimization problem of the household in period \((t_B + 1)\).

In order to compute this initial condition, we start by taking the budget constraint for period \( t_B \) [equation 36]. Imposing on that equation the market clearing conditions in the shares market in period \( t_B \) [equations 12 and 13 written with \( 0 \) replaced by \( t_B \)] and assuming that the economy starts with every household owning the same share of each firm and each household owning the same share of each bank so that we also have \( z^{f}_{t_B} = \frac{1}{H} \) and \( z^{bank,f}_{t_B} = \frac{1}{H} \), we obtain

\[
\frac{B_{t_B+1}}{1 + R_{t_B}} = P_{t_B} c_{t_B}
\]

We now write the following tautology

\[
B_{t_B+1} = \frac{B_{t_B+1}}{1 + R_{t_B}} (1 + R_{t_B}) \Leftrightarrow B_{t_B+1} = \frac{B_{t_B+1}}{1 + R_{t_B}} + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B}
\]

Using equation 38, this is equivalent to writing

\[
B_{t_B+1} = P_{t_B} c_{t_B} + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B}
\]

Using the market clearing condition in the goods market in period \( t_B \) [equation 8 written with \( 0 \) replaced by \( t_B \)], we obtain

\[
B_{t_B+1} = \frac{F}{H} \left[ P_{t_B} A_{t_B} F(k_{t_B}, n_{t_B}^d) - P_{t_B} [k_{t_B+1} - (1 - \delta)k_{t_B}] \right] + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B}
\]
Using the definition of profits of the firm $f$ in period $t_B$ in here, we obtain

$$B_{t_B+1} = \frac{F}{H} \left[ \Pi^f_{t_B} + W_{t_B} n^d_{t_B} \right] + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B} \Leftrightarrow B_{t_B+1} = W_{t_B} \frac{F}{H} n^d_{t_B} + \frac{F}{H} \Pi^f_{t_B} + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B}$$

With the market clearing condition in the labour market, this becomes

$$B_{t_B+1} = W_{t_B} n^s_{t_B} + \frac{F}{H} \Pi^f_{t_B} + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B}$$

Using the market clearing condition from the bank loans market, we obtain

$$B_{t_B+1} = W_{t_B} n^s_{t_B} + \frac{F}{H} \Pi^f_{t_B} + \frac{L}{H} \frac{1}{p_{leg}} RES_{t_B} R_{t_B}$$

Using the definition of profits of bank $l$ in period $t_B$ here, we obtain

$$B_{t_B+1} = W_{t_B} n^s_{t_B} + \frac{F}{H} \Pi^f_{t_B} + \frac{L}{H} \Pi^l_{t_B}$$

Finally, using the market clearing conditions in the shares market, this can be written as

$$B_{t_B+1} = W_{t_B} n^s_{t_B} + \sum_{f=1}^{f=F} z^f_{t_B+1} \Pi^f_{t_B} + \sum_{l=1}^{l=L} z^l_{t_B+1} \Pi^l_{t_B}$$

This equation tells us that, in equilibrium, the amount the household borrows at the beginning of period $t_B$ is the sum of the wage earnings and dividend earnings that she will receive at the end of that period. Hence, when we go on to consider the typical household’s optimization problem at the beginning of period $(t_B + 1)$ we should add this equation as an initial condition (describing the debt she carries from the previous period). Therefore, at the beginning of period $(t_B + 1)$ the household faces the following constraints

$$W_{t_B+i} n^s_{t_B+i} + \frac{B_{t_B+i+2}}{1 + R_{t_B+i+1}} + \sum_{f=1}^{f=F} z^f_{t_B+i+1} \Pi^f_{t_B+i} + \sum_{l=1}^{l=L} z^l_{t_B+i+1} \Pi^l_{t_B+i} =$$

$$= P_{t_B+i+1} c_{t_B+i+1} + B_{t_B+i+1} + \sum_{f=1}^{f=F} Q^f_{t_B+i+1} (z^f_{t_B+i+2} - z^f_{t_B+i+1}) + \sum_{l=1}^{l=L} Q^l_{t_B+i+1} (z^l_{t_B+i+2} - z^l_{t_B+i+1})$$

for $i = 0, 1, 2, 3, \ldots$

plus the initial condition
Using the initial condition in the budget constraint written with \( i = 0 \), the budget constraint for \( i = 0 \) becomes

\[
\frac{B_{t_B + 2}}{1 + R_{t_B + 1}} = P_{t_B + 1} c_{t_B + 1} + \sum_{f=1}^{F} Q_{t_B + 1}^f (z_{t_B + 2}^f - z_{t_B + 1}^f) + \sum_{l=1}^{L} Q_{t_B + 1}^{bank,l} (z_{t_B + 2}^{bank,l} - z_{t_B + 1}^{bank,l})
\]

