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Understanding the Fiscal Theory of the Price Level

by Lawrence J. Christiano and Terry J. Fitzgerald

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I. Introduction to the Fiscal Theory of the Price Level

Price stability is an important goal of public policy. To reach this goal, two key questions must be addressed:

- How can price stability be achieved? And,
- How much price stability is desirable?

Standard monetarist doctrine offers a simple answer to the first question: Make sure the central bank has an unwavering commitment to price stability. Recently, though, some economists have begun to rethink the foundations of this doctrine, giving rise to an alternative view in which a tough, independent central bank is not sufficient to guarantee price stability. In this view, price stability requires not only an appropriate monetary policy, but also an appropriate *fiscal* policy.[1] Because fiscal policy receives so much attention in this new view of price-level determination, Michael Woodford has called it the *fiscal theory of the price level*.[2] Throughout this review, we refer to it as the *FTPL*.

Monetarist doctrine also recognizes that both fiscal and monetary policy must be selected appropriately if price stability is to be achieved. However, this doctrine holds that if the central bank is tough, the fiscal authority will be *compelled* to adopt an appropriate fiscal policy.[3] The FTPL denies this. It says that unless steps are taken to ensure appropriate fiscal policies, the goal of price stability may remain elusive no matter how tough and independent the central bank is.

The FTPL has significant implications for the way central banks conduct business. The conventional view prescribes that central bankers should stay away from fiscal authorities to reduce the likelihood of being pressured into

1 Cochrane (2000) goes so far as to say that monetary policy may be *irrelevant to* price determination. In his view, government-provided transactions assets are a vanishing component of all financial assets traded.

2 Benhabib, Schmitt-Grohe, and Uribe (2000), Cochrane (1998a, 2000), Dupor (2000), Leeper (1991), Sims (1994, 1999), and Woodford (1994, 1995, 1996, 1998a,b,c, 1999) all advocate the FTPL, while Buiter (1999), Carlstrom and Fuerst (2000), Kocherlakota and Phelan (1999), and McCallum (1998) provide critical reviews.

3 This classic statement is from Sargent and Wallace (1981); see especially the last paragraph of their paper.

making poor monetary policy decisions. The FTPL implies that central bankers with a mandate to foster price stability must do more than simply make sure their own house is in order; they also must convince the fiscal authority to adopt an appropriate fiscal policy.

The FTPL literature also draws attention to the second question—how much price stability is desirable?—which is both important and difficult. Sims (1999) and Woodford (1998a) point out that allowing the price level to fluctuate with unexpected shocks to the government budget constraint produces public finance benefits.[4] For example, a bad fiscal shock such as a war or natural disaster drives up the price level; this is equivalent to taxing the holders of the government's nominal liabilities. This promotes efficiency to the extent that it allows the authorities to keep labor tax rates smooth. In practice, this benefit is likely to be mitigated by whatever distortionary costs may be associated with price instability.[5] Cochrane (1998b) is mindful of these costs when, in his analysis, he simply takes for granted that complete price stability is a fundamental social objective.[6] Sims (1999) claims that public finance benefits overwhelm the distortionary costs associated with volatile prices, and so he conjectures that complete price stability is nonoptimal. A convincing answer to the second question awaits a quantitative study that carefully balances benefits and costs.

This paper explains the FTPL and elaborates on its implications for the two questions posed above, as well as for other issues. In the remainder of this introduction, we provide an overview of our analysis. We first discuss the crucial assumption that differentiates the FTPL from the conventional view. Next, we summarize some of the key issues that any assessment of the FTPL must confront, then briefly describe other issues addressed in the FTPL literature. Finally, we emphasize the connection between the FTPL and the traditional Ramsey literature on optimal monetary and fiscal policy.

What Distinguishes the FTPL?

The difference between the conventional view and the FTPL does not lie in any error of logic.[7] Instead, the two differ in their views of the government's intertemporal budget equation. That equation says the value of government debt is equal to the present discounted value of future government tax revenues net of expenditures (that is, surpluses), where both debt and surpluses are denominated in units of goods. This equation is expressed as

$$\text{(1.1)} \quad \frac{B}{P} = \text{present value of future surpluses},$$

where B is the outstanding nominal debt of the government and P is the price level. The conventional view holds that this equation is a constraint on the government's tax and expenditure policy;[8] that is, policy must be set so the right side equals the left, whatever the value of P. According to this view, when equation (1.1) is disturbed, the government must alter its expenditures or its taxes to restore equality. FTPL advocates, however, argue there is no inherent requirement that governments treat this equation as a constraint on policy. In their view, the intertemporal budget equation is instead an equilibrium condition: When something threatens to disturb the equation, the market-clearing mechanism moves the price level, P, to restore equality.

