ON THE WELFARE COST OF INFLATION

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1. Introduction

This paper provides new estimates of the welfare cost of inflation in the U.S. economy. The estimation is based on exactly the thought-experiment underlying Bailey's (1956) original study: Two economies with the same preferences and technology are compared, both in a deterministic steady state with constant rates of money growth and price inflation, and constant nominal interest rates equal to the common real rate plus the inflation rate. In one economy monetary policy induces a steady deflation at the Friedman (1969) optimal rate corresponding to a nominal interest rate of zero; in the other money grows so as to induce a positive nominal interest rate r. We define $w(r)$ as the fraction of income people would be willing to forego to move from the second economy to the first, and call this function $w(r)$ the <u>welfare cost</u> of the inflation rate corresponding to r.

One can at least imagine experimentally tracing out the function $w(r)$ by confronting people with differing income-interest rate combinations and observing their choices, but in practice estimating welfare costs involves using a theoretical model of a monetary economy to interpret available data on money, income, and interest rates. Bailey invoked a consumers' surplus argument to use the area under a semi-log money demand function as an estimator of $w(r)$. The same logic applied to other money demand functions yields other estimates. In the next section, I review the evidence on money demand from U.S. time series, and the implied behavior for welfare costs. We will see that the estimated costs of moderate inflations hinge critically on the way this evidence is interpreted.

Section 3 provides one possible general equilibrium rationale for the welfare estimates reported in Section 2, using a simplified version of Sidrauski's (1967a,b) model. Section 4 provides a second rationale, using as context a special case of a model of a transactions technology proposed by McCallum and Goodfriend (1987). Both of these models can be viewed as providing theoretical interpretations of money demand evidence and thus as justifications of the consumers' surplus formulas used in Section 2. The transactions-time approach of McCallum and Goodfriend also suggests interesting connections with the earlier inventory-theoretic literature on money demand, connections that imply an interpretation of the welfare cost of inflation as wasted time.

The estimates of Sections 2-4 are all obtained by treating the nominal interest rate as a policy variable, assuming that the monetary policy that implements any given interest rate can be carried out with lump-sum fiscal transfers. Section 5 re-examines the estimation under the alternate assumption that only flat rate income taxes can be used, and that a government sector of given size must be financed either with inflation taxation or with income taxation. This modification introduces theoretical complications but does not, I argue, lead to major quantitative differences from the conclusions of earlier sections. Section 6 concludes the paper.

2. Money Demand and Consumers' Surplus

To review Bailey's argument and to consider alternatives, I take as a primitive in this section a demand function for real balances of the form $M/P = L(r,y)$, where M is money, P is the price level, r is the nominal interest rate, and y is real income. Moreover, I assume that this function L takes the form $L(r,y) = m(r)y$. This unit income elasticity assumption, which will be maintained in various forms throughout the paper, greatly simplifies matters and is well-supported by U.S. time series evidence.[1]

Figure 1 displays a plot of U.S. observations on the ratio of M1 to NNP (m_t) against observations on a long-term interest rate (r_t) for the years 1900-85. (These pairs are the circles in the figure.) The figure also plots the curves $m = Ar^{-\eta}$ for the η-values .3, .5, and .7, where A is selected so the curve passes through the geometric means of the data pairs. Within this parametric family, it is evident that $\eta = .5$ gives the best fit.

Figure 2 presents the same data, this time along side the curves $m = Be^{-\xi r}$ for the ξ-values 5, 7, and 9. Again, all three curves pass through the geometric means. Within this parametric family, $\xi = 7$ appears to give the best fit. It also clear, I think, that the semi-log function plotted here provides a description of the data that is much inferior to the log-log curve in Figure 1.

In order to provide some perspective on these estimates, Figure 3 plots actual U.S. real balances (not deflated by income) against the real balances predicted by the log-log demand curve: $Ar_t^{-.5}y_t$. One sees that the fitted values track well the secular decline in velocity prior to World War II, including the acceleration of this decline in the 1930s and 40s, and also track well the increase in M1-velocity in the last 35 years. One also sees, however, that the fitted series exhibits some large, shorter-term, fluctuations that do not appear at all in the actual series. The interest elasticity needed to fit the long-term trends (and very sharply estimated by these trends) is much too high to permit a good fit on a year-to-year basis. Of course, it is precisely this difficulty that has motivated much of the money demand research of the last 30 years, leading to distributed lag formulations of money demand that attempt to reconcile the evidence at different frequencies. In my opinion, this reconciliation has not yet been achieved, but in any case, it is clear that the functions I have fit contribute nothing toward the resolution of this problem.