Therefore the complete description of constraints faced by the household at the beginning of period \( (t_B + 1) \) is

\[
\frac{B_{t_B + 2}}{1 + R_{t_B + 1}} = P_{t_B + 1} c_{t_B + 1} + \sum_{f=1}^{F} Q_{t_B + 1}^f (z_{t_B + 2}^f - z_{t_B + 1}^f) + \sum_{l=1}^{L} Q_{t_B + 1}^{bank,l} (z_{t_B + 2}^{bank,l} - z_{t_B + 1}^{bank,l})
\]

\[
W_{t_B + 1} n_{t_B + 1} + \frac{B_{t_B + 2 + i}}{1 + R_{t_B + 1 + i}} + \sum_{f=1}^{F} z_{t_B + 1 + i}^f \Pi_{t_B + 1 + i}^f + \sum_{l=1}^{L} z_{t_B + 1 + i}^{bank,l} \Pi_{t_B + 1 + i}^{bank,l} =
\]

\[
= P_{t_B + 1 + i} c_{t_B + 1 + i} + B_{t_B + 1 + i} + \sum_{f=1}^{F} Q_{t_B + 1 + i}^f (z_{t_B + 2 + i}^f - z_{t_B + 1 + i}^f) + \sum_{l=1}^{L} Q_{t_B + 1 + i}^{bank,l} (z_{t_B + 2 + i}^{bank,l} - z_{t_B + 1 + i}^{bank,l})
\]

for \( i = 1, 2, 3, \ldots \)

But these two equations are identical in form to the ones the household was facing at the beginning of period \( t_B \). Therefore we can repeat the reasoning and derive an initial condition for the household’s optimization problem at the beginning of period \( (t_B + 2) \). We will obviously obtain a condition which has the same form: we obtain

\[
B_{t_B + 2} = W_{t_B + 1} n_{t_B + 1} + \sum_{f=1}^{F} z_{t_B + 2}^f \Pi_{t_B + 1}^f + \sum_{l=1}^{L} z_{t_B + 2}^{bank,l} \Pi_{t_B + 1}^{bank,l}
\]

(and so on and so forth for all periods ahead). The conclusion to be drawn is that, when we start our analysis of the economy in the middle of History (period 0, for example), we should add to the household’s optimization problem the following initial condition

\[
B_0 = W_{-1} n_{-1} + \sum_{f=1}^{F} z_0^f \Pi_{-1}^f + \sum_{l=1}^{L} z_0^{bank,l} \Pi_{-1}^{bank,l}
\]
This is exactly the initial condition we have used in this paper. This initial condition was derived by thinking an economy where firms don’t borrow back to its initial moment. If instead we think an economy where firms borrow the wages at the beginning of the period back to its initial moment, the initial condition we obtain is

\[ B_0 = \sum_{f=1}^{F} z_0^f \Pi_{-1}^f + \sum_{l=1}^{L} z_0^{bank,l} \Pi_{-1}^{bank,l}. \]
References


[29] Ravn, M. O. and Sola, M. 1995. Stylized facts and regime changes: are prices procyclical?


Table 1. Stochastic simulation. Shocks in the firms’ technological parameter with a constant rate of growth of the money supply.

A. Standard deviations (s.d.)

<table>
<thead>
<tr>
<th>Variable</th>
<th>s.d.</th>
<th>s.d. of the variable divided by the s.d. of $\hat{y}_t$</th>
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<td>$\hat{y}_t$</td>
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<tr>
<td>$\hat{c}_t$</td>
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<td>$\hat{i}_t$</td>
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<tr>
<td>$\hat{p}_t$</td>
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<td>0.46</td>
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B. Cross-correlations

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<th>$\hat{y}_{t-4}$</th>
<th>$\hat{y}_{t-2}$</th>
<th>$\hat{y}_{t-1}$</th>
<th>$\hat{y}_t$</th>
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<th>$\hat{y}_{t+2}$</th>
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<tr>
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Table 2. Stochastic simulation. Shocks in the firms’ technological parameter and shocks in the rate of growth of the money supply.

A. Standard deviations (s.d.)

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B. Cross-correlations

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Figure 1: Output as % dev. from steady-state

Figure 2: Consumption as % dev. from steady-state

Figure 3: Investment as % dev. from steady-state

Figure 4: Hours of work as % dev. from steady-state
Figure 9
1% shock in A
Inflation rate