Michael Woodford has called this assumption—that government policy is not calibrated to satisfy the intertemporal budget equation for all values of P—the *non-Ricardian assumption*. Another way of stating this assumption is that if the real value of government debt were to grow explosively, no adjustments to fiscal and monetary policy would be made to keep it in line.[9]

4 For previous discussions, see Chari, Christiano, and Kehoe (1991), Judd (1989), and Lucas and Stokey (1983).

5 See Woodford (1998a, pp. 59–60) for an elaboration of this point.

6 Cochrane (1998b) emphasizes the need for some type of government security whose payoff fluctuates with shocks to the government budget constraint but does not generate the sort of distortionary costs associated with a fluctuating price level.

7 Some authors are concerned with the possibility the FTPL may be logically incoherent (see Buiter [1999]). We address these concerns, in part, by presenting a class of economic environments in which the FTPL is logically coherent.

8 Our notion of taxes includes seignorage revenues and taxes on the return to government debt, that is, default.

9 Technically, we are exploiting the equivalence between the intertemporal budget equation and a certain transversality condition. We discuss this equivalence later in the text and in the appendix.

The FTPL does not anticipate exploding debt. Rather, as long as there is absolutely no doubt about the government's commitment to *not* adjusting fiscal policy in the face of exploding debt, then prices will respond in such a way that the debt will not explode in the first place.

Assessing the FTPL

To evaluate the FTPL, it is useful to focus on the following positive and normative issues. Is the non-Ricardian assumption empirically plausible?[10] Does the FTPL offer a compelling explanation for episodes of high inflation? And finally, does the FTPL provide useful input into the design of socially efficient policies?

Clearly, the non-Ricardian assumption is not a good characterization of policy in *all* times and places. Often governments do seem ready to adjust fiscal policy when the debt gets too large. For instance, when the U.S. government debt began to increase in the 1980s and 1990s, there was considerable pressure for some combination of a tax increase and expenditure decrease to bring the debt back in line.[11] Likewise, according to the Maastricht Treaty, members of the European Union formally record their intention to adjust fiscal policy in the event their debts grow too large. Another example is provided by the International Monetary Fund. That organization uses an array of sanctions and rewards to encourage member countries to keep their debts in line by suitably adjusting fiscal policy.

For the FTPL to be an interesting positive theory, it need not hold in all situations. As Woodford (1998b) emphasizes, it may provide a useful characterization of actual policies in some contexts, even if it does not in others. For example, the government budget constraint was essentially absent from standard macroeconomic models of the 1960s and 1970s, and it played little role in Keynesian policy analysis (Sargent [1987, p. 112]). As a result, it is perhaps reasonable to suppose the non-Ricardian assumption held for that period.[12] Loyo (1999) argues that Brazilian policy in the late 1970s and early 1980s was non-Ricardian and that the FTPL provides a compelling explanation for Brazil's high inflation during that time.[13]

Even if, in practice, policy has never been non-Ricardian, the FTPL might still hold interest as a normative theory, for two reasons. First, optimal policies might themselves be non-Ricardian.[14] Second, the FTPL could serve as useful input into policy design, even if non-Ricardian policies are, in practice, *bad.* To see why they might be bad, consider legislators living in a non-Ricardian regime. Understanding that tax cuts or increases in government spending do not necessarily have to be paid for with higher taxes later, they might be tempted to embrace policies that imply too much spending and too much debt. Restricting fiscal policy by limiting government debt may be an effective way to deal with this problem.[15] By establishing the logical possibility of non-Ricardian policy, the FTPL implies that such policies could occur, in the absence of specific measures to rule them out. As a result, the FTPL can be used to articulate a rationale for the type of debt limitations imposed by the IMF and by the Maastricht Treaty.

Other Issues Addressed by the FTPL

Although we stress the implications of the FTPL for the two questions posed above, they are not the exclusive or even primary focus of the literature. The FTPL has plenty to offer, even for those with no interest in our two questions. FTPL advocates emphasize the value of their framework for understanding price-level determination when traditional quantity-theoretic reasoning breaks down or does not apply. This

10 The empirical plausibility of the non-Ricardian assumption poses a special challenge, because it cannot be assessed based on time series alone. For further discussion, see "Is the Non-Ricardian Assumption Empirically Plausible?" on page 8 of the present article.

11 Woodford (1998b) acknowledges that the political reaction to growing debt in the 1980s and 1990s (as well as other considerations) indicates U.S. policy during the past two decades is probably not well-characterized as non-Ricardian (see also our section "The FTPL and the Control of Average Inflation" on page 18). He argues that earlier episodes in U.S. postwar history—for example, the 1965–79 period—might be better characterized in this way.

12 See Woodford (1998b) for an elaboration of the argument that U.S. policy may have been non-Ricardian in the 1960s and 1970s.

13 One wonders whether it makes sense to assume that a theory like the FTPL, which focuses on the long-run properties of fiscal policy, holds for some periods and not others.