For my purposes, as described in the introductory section, I do not see the empirical difficulties exhibited in Figure 3 as particularly serious. What one would like to have, for the hypothetical experiment described in Section 1, are long time series from many different economies, similar except in their average rates of money growth and hence in their nominal interest rates, so we could trace out the empirical relation $m(r)$ cross-sectionally, ignoring all but <u>very</u> low frequency information. In lieu of such an ideal natural experiment, it seems to me best to focus on low-frequency information from the single U.S. series. As Figure 1 shows, this information is very good.

To translate the evidence on money demand into a welfare cost estimate, let $\psi(m)$ be the inverse function of $m(r)$ and define $w(r)$ by

$$w(r) = \int_{m(r)}^{m(0)} \psi(m)\,dm = \int_0^r m(x)\,dx - rm(r), \tag{2.1}$$

the area under the inverse demand function from actual real balances $m(r)$ to the optimal balance $m(0)$ (which may be infinite).

For the log-log demand function $Ar^{-\eta}$, (2.1) implies $w(r) = A\frac{\eta}{1-\eta}r^{1-\eta}$. For $\eta = .5$, this is just a square root function. It is plotted in Figure 4. For the semi-log function $Be^{-\xi r}$, (2.1) implies $w(r) = \frac{B}{\xi}[1 - (1+r\xi)e^{-\xi r}]$. This curve is also plotted, for $\xi = 7$, in Figure 4. This is the parameterization used by Bailey.

Note that the two demand curves imply very different estimates for the welfare cost of moderate inflations. At a 6 percent interest rate, for example, the log-log curve implies a welfare cost of about one percent of income, while the semi-log curve implies a cost of less that 0.3 percent. On the other hand, the two curves are nearly parallel between interest rates of 4 and 10 percent, so they imply similar estimates of the cost of exceeding an inflation rate of zero by moderate amounts. The main difference is that log-log demand implies a substantial gain in moving from zero inflation to the Friedman optimal deflation rate needed to bring nominal interest rates to zero, while under semi-log demand this gain is trivial.

3. The Sidrauski Framework

The general equilibrium model of Sidrauski (1967a,b) provides one framework that can provide an explicit rationale for the consumers' surplus formula (2.1). Since (2.1) is at best only an approximation, is essential to have some basis for assessing its accuracy. For this purpose, it will be adequate to use a simplified version of Sidrauski's model that abstracts from capital accumulation.

Consider a deterministic, representative agent model, in which households gain utility from the consumption c_t of a single, non-storable good, and from their holdings m_t of real balances. Household preferences are:

$$(3.1) \qquad \sum_{t=0}^{\infty} (1+\rho)^{-t} U(c_t, m_t) \quad ,$$

where:

$$U(c,m) = \frac{1}{1-\sigma}([c\varphi(\frac{m}{c})]^{1-\sigma} - 1) .$$

These homothetic preferences are consistent with the absence of trend in the ratio of real balances to income in U.S. data, and the constant relative risk aversion form is consistent with balanced growth when the endowment grows at a constant rate.

Each household is endowed with one unit of time, which is inelastically supplied to the market and which produces $y_t = y_0(1+\gamma)^t$ units of the consumption good in period t. Hence one equilibrium condition is

$$c_t = y_t = y_0(1+\gamma)^t . \tag{3.2}$$

Households begin period t with M_t units of money, out of which they pay a lump sum tax H_t (or, if $H_t < 0$, receive a lump sum transfer). The price level is P_t, so the cash flow constraint for households is:

$$M_{t+1} = M_t - H_t + P_t y_t - P_t c_t$$

in nominal terms and thus:

$$(1+\pi_{t+1})m_{t+1} = m_t - h_t + y_t - c_t \tag{3.3}$$

in real terms, where $h_t = H_t/P_t$ and $1 + \pi_t = P_t/P_{t-1}$.