14 See Sims (1999) and Woodford (1998a).

15 Chari and Kehoe (1999) describe a model in which countries form a monetary union, and, absent debt constraints, the result is excessive debt. Woodford (1996) argues that a union without debt constraints is likely to end up with excessive price volatility. His reasoning uses the kind of logic surveyed in this paper. He notes that if policy is non-Ricardian, then fiscal shocks must show up as shocks to the price level, regardless of monetary policy. (We call this "Woodford's really unpleasant arithmetic" in "The FTPL with Stochastic Fiscal Policy" on page 15.) He argues that price-level instability arising from this source is likely to be excessive in a monetary union that adopts a non-Ricardian policy. It is precisely because he believes non-Ricardian policy is a realistic possibility—a possibility which, in this case, he thinks is bad—that he approves of the explicit debt restrictions incorporated into the Maastricht Treaty. For another similar discussion of the potential dangers of non-Ricardian policy, see Woodford (1998a, p. 60).

could happen, for example, if the monetary authority adopted a policy of pegging the rate of interest, so that the money supply would respond passively to demand. This case deserves emphasis because interest rate targets are thought to play an important role in monetary policy in practice (see Taylor [1993]).[16] Another scenario of interest occurs when private transactions involve no use of government-provided money. This review presents examples to illustrate the interest rate pegging and cashless economy scenarios.

Frank Ramsey and the FTPL

Our cashless economy example highlights the parallels between the FTPL and the traditional literature on optimal monetary and fiscal policy inspired by Frank Ramsey (1927) and reintroduced into macroeconomics by Lucas and Stokey (1983).[17] In the Ramsey literature, government "policy" is a sequence of actions (tax rates, expenditures, etc.) indexed by the date and (in models with uncertainty) by the realized value of shocks. Because these policies are not functions of past prices, prices exist such that the debt explodes and households refuse to buy it. Such possibilities are of no concern in the Ramsey literature because the government is viewed as having selected its policy before prices are determined, and it is taken for granted that only equilibrium prices occur.[18] In equilibrium, demand equals supply in all markets, including those for government debt.[19] We think of non-Ricardian policies as corresponding to the type contemplated in the Ramsey literature.

In the Ramsey literature, there is a concern that policies may not be time consistent, in the sense that they are not consistent with the government's incentives to implement them in real time. We believe these concerns may also apply to the FTPL. Consider again the situation in which the price level rises when there is a bad shock to the government budget constraint. In this situation, private agents may suspect the government will resort to high prices as an easy way to renege on debt. In this case, the policy backfires, with agents refusing to accumulate government debt in the first place. To avoid this outcome, it is necessary to convince potential holders of government debt that they will receive subsidies when good things happen to the government constraint.[20] However, there may be times and places in which the institutional and other social structures needed to achieve the required degree of credibility do not exist.

The remainder of this review is organized as follows: Part II makes most of our points in a one-period environment. By adopting such a simple setup we are able to get to the basic ideas without technical complications. At the same time, some issues simply cannot be discussed in a one-period environment; we defer these to part III. Part IV presents a simple model for thinking about the desirability of price fluctuations under the FTPL in an environment with no government-provided money. Part V provides concluding remarks.

■ **16** For a recent analysis of the case in which the interest rate is not pegged but is allowed to move around with variations in the state of the economy, see Benhabib, Schmitt-Grohe, and Uribe (2000).

■ **17** Sims (1999) and Woodford (1998a) emphasize these parallels.

■ **18** Implicitly, we are taking a particular stand on what constitutes government policy in a Ramsey–optimal policy setting. Strictly speaking, Ramsey theory tells us only what government actions are taken in the best equilibrium. There may well be many government policy rules that result in the same Ramsey equilibrium, where a policy rule specifies government actions as a function of the realization of exogenous and endogenous variables. Different policy rules imply different policy actions out of equilibrium. Woodford (1998a, 1999) clarifies the distinction between the government's policy rule and the Ramsey-equilibrium outcomes. He computes Ramsey-equilibrium outcomes and then searches for Taylor-like interest rate rules which support those outcomes as an equilibrium. In this paper, we take the position that the government's policy actions in a Ramsey equilibrium are the government's policy rule.

■ **19** In Ramsey theory and in the FTPL, market prices exist where government policy commits to infeasible actions. For example, there may be prices where the government commits to paying for goods with money financed from new debt issues, which no one buys. By focusing on equilibrium prices only, standard practice ignores these possibilities. Bassetto (2000) argues this is a mistake and proposes alternative equilibrium concepts to deal with the problem.

■ **20** See Chari, Christiano, and Kehoe (1991) for a detailed analysis.

II. Fiscal Theory in a One-Period Economy

The FTPL is defined by its non-Ricardian assumption on fiscal policy. The best way to understand this assumption and to quickly get to the heart of the FTPL is to examine a one-period model.[21] That's what we do here.

We begin with the conventional wisdom, the classic Sargent and Wallace (1981) analysis. We go on to explain how the FTPL differs from this conventional wisdom: Sargent and Wallace adopt the Ricardian view, while the FTPL adopts the non-Ricardian view of policy. We explore several interpretations of these assumptions and conclude that the non-Ricardian assumption requires that the government is able to commit to its policy actions in advance. We then ask, how can we assess the empirical plausibility of the non-Ricardian assumption? Finally, we explain how the FTPL can be used to study the price level in an economy with no government-provided fiat money.

Sargent and Wallace's Unpleasant Monetarist Arithmetic

Suppose that in the morning of the only day in this model, private agents hold a given amount of government debt, b. Here and throughout this review, we assume government debt is non-negative: Agents cannot borrow from the government. In the Sargent and Wallace model, debt is fixed in real terms; it represents a commitment to pay a fixed real amount of goods—for instance, corn.

The government's budget constraint is given by

$$b' + s^f + s^m = b.$$

The left and right sides of this equation summarize the sources and uses, respectively, of corn to the government. The first source of funds, b', is corn the government receives from households that purchase new debt in the evening. The second term, s^f, denotes taxes net of spending, and the third term, s^m, is seignorage from government-supplied fiat currency. The right side of the budget constraint, b, is the principal and interest on past government debt.

Optimizing households will obviously never choose $b' > 0$, and they are constrained from setting $b' < 0$ by assumption. Therefore, household optimization implies that b' must be zero. By imposing this result, we get the intertemporal government budget equation,

(2.1) $$b = s^f + s^m.$$

Sargent and Wallace's main conclusions can be understood from this equation. Suppose a "loose" fiscal policy is adopted—that is, s^f is reduced. Simple arithmetic dictates the monetary authority must increase s^m. Under normal circumstances, this translates into an increase in inflation.[22] In a multiperiod model, there is some discretion over timing. The rise in inflation can occur sooner or later, or it could be spread out over time. Whatever the timing, though, if the fiscal authority reduces s^f, the arithmetic necessitates inflation must go up at some point. Hence the title of Sargent and Wallace's famous paper, "Some Unpleasant Monetarist Arithmetic."

The same arithmetic suggests a solution to the inflation problem: Design central banks so they can credibly commit to not "caving in" to an irresponsible fiscal authority that sets s^f too low. Governments around the world have sought to implement this solution by making central banks independent and directing them to assign a high priority to inflation. With the monetary authority completely committed to a fixed value for s^m, the arithmetic forces the fiscal authority to adopt a fiscal policy consistent with that s^m. This is the basis for the current conventional view that to achieve a stable price level, it is sufficient to have a tough, independent central bank that is focused on prices.

The Fiscal Theory

According to FTPL advocates, the Sargent and Wallace framework may not be relevant for economies like the United States. In practice, government debt is a commitment to deliver a

21 We are grateful to Marco Bassetto for suggesting this to us.

22 By "normal," we mean that the economy is on the "right" side of the Laffer curve. Seignorage is the nominal increase in the money stock divided by the price level. A convenient formula becomes available if we temporarily reinterpret our model as a multiperiod model in a steady state. Let the demand for real balances be $m = \exp(-\alpha\pi)$, where $\alpha > 0$, π is the gross inflation rate from this period to the next, and m is the money stock divided by the price level. Seignorage, then, is just $m(1-1/\pi) = \exp(-\alpha\pi)(1-1/\pi)$. For inflation rates below $\pi^* = \left(\alpha + \sqrt{\alpha^2 + 4\alpha}\right)/(2\alpha)$, seignorage is increasing in inflation, and for inflation rates above this point, seignorage is decreasing. The "right" side of the Laffer curve refers to inflation rates below π^*.

certain amount of domestic currency, not goods. This creates new possibilities.

Let's revise the previous analysis, replacing b with B, nominal debt. The government's budget constraint becomes

$$(2.2) \quad B' + P(s^f + s^m) = B.$$

As before, optimizing households will not buy government debt in the evening. With demand at zero, equilibrium can be reached only at $B' = 0$. Equation (2.2), with $B' = 0$, is the government's intertemporal budget equation,

$$(2.3) \quad B = P(s^f + s^m).$$

Now, P is an endogenous variable. If the fiscal authority makes s^f small, there is no arithmetic to compel the monetary authority to raise s^m. If the monetary authority holds fast to s^m while the fiscal authority reduces s^f, the equation can be satisfied as long as P jumps. This is what FTPL advocates expect would happen.

Interpreting Ricardian and Non-Ricardian Fiscal Policy

At this point, we must clarify two key concepts. Fiscal and monetary policy are said to be *non-Ricardian* if $s \equiv s^f + s^m$ is chosen in a way that does not guarantee the intertemporal budget equation (2.3) is satisfied for all possible prices. In contrast, s is a Ricardian fiscal policy if it is chosen so that the intertemporal budget equation is satisfied no matter what P is realized. In our single-period model, this can happen only if s is a particular function of the price level, $s(P) = B/P$. The assumption that fiscal and monetary policy are non-Ricardian defines the FTPL.