Households maximize (3.1) subject to (3.3). Among the first order conditions for this problem is:

$$(3.4) \qquad 1 + \frac{U_m(c_t,m_t)}{U_c(c_t,m_t)} = (1+\rho)(1+\pi_t)\frac{U_c(c_{t-1},m_{t-1})}{U_c(c_t,m_t)} .$$

In any equilibrium, $c_t = y_t$ for all t, from (3.2). Now consider a balanced growth equilibrium in which the money growth rate is constant at μ, so that the inflation factor $1+\pi_t$ is constant at the value $(1+\mu)/(1+\gamma)$, and the real balances-income ratio $m_t/y_t = m_t/c_t$ is constant at m. In this case, (3.4) becomes:

$$(3.5) \qquad 1 + \frac{\varphi'(m)}{\varphi(m)-m\varphi'(m)} = (1+\rho)(1+\mu)(1+\gamma)^{-1+\sigma} = 1+r .$$

The right side of (3.5) is just one plus the nominal interest rate, which I call r. Then (3.5) is conveniently rewritten as:

$$(3.6) \qquad \varphi'(m) = r[\varphi(m) - m\varphi'(m)] .$$

Let $m(r)$ denote the m value that satisfies (3.6), expressed as a function of the interest rate. Throughout the paper, it is this steady state equilibrium relation $m(r)$ that I call a "money demand function," and that I identify with the curves shown in Figures 1 and 2.

The flow utility enjoyed by the household in the steady state is $U(1,m(r))$. Provided $m'(r) < 0$, this utility is maximized over non-negative nominal interest rates at $r = 0$: the Friedman (1969) rule of a deflation equal to the real rate of interest.[2] In this section, I define the welfare cost $w(r)$ of a nominal rate r to be the percentage income compensation needed to leave the household indifferent between r and 0. That is, $w(r)$ is defined as the solution to:

$$U[e^{w(r)}, m(r)] = U[1, m(0)] \;. \tag{3.7}$$

Our objective is to use an estimated $m(r)$ to obtain a quantitative estimate of the function $w(r)$.

This can be done as follows. Let $m(r)$ be given and let $\psi(m)$ be the inverse function. Then (3.6) implies that φ satisfies the differential equation

$$\varphi'(m) = \frac{\psi(m)}{1 + m\psi(m)} \varphi(m) \;. \tag{3.8}$$

The solution to (3.8) is:

$$\varphi(m) = \varphi(m_0)\exp\left(\int_{m_0}^{m} \frac{\psi(x)}{1 + x\psi(x)} dx \right) \;. \tag{3.9}$$

With the preferences I have assumed, the definition (3.7) of $w(r)$ implies

$$e^{w(r)}\varphi[e^{-w(r)}m(r)] = \varphi(m(0)) \;. \tag{3.10}$$

Taking $m_0 = m(0)$ in (3.9) and combining (3.9) and (3.10) gives:

$$e^{w(r)}\exp\left(-\int_{e^{-w(r)}m(r)}^{m(0)} \frac{\psi(x)}{1 + x\psi(x)} dx\right) = 1 \;. \tag{3.11}$$

Taking logs gives:

$$(3.12) \qquad w(r) = \int_{e^{-w(r)}m(r)}^{m(0)} \frac{\psi(x)}{1 + x\psi(x)} dx \quad .$$

Equation (3.12) is equivalent to the differential equation:

$$(3.13) \qquad w'(r) = - [1 + e^{-w} m\psi(e^{-w}m)]^{-1} e^{-w}\psi(e^{-w}m)m'(r)$$

with the initial condition $w(0) = 0$.

Given any money demand function m (and inverse ψ), (3.13) is readily solved numerically for an exact welfare cost function $w(r)$. But comparing (3.12) and (2.1), one can guess that for small values of r--and hence $w(r)$--the solution to (3.12) or (3.13) must be very close to the value implied by the consumers' surplus formula. In fact, on a plot such as Figure 4, the exact and the approximate solutions cannot be distinguished by the eye. (See also Figure 5 in the next section.)