How are we to interpret non-Ricardian policy? In principle, two interpretations are possible. The first may seem natural, initially; however, on further reflection it makes no sense. In this interpretation, the government is unconcerned with the intertemporal budget equation when it chooses s: Either it is unaware of its existence, or it simply does not care. If the government were completely unconcerned with intertemporal budget balance, it would be impossible to understand why we have taxes. Absent concerns that stem from the existence of the intertemporal budget equation, borrowing is always more appealing than raising taxes because the latter produces deadweight losses. If governments didn't raise taxes, however, s would be negative and there would be no positive value of P to satisfy the intertemporal budget equation. If we adopt this interpretation of non-Ricardian policy, the apparent existence of equilibrium is a puzzle. This interpretation deserves no further consideration.

Can the government's concern for intertemporal budget balance be reconciled with the notion that s is set exogenously, not as a function of P? Yes, if we imagine the government commits to s in advance, before P is determined. We can illustrate this in two ways. The first is based on the parable of the Walrasian auctioneer, who helps the economy find the equilibrium price level. Under the non-Ricardian assumption, the government announces s before the Walrasian auctioneer finds the market-clearing price level. When the government selects s, it fully understands that households will buy zero B' in equilibrium. However, because of its first-move advantage, the government knows it can force the auctioneer to choose P so that $P = B/s$.

Our second illustration is drawn from everyday life. A pedestrian who wants traffic to stop at a crosswalk will sometimes step into the street, making a show of being unconcerned about oncoming cars. Is such a person really unconcerned with the prospect of being struck and killed? Of course not. He expects the oncoming cars, seeing his commitment to cross regardless of the consequences, to stop rather than suffer the horror of an accident. Under a non-Ricardian fiscal policy, the government's approach is analogous to that of the pedestrian. The government's "policy" is simply an action, s. In principle, a value of P could occur that would put the government in the fiscally explosive situation of offering debt that the market refuses to absorb—that is, $B' > 0$. However, if the market is completely convinced of the government's commitment to s, then, like the car that stops for the pedestrian, the market will generate a value of P to guarantee debt is not excessive (in this case, "excessive" simply means greater than zero). The non-Ricardian government banks on the idea that the market abhors non-equilibrium P as much as drivers abhor hitting pedestrians.[23]

Although the word "commitment" in this context is consistent with the technical economics literature, it may nevertheless confuse the reader because it has so many meanings in everyday language. By saying the government has commitment, we mean only that it moves first, before prices are set. We do not mean to imply the government's motives are laudable, or its ability to move first reflects strength of character on the part of policymakers. For example, a government that is perpetually in gridlock because legislators cannot reach agreement acts with commitment, in our usage of the term.[24]

Now consider Ricardian policies. For the purpose of our analysis, we take no position on the relationship between these policies and the government's ability to commit. Still, we suspect that Ricardian policies are consistent with any degree of commitment.

Although Sargent and Wallace don't use this language, it seems fair to say they adopt a Ricardian specification of policy. If we think of their analysis as applying to a realistic modern economy, then we must think of real government debt in their model (that is, b in equation [2.1]) as B/P. For different values of P, the value of b changes, leading to adjustments in $s^m + s^f$ under the Sargent and Wallace analysis. Therefore, we interpret Sargent and Wallace as adopting the Ricardian assumption.

Is the FTPL *Sensible?* An Analogy to Microsoft

Under the FTPL, the price level is determined by equation (1.1) or equation (2.3). A reasonable question at this point is, is there any sensible interpretation of the FTPL? At first glance, determining P in this way may seem like accounting gimmickry without substantive interest. But this is not the case. As Cochrane (2000) emphasizes, the price of Microsoft shares is determined the same way! Under the FTPL, the government's relationship to its bondholders is somewhat like Microsoft's relationship to its equity holders.

Microsoft works to set aside real output for equity holders, though its motives for doing so are different from the government's. Microsoft does not calibrate its dividend stream to guarantee the present-value formula for its stock price will hold for all possible stock prices. Instead, the mechanism operates in precisely the opposite direction: Market traders forecast what Microsoft will generate for them, then calculate the ratio of that amount to the number of shares outstanding, and that's the stock price! FTPL advocates argue the price level in an actual economy is determined in exactly the same way. The government does not calibrate s to ensure its present-value budget equation (2.3) for all values of P. Instead, bondholders figure out how many goods (s) the government is setting aside for them and then calculate the price level as the ratio of B to s.

Is the Non-Ricardian Assumption Empirically Plausible?

In assessing the FTPL as a positive theory for a particular time period, the plausibility of the non-Ricardian assumption must be considered. A simple examination of time-series data will not help. Under both the non-Ricardian and the Ricardian assumptions, we expect to see $s = B/P$. The only direct way to distinguish the two assumptions is to see how s responds when the economy is out of equilibrium. According to the Ricardian assumption, s adjusts with P to

23 As the analogy suggests, there can be trouble if commitment is not credible. If the oncoming traffic is not completely convinced of the pedestrian's commitment (drivers believe the pedestrian is sneaking glances at the oncoming traffic, ready to make adjustments in case something goes wrong), then miscalculations can lead to a tragic collision. We argue (see especially "Summary" on page 24) that this is possible if private agents are not completely convinced of the government's commitment to non-Ricardian fiscal policy. In this case, markets might produce the "wrong" prices, leading to excessive government debt in that the private sector refuses to purchase it. Buiter (1999) seems to be concerned with this kind of outcome: He refers to the "painful" fiscal adjustments that must be made when the "Ricardian reality dawns" and the private sector refuses to buy government debt. We do not mean to suggest that catastrophe will always occur if there is uncertainty about government policy. As "The FTPL with Stochastic Fiscal Policy" (page 15) shows, the non-Ricardian assumption is perfectly consistent with stochastic fiscal policy.

24 In private communication, Christopher Sims pointed out that our notion of commitment encompasses "the commitment of two pedestrians who enter a crosswalk while engaged in a fist fight. Few doubt that they are not watching traffic."

preserve $s = B/P$. According to the non-Ricardian assumption, s is like a utility-function parameter: Its value remains unchanged, so that $s \neq B/P$ out of equilibrium. This sounds like an easy thing to check—just compare s and B/P out of equilibrium. The problem is that, according to the theories considered here, only equilibrium values of s are recorded in the data.[25]

This does not mean there is no way to choose between the non-Ricardian and the Ricardian assumptions. In fact, we think there are two ways to go. One is to extrapolate what is reasonable out-of-equilibrium behavior, based on what we see in equilibrium.[26] Another way is to view the FTPL as a starting point for a natural set of auxiliary assumptions that restrict time-series data and then test those assumptions.[27] If the non-Ricardian assumption leads to a useful set of theories, this would tip the balance in favor of that assumption. We now discuss these two approaches.

Extrapolating Out-of-Equilibrium Behavior from Equilibrium

According to the non-Ricardian assumption, the government's policy is a commitment to a particular action, s. Under the Ricardian assumption, policy is a strategy for setting s as a function of real debt. If governments directly recorded their policies in writing, we could better discriminate between the two assumptions. There are two situations where this seems to have occurred, and, with one important caveat, the results appear to favor the Ricardian over the non-Ricardian assumption. The Maastricht Treaty requires members of the European Union to adjust their fiscal variables when their real debt gets too large. The IMF works in the same way, pressuring its members to adjust fiscal variables if their real debt gets out of hand. We think it is fair to say that if a non-equilibrium P were somehow called out, these arrangements would generate an adjustment in s. Casual examination of the (admittedly, equilibrium) time-series data suggests the same. In practice, when the debt gets large, political pressures come into play to adjust the surplus to bring the debt back in line. This happened in the United States in the late 1980s and 1990s, when the federal debt began to grow significantly, producing political support for raising taxes and/or reducing spending.

Now, for the caveat: These examples suggest the non-Ricardian assumption may be an implausible characterization of current policy in Europe, the United States, and some emerging-market economies. However, as our introduction emphasized, these examples do not establish the non-Ricardian assumption as implausible for *all* times and places.

Is the Non-Ricardian Assumption a Good Starting Point?

Another way of assessing the empirical value of the non-Ricardian assumption asks how good a platform it is for developing useful, testable restrictions. Space does not permit us to pursue this idea here, beyond mentioning that interesting work is under way. In particular, Canzoneri, Cumby, and Diba (1998), Cochrane (1998a, b), Loyo (1999), and Woodford (1998b) have pursued the assumption of statistical exogeneity of the government surplus. This is not an implication of the non-Ricardian assumption per se, though that assumption does naturally lead one to it.

This approach can best be understood by an analogy attributed to Benjamin Friedman (see Cochrane [1998a]). Consider the equation of exchange, $MV = PY$, where M is money, V is velocity, and Y is output. As it stands, this equation has no testable implications; without additional assumptions, it simply defines V. Still, if incorporating simple, plausible assumptions converts the equation into a theory that allows us to understand the data better, then the equation of exchange is empirically useful.[28] Similarly, the non-Ricardian assumption may be a good starting point for identifying simple auxiliary assumptions that convert the FTPL into a useful, testable theory. If so, this would help vindicate the non-Ricardian assumption as a useful empirical assumption.

25 This result holds even if there are multiple periods and uncertainty.

26 There are examples of models in which the equilibrium time-series data contain information about what happens out of equilibrium. For example, in Green and Porter (1984), limited information has the consequence that events occur in equilibrium that are observationally equivalent to agents' having deviated from the equilibrium. Although agents don't actually deviate in the equilibrium, they must, nevertheless, be punished as though they had as a credible signal of what would happen if a deviation really did occur. In this sense, the events in equilibrium provide evidence of what would happen out of equilibrium.