It is interesting to note that for the particular demand function $m(r) = Ar^{-1/2}$, the integral (3.9) can be evaluated directly to give the utility function: $\varphi(m) = [1 + Bm^{-1}]^{-1}$, where $B = A^2$. Since the value of A empirically is about .042 (see Figure 1), the Sidrauski utility function is a CES:

$$U(c,m) = [c^{-1} + (.0018)m^{-1}]^{-1} \ .$$

The elasticity of substitution is 0.5.

4. <u>The McCallum-Goodfriend Framework</u>

McCallum and Goodfriend (1987) have proposed a variation on the Sidrauski model in which the use of cash is motivated by an assumed transactions technology, rather than by an assumption that real balances yield utility directly. Their model provides another framework for assessing the costs of inflation, and also provides an interpretation of these costs as wasted time.

In this model, household preferences depend on goods consumption only:

$$\sum_{t=0}^{\infty} (1+\rho)^{-t} \frac{1}{1-\sigma}(c_t^{1-\sigma} - 1) \quad . \tag{4.1}$$

Each household is endowed with one unit of time, which can be used either to produce goods or to carry out transactions. Call s_t the fraction devoted to transacting. The goods production technology is assumed to be

$$c_t = (1-s_t)y_t = (1-s_t)y_0(1+\gamma)^t \quad . \tag{4.2}$$

The cash flow constraint in real terms is:

$$(1+\pi_{t+1})m_{t+1} = m_t - h_t + (1-s_t)y_t - c_t \quad . \tag{4.3}$$

The new element in the model is a transactions constraint, relating household holdings of real balances and the amount of household time devoted to transacting to the spending flow the household carries out. I assume that this constraint takes the form:

$$c_t = m_t f(s_t) \quad , \tag{4.4}$$

which will be consistent with a unit elasticity of money demand.

Households maximize (4.1) subject to (4.3) and (4.4). The first order conditions include:

$$(4.5)\qquad 1 + \frac{y_t f(s_t)}{m_t f'(s_t)} = (1+\rho)(1+\pi_t)\frac{U'(c_{t-1})}{U'(c_t)}\left[\frac{X_t}{X_{t-1}}\right] ,$$

where $X_t = 1 + y_t[m_t f'(s_t)]^{-1}$.

As in the last section, I evaluate this first order condition on a balanced equilibrium path on which the money growth rate μ, the inflation rate π, the transactions time s, and the ratio $m = m_t/y_t$ are all constant. In this case, (4.5) becomes:

$$(4.6)\qquad f(s) = rmf'(s) ,$$

where the nominal interest rate r is again defined by:

$$1+r = (1+\rho)(1+\mu)(1+\gamma)^{-1+\sigma} .$$

The other equilibrium condition follows from (4.2) and (4.4):

$$(4.7)\qquad 1-s = mf(s) .$$

Given f, we can solve (4.6) and (4.7) for s and m as functions of r.

In this model, the time spent economizing on cash use, $s(r)$, has itself the dimensions of a percentage reduction in consumption, and hence is a direct measure of the welfare cost of inflation. My objective again is to estimate this function $s(r)$, using the money demand curves described in Section 2 as estimates of the function $m(r)$. To do this, we need to work backward from $m(r)$ to the transactions technology function f. As in the last section, we do this by finding a first order differential equation in the welfare cost $s(r)$.

Given φ, let $m(r)$ and $s(r)$ satisfy (4.6) and (4.7). Then differentiating (4.7) through with respect to r and using (4.6) and (4.7) to eliminate $f(s)$ and $f'(s)$ yields:

$$(4.8) \qquad s'(r) = -\frac{rm'(r)(1-s(r))}{1-s(r)+rm(r)} .$$

Figure 5 plots the solution $s(r)$ with initial condition $s(0) = 0$ for the log-log and semi-log demand cases, for interest rates ranging from 0 to 2 (200%). Also plotted are the areas under the two demand curves, as in Figure 4. For the semi-log case, the exact and approximate welfare cost estimates cannot be distinguished. (Indeed, since approximations are also involved in calculating the solution to (4.8), I do not know which of the curves for the semi-log case in Figure 5 better approximates the solution to (4.8), though this question is answerable at greater expense.) For the log-log case, the two curves are also virtually identical at interest rates below 20%. Thus the McCallum-Goodfriend model yields simply a new interpretation of estimates already obtained.