27 For a thorough discussion of this strategy, see Woodford (1998b).

28 An example of such an assumption is the specification that V has a simple functional relationship to the nominal rate of interest.

Although we are inclined to be skeptical of the non-Ricardian assumption, the FTPL is still very much in its infancy. It remains to be seen where the FTPL will take us and what observations it will help us to explain. The initial results are promising, though not uncontroversial. Cochrane (1998a,b) argues that an FTPL that assumes a statistically exogenous surplus process helps us understand the dynamics of U.S. inflation in the 1970s, and Loyo (1999) argues that it is useful for understanding Brazil's high inflation in the 1980s.

Another literature, begun by Calvo (1978), Kydland and Prescott (1977), and Barro and Gordon (1983), posits that the absence of commitment in government policy can account for the high-inflation episodes mentioned in the previous paragraph. One way to assess the FTPL is to compare its ability to account for such experiences with that of the time-consistency literature. The outcome of this comparison is not obvious. McCallum (1997), among others, argues that time inconsistency is *not* a useful explanation for high-inflation episodes. Ireland (1998) argues the other way, that absence of commitment is useful.[29]

The Price Level in a World with No Government-Provided Money

Some FTPL advocates claim that an important virtue of the theory is that it provides a way of thinking about the price level that works even in a world where supply and demand for government fiat money are nonexistent. Cochrane (1998a, 2000) argues this is of interest because, to a first approximation, we have already reached that point.[30]

The basic pieces of the argument are already in place: Under the non-Ricardian assumption that $s^f + s^m$ is exogenous, equation (2.3) determines the price level. This conclusion can be reached without reference to money or whether it is even present in the economy. That's the tip-off for the result to come: The price level can be pinned down, even if there is *no* government-provided money in the economy. To see this, imagine that trade in the economy is carried out by barter. Equivalently, one could think of a scenario in which trades are financed with the exchange of financial claims on privately held assets. These trades could even be denominated in "dollars," even though government-provided money ("dollars") does not exist.

What is nominal, dollar-denominated government debt in this world with no dollars? Clearly it is not a pledge to deliver government-provided money, because there is none! Instead, it is a pledge to deliver B dollars' worth of goods to the bearer of B. The formal obligation leaves open exactly how many goods B dollars corresponds to, because the price level is unspecified. In this sense, it is like real-world U.S. government debt.[31] The price level that is realized is determined by the government's fiscal decisions. Fiscal decisions result in real surplus, s, which is what the government actually has available for paying off bondholders. With the amount of goods available to pay bondholders equal to s and the nominal value of debt equal to B, the natural definition of the price level is $P=B/s$.

At this point, the price level in a world without government-provided money may seem a useless appendage. In part IV, we shall see that the price level in such an economy can play an important role, helping to implement an efficient fiscal policy.

29 For further discussion, see Albanesi, Chari, and Christiano (1999) and Christiano and Gust (2000a,b).

30 According to Woodford (1998a, b, c), the assumption that money demand and supply are *literally* nonexistent is too extreme. He prefers to analyze a "cashless limit." This is an economy in which the demand for money is so small that seignorage is negligible and can, to a first approximation, be safely ignored in the government's budget constraint (both the flow-budget constraint and the intertemporal budget equation). However, the demand for money is sufficiently large that the central bank can still control the rate of interest.

31 The U.S. government also offers indexed debt, which corresponds to a commitment to deliver a specific amount of a basket of goods. Indexed debt is a small portion of the government's portfolio of liabilities.

III. Fiscal Theory in General Equilibrium

Here we address issues that could not be addressed in the one-period example. The first issue will likely concern any reader who has made it this far. Part II showed how an equation that is not usually used in the context of price determination—the government's intertemporal budget equation—can pin down the price level. But don't we already have a theory of the price level? If we adopt the non-Ricardian assumption on policy, won't the price level be *overdetermined?* It might be, depending on how we flesh out the rest of the economy. If the price level were overdetermined, there would be no equilibrium, except in the fortuitous case in which the government happens to pick just the right value for s. If this were the situation for all reasonable ways of modeling the rest of the economy, the FTPL would be in trouble: It would not be a logically coherent macroeconomic model. But this is not the case. In the following sections, we flesh out the economy in what appears to be a reasonable way, and we find the price level is uniquely determined. (This issue is addressed more rigorously in the appendix.)

We then turn to an issue of greater concern. We present evidence suggesting that to use the FTPL, one must take the non-Ricardian assumption *very* seriously. Seemingly minute departures from that assumption—in the direction of allowing for some sensitivity in the surplus to the real debt—collapses the FTPL's ability to pin down the price level.

The final section revisits the central bank's ability to control average inflation under the FTPL. It shows that conventional views about how to control average inflation could actually spark an explosive hyperinflation under the FTPL. So, although the central bank can feasibly control average inflation, its method of doing so must be designed with care.

Is the Price Level Overdetermined in the FTPL?