Given a monotonic $s(r)$, the implied transactions time function f can be calculated using the inverse function $r(s)$ and (4.7):

$$(4.9) \qquad f(s) = \frac{1-s}{m(r(s))} .$$

This function is given, for the semi-log and log-log cases, in Figure 6. To fit a semi-log money demand, the transactions time function must take a positive value at $s = 0$: It must be possible to conduct one's affairs using cash and no time at all. At some point, beyond the range of Figure 6, this function becomes infinite, reflecting the upper limit on transactions time shown on Figure 5.

For the log-log case, the implied transactions time function is simply a straight line through the origin, $f(s) = ks$, for some constant k. This case has, I think, a helpful interpretation in light of the inventory-theoretic literature on money demand that dates from the work of Baumol (1952), Tobin (1956), Patinkin (1965), Dvoretsky (1965), and Miller and Orr (1966). This literature was designed specifically to be useful in thinking about the use of time and cash to carry out transactions. In Baumol's original formulation, a single consumer spends out his cash holdings at a constant rate C per unit of time. When his cash is exhausted, it must be replenished at a cost that is independent of the amount of cash obtained. If the cost is incurred at intervals of length τ, then $M = C\tau$ must be withdrawn each time, and the withdrawal cost per unit of time will be proportional to $1/\tau = C/M$. This reasoning leads directly to a transactions-time technology of the form $s_t = km_t s_t$, where the constant k depends on the time cost of cash management.[3]

Miller and Orr (1966) introduced stochastic elements into this inventory problem by considering a firm or entrepeneurial household which has stochastic cash inflows as well as outflows. They consider a situation in which in any short time interval, either a receipt of size x arrives or a payment of size x comes due, each event occuring with probability 1/2. When the cash balance hits zero, a trip to the bank is taken and jx

dollars are withdrawn, for some integer value j. When cash reaches the value nx, for some value $n > j$, a trip to the bank is taken to deposit $(n-j)x$, returning the cash balance to jx. Given the two parameters (n,j), cash holdings form a Markov process with states $x, 2x, \ldots, (n-1)x$. Miller and Orr calculate the invariant distribution of this process. The average cash balance, the mean of this distribution, is $\frac{n+j}{3}x$. The probability that a trip occurs in any period is $\frac{1}{n(n-j)}$. A policy (n,j) that minimizes the expected number of trips subject to an upper bound on average balances turns out to satisfy $j = \frac{n}{3}$, independent of the bound on cash. It follows that for a given cash flow, average balances are proportional to n, say $m = An$, and the number of trips is inversely proportion to n^2, say $s = B/n^2$. Eliminating n between these equations, one obtains the formula for the implied transactions technology

$$c \leq Dms^{1/2} , \tag{4.10}$$

where D is another constant of proportionality.

These two examples suggest a parameterization of the function f of the form $f(s) = ks^{\gamma}$, $0 < \gamma \leq 1$, admitting the Baumol case, $\gamma = 1$, the Miller-Orr case, $\gamma = .5$, and the other possibilities in between. Specialized to this case, (4.6) and (4.7) imply:

$$s = \gamma rm , \tag{4.11}$$

$$1 - s = mks^{\gamma} . \tag{4.12}$$

Eliminating m between (4.11) and (4.12) gives

(4.13) $$s = 1 - \frac{k}{r\gamma} s^{1+\gamma} .$$

When the solution s to (4.13) is small, its value is well approximated by:

(4.14) $$s(r) = (\gamma r/k)^{\frac{1}{1+\gamma}} ,$$

and the implied money demand function is:

(4.15) $$m(r) = k^{-\frac{1}{1+\gamma}} (\gamma r)^{-\frac{\gamma}{1+\gamma}} .$$

Thus the log-linear money demand curve corresponds in a very direct way to inventory models of cash management, provided the interest elasticity η is less than one-half, and the case $\eta = 0.5$ corresponds to the simplest of these models, the Baumol model. Indeed, with $\gamma = 1$, (4.15) is the celebrated square root rule of money demand. The corresponding welfare cost formula (4.14) is also a kind of square root rule.

To summarize, the McCallum-Goodfriend model of this section and the Sidrauski model of Section 3 lead to essentially the same estimate of the welfare costs of inflation: they simply provide different interpretions of the area under a money demand curve. It seems to me a definite advantage of the McCallum-Goodfriend framework that it enables us to interpret this estimate in a way that makes a connection with the inventory-theoretic literature. This connection helps us to see what is at stake in assuming one or another specific functional form for m(r), and helps us to judge the plausibility of the theory's predictions at interest rates that are outside the range we have observed. For welfare estimates, the behavior of money demand as the interest rate approaches zero is crucial. We can see from Figures 1 and 2 that the log-log form of $m(r)$ fits the data at low interest rates much better than does the semi-log form. I think inventory theory supports this empirical conclusion: Managing an inventory always requires <u>some</u> time, and a larger average stock must always reduce this time requirement, no matter how small it is.

5. Fiscal Considerations

In the analysis to this point the nominal interest rate r has been treated as a policy variable, and the welfare cost of inflation has been defined by a comparison of resource allocations when $r > 0$ to a benchmark case of $r = 0$. In fact, of course, any particular interest rate policy must be implemented by a specific money supply policy, and this monetary policy must in turn be implemented by a policy of fiscal transfers, open market operations, or both. This fact raises no difficulties as long as the necessary transfers can be effected through lump-sum payments or assessments, but if this is not possible the optimality of the Friedman rule can cease to obtain. Aspects of this question have been examined by Phelps (1973), Bewley (1983), Kimbrough (1986 a,b), Lucas and Stokey (1983), Woodford (1990), and most recently by Guidotti and Vegh (1993) and Chari, Christiano, and Kehoe (1993). This section addresses some of these fiscal questions in the context of the Sidrauski model of Section 3.

Let $m(r)$ be steady state real balances (the solution to (3.5)). Define the parameter δ by $1+\delta = (1+\rho)(1+\gamma)^{\sigma-1}$, so that $\delta \cong \rho + \sigma\gamma - \gamma$ is the amount by which the real interest rate exceeds the growth rate of production. Then the consumer budget constraint (3.3) and the resource constraint (3.2) together imply that to implement an interest rate r, the fraction

(5.1) $$\left(\frac{\delta - r}{1+\delta}\right)m(r) .$$

of income y_t must be transfered from the private sector to the government in a steady state, in the form of real balances withdrawn from circulation. (If $\delta < r$, the negative of this magnitude is seigniorage revenue, relative to income.)

For the function $m(r) = (.042)r^{-1/2}$ that fits U.S. data, $m(r) \to \infty$ as $r \to 0$, so if the flow (5.1) must be withdrawn using a fractional tax on income, the policy $r = 0$ is not feasible. The need to resort to income taxation thus places a positive lower bound on r. But with $\delta = .02$, an income tax rate of unity would implement an interest rate of $.06 \times 10^{-5}$! The needed qualification to the Friedman rule does not, in this case, seem to be of any quantitative interest.

The cases considered by most of the authors cited above, however, have the additional complications that labor is elastically supplied, so an income tax distorts resource allocation, and there is a positive amount of government consumption, necessitating a resort to distorting taxation of <u>some</u> kind. In the rest of this section, I add these two features to the model of Section 3, and re-do the welfare cost calculations described there. The results of these calculations are given in Figures 7 and 8.

Let x denote consumption of leisure, and assume preferences of the form

$$U(c,m,x) = \frac{1}{1-\sigma}\left(\left[c\varphi\left(\frac{m}{c}\right)h(x)\right]^{1-\sigma} - 1\right) .$$

Modify the resource constraint to include government consumption, g_t:

$$c_t + g_t = (1-x_t)y_t = (1-x_t)y_0(1+\gamma)^t .$$

Modify consumers' budget constraints to reflect income taxation at a flat rate τ:

$$(1+\pi_{t+1})m_{t+1} = m_t + (1-\tau)(1-x_t)y_t - c_t .$$

If government consumption is a constant ratio g to full income y_t, this model has an equilibrium path with constant ratios of consumption and real balances to income and with leisure constant as well. On this balanced path, let $c = c_t/y_t$ and $m = m_t/y_t$. The steady state equilibrium conditions are:

(5.2) $$\varphi'(\tfrac{m}{c}) = r[\varphi(\tfrac{m}{c}) - (\tfrac{m}{c})\varphi'(\tfrac{m}{c})] ,$$

(5.3) $$c\varphi(\tfrac{m}{c})\frac{h'(x)}{h(x)} = (1-\tau)[\varphi(\tfrac{m}{c}) - (\tfrac{m}{c})\varphi'(\tfrac{m}{c})] ,$$

(5.4) $$c + g + x = 1 ,$$

(5.5) $$(\tfrac{r-\delta}{1+\delta})m = (1-\tau)(1-x) - c .$$

Condition (5.2) just repeats (3.6), but consumption need not, in this section, equal full income. Condition (5.3) equates the marginal rate of substitution between goods and leisure to the after tax real wage, $1-\tau$.

Conditions (5.4) and (5.5) are the resource and consumer budget constraints; together, they imply the government budget constraint.

For any given nominal interest rate r and government consumption rate g, (5.2)-(5.5) are four equations that can be solved for the steady state allocation (c,x,m) and the income tax rate τ. Any monetary policy dictates a tax policy, depending on the extent to which seigniorage revenues help to finance g, or the extent to which the need to withdraw cash from the public adds to the burden on the tax system. Figures 7 and 8 tabulate a welfare cost function $w(r)$, defined as

$$U[e^{w(r)}c(r),m(r),x(r)] = U[c(\delta),m(\delta),x(\delta)] .$$

I use $r = \delta$ as a benchmark rather than $r = 0$ because, depending on the assumed functions φ and h, the system (5.2)-(5.5) may not have a solution at $r = 0$.

The figures are based on the following parameterization. For the function φ, I used $\varphi(m) = (1 + Bm^{-1})^{-1}$, which follows from the money demand function $m(r) = Ar^{-1/2}$ with $B = A^2$. A was set equal to .042, to fit the U.S. data. For the function h, I used $h(x) = x^{\beta}$, and let β range over the values .0001, .3, .6, and .9. Reading from bottom to top, these are the four curves plotted on Figures 7 and 8. I set $g = .35$, matching the share of income channeled through government at all levels in the U.S. today, including transfers of all kinds. Then the right side of the resource constraint (5.4) was set equal to 1.35, not unity, so that if $x = 0$, $c = 1$. Finally, I set $\delta = .02$. With these assumed functional forms and parameter values, $w(.02) = 0$, and the curve corresponding to $\beta = .0001$

is indistinguishable, over the range of interest rates used, from the function

$$w(r) = -\ln(1 - Ar^{1/2}) + \ln(1 - A\delta^{1/2}) ,$$

which is the welfare cost function plotted in Figure 4, shifted down by a constant.

Figure 7 covers interest rates from near zero to two percent. One can see that above about half a percent, estimated welfare costs are the same as in the inelastic labor supply, lump sum tax case studied in earlier sections. The effects of distorting taxation appear only at extremely low interest rates. To see these effects more clearly, Figure 8 plots the same four curves at interest rates from $r = .0001$ (.01 percent) up to .003. Thus for a leisure elasticity of $\beta = .3$, the optimal interest rate is about .03 percent, while at $\beta = .9$, it is about .04 percent. For any $\beta > 0$, the optimal r is strictly positive, but the deviations from $r = 0$ are trivial. The differences in welfare are small too. The minimized welfare costs are in all cases less that -.0045, while the intercept of the benchmark curve, $- w(\delta)$, is -.006, a difference of .0015 times income.

These second-best tax problems have so many logical possibilities that I thought it would be useful to work one case through, quantitatively, to see what kind of magnitudes are at stake. But the case I selected for study is, in some respects, arbitrary, and the literature cited above is helpful in isolating crucial assumptions. The model underlying Figures 7 and 8 is a special case of the model analyzed in Section 2 of Chari, Christiano, and Kehoe (1993), where it is shown that the Friedman $r = 0$ policy is optimal in the sense of Ramsey, <u>provided</u> that the private sector begins with a net nominal position (money plus nominal debt) of zero. If, on the other hand, the net nominal position of the private sector is positive, a monetary-fiscal policy that is efficient in Ramsey's sense entails an initial hyperinflation to exploit the capital levy possibilities. In my analysis, there is no debt and the public holds a positive initial nominal position (its cash), but I have constrained the money growth rate and the tax rate to be constant, precluding a capital levy. Under these assumptions, Woodford (1990) shows that $r = 0$ is not optimal, a fact that Figures 7 and 8 reflect.

In short, the optimality of the Friedman rule can be studied in a very wide variety of second-best frameworks, with a wide variety of different qualitative conclusions. In the specific context I have used, the Friedman rule needs qualification, but the magnitude of the needed amendment is trivially small. I would conjecture that the same two conclusions would be reached within the McCallum-Goodfriend context. The fact is that real balances are a very minor "good" in the U.S. economy (and I have probably magnified their importance by working with M1 rather than with the monetary base), so the fiscal consequences of even sizeable changes in inflation and interest rates are just not likely to be large.

6. Conclusion

In a monetary economy, it is in everyone's private interest to try to get someone else to hold non-interest bearing cash and reserves. But someone has to hold it all, so all of these efforts must simply cancel out. All of us spend several hours per year in this effort, and we employ thousands of talented and highly-trained people full-time in the financial industry to help us. These person-hours--many billions of dollars worth--are simply thrown away, wasted on a task that should not have to be performed at all.

In attempting to estimate the magnitude of this waste, I have adopted the original Bailey (1956) framework, within which the cost of inflation is identified entirely with the cost of higher-than-necessary long run average nominal interest rates. Many economists, I know, believe that this framework misses many of the costs of inflation, which are thought to arise from price variability or from other side effects or statistical correlates of high average inflation rates. Perhaps so: Inflationary finance is a symptom of a poorly functioning society, and one is not surprised to find it associated, in the data, with a variety of other social ills. But I think the proper task of welfare economics is to use theory to trace social problems back to their sources in bad policies, and try to work through the pure economics of the effects of specific changes in these policies, one at a time. From this point of view, I think the question Bailey asked is exactly the right one.

Within this framework, and with the log-log money demand function that fits twentieth century U.S. time series best, the welfare cost of a 10 percent nominal interest rate is about .013 times of GDP.[4] Taking second-best, fiscal considerations into account might lead to a reduction of this estimate to .012, depending on the elasticity of labor supply. To readers unfamiliar with quantitative welfare analysis, these numbers may seem small. But the United States is a six trillion dollar economy, so to convert the stocks and flows in the paper to dollars one needs to multiply all of them by $\$6\times10^{12}$. One percent of U.S. GDP is $60 billion. The estimates in this paper imply that by pursuing a suitably deflationary monetary policy, the

U.S. could enjoy an additional income flow of this magnitude, growing at 3 percent per year, in perpetuity. This is real money.

1. Estimates of the income elasticity of money (M1 or M2) demand obtained from long U.S. time series tend to be around unity: Meltzer (1963a), Laidler (1977), Lucas (1988), Stock and Watson (1991). Meltzer (1963b) reports estimates near one for sales elasticities in a cross-section sample of firms. Estimates from post-war quarterly data are generally below one: Goldfeld (1987). Recent estimates by Mulligan and Sala-i-Martin (1992) from panel data on U.S. states are higher, around 1.3. These estimates may not seem central to the experiments with a constant-income endowment economy reported in this paper, but the estimates of interest elasticities that obviously are central are obtained together with estimated income elasticities, so errors in one will affect the other.

2. Depending on the way the holding of real balances in motivated, the equilibrium in the limiting economy where $r = 0$ may be ill-defined, or there may be equilibria with $r = 0$ that are not close to equilibria with r positive but arbitrarily small. I will confine attention here to economies with $r > 0$, and by referring to 0 as the optimal rate I mean that reducing r is welfare improving, for any $r > 0$.

3. Karni (1973), Kimbrough (1986a,b), Den Haan (1990), and Gillman (1993) have also used monetary models featuring a time-using technology for transactions. Karni is explicit about the links with the inventory-theoretic literature that I am here using to motivate a specific form for this technology. The construction of an explicit general equilibrium model in which agents solve Baumol-like cash management problems has not been carried out in any of these papers, nor is it in this one. See

Fusselman and Grossman (1989) or Grossman (1987) for the most interesting results along this line.

4. The major single difficulty with these estimates, in my opinion, is that they are based on a model that does not distinguish between the separate payments roles of currency and deposits. For this, one needs a model of a banking system. See Yoshino (1991) for an interesting treatment of this issue.

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