We begin this section by providing a general discussion of the issues involved in determining the price level. We then turn to a specific example in which the price level is uniquely determined by the FTPL.

General Discussion

It is easy to find examples of the FTPL in which the price level is overdetermined. Recall the equation of exchange, discussed previously. For convenience, we reproduce it here: $MV = PY$. In traditional, old-fashioned monetarism, V is assumed to be fixed by technology, Y is determined exogenously, and monetary policy takes the form of a choice of M. In this model, P is obviously determined by the equation of exchange. If the rest of the economy were characterized by these assumptions, then a logically coherent FTPL would be impossible.[32]

Each of these assumptions, however, has been rejected on empirical grounds. First, V exhibits substantial fluctuations in the data; the assumption that V is fixed is replaced in modern models by the assumption that V is an increasing function of the nominal rate of interest. Additionally, expected inflation plays an important role in determining R. With these two features, a logically coherent FTPL is possible. These changes cause expected future values of P to enter the equation of exchange through V. This creates the possibility that there are many P processes that can satisfy the equation, leaving room for the non-Ricardian assumption to pin down one of them. This possibility is illustrated through an example in the appendix.[33]

Second, the assumption that Y is exogenous has been questioned. There is general agreement that at least short-run movements in Y are influenced by movements in V, P, and M. When models are constructed to capture this, expected future values of P enter into the determination of Y. As in the example of the

■ **32** Finite-horizon models in which the simple quantity equation holds in the last period, but in which velocity is an increasing function of the nominal interest rate in the previous periods also have the property that the price level is overdetermined (see Buiter [1999]). The reasoning is similar to that used in our text. In those models, the equilibrium price level must satisfy a first-order difference equation. All equilibrium prices are then pinned down by the fact that the price level is pinned down in the last period. The likelihood that this price level coincides with the one produced by the intertemporal budget equation of the government seems small. The most likely outcome is that the price level will be overdetermined.

■ **33** Brock (1975), Obstfeld and Rogoff (1983), and Matsuyama (1991) present similar examples. An example taken from Woodford (1994) can be found in the appendix.

previous paragraph, there can be multiple P processes that satisfy the equation of exchange. Again, this leaves room for the non-Ricardian assumption to pin down one of them.[34]

Third, there is a nearly universal consensus that exogenous M poorly characterizes monetary policy. For example, Taylor (1993) has argued that, in practice, monetary policy is best thought of as a rule for setting the rate of interest. In this case, M becomes an endogenous variable. We can see in the equation of exchange that if R is the exogenous policy variable (as opposed to M), then V is pinned down. But now there are two endogenous variables, M and P, in this equation. Generally, under these circumstances, P and M are not pinned down. There is, in a sense, a missing equation. Again, there is room for the FTPL to fit in.

An Example

Next, we present a simple, multiperiod model economy in which the price level is uniquely determined in the FTPL. There is no last period, and time is indexed by $t = 0, 1, 2, \ldots$. Suppose that output, Y, is the same for each date, t. Money demand depends on the rate of interest,

$$(3.1) \quad \frac{M_t}{P_t} = AR_t^{-\alpha}, \ \alpha > 0.$$

The parameter A captures other factors (like income) that affect money demand but are assumed to be constant here. M_t is the money stock at the beginning of period t; P_t is the price level during period t; and R_t is the nominal rate of interest on government bonds held from the beginning of period t to the beginning of period $t+1$. The Fisher equation holds

$$(3.2) \quad 1+r = (1+R_t)\frac{P_t}{P_{t+1}}.$$

The expression on the right $(1+r)$ is the real rate of interest on bonds paying a nominal rate of return, R_t, and $r > 0$ is the rate at which households discount future utility. This pins down the real rate of interest.

A reasonable specification of monetary policy is that the central bank targets the nominal rate of interest. For purposes of exposition, we adopt an extreme version of this specification, in which the central bank pegs the rate of interest to a constant, $R > 0$. The central bank accomplishes this by supplying whatever amount of money the private economy demands at this rate of interest.

The interest rate peg pins down seignorage:

$$s_t^m \equiv \frac{M_t - M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{P_{t-1}}{P_t}\frac{M_{t-1}}{P_{t-1}}.$$

Imposing the money-demand and Fisher equations [(3.1) and (3.2)] and the policy rule $R_t = R$, we find[35]

$$(3.3) \quad s^m = AR^{-\alpha}\frac{R-r}{1+R}, \ t = 0, 1, 2, \ldots.$$

Consistent with the FTPL, we assume the primary budget surplus, s_t^f, is non-Ricardian. We adopt the simplest such policy, one in which s_t^f is simply a constant, s^f. Thus, net government revenues from all sources, excluding interest payments, are given by

$$(3.4) \quad s_t = s = s^f + s^m > 0.$$

To complete the description of the government, we present the period-t budget constraint. We assume government debt is composed of one-period discount bonds; that is, the amount of borrowing in period t is $B_{t+1}/(1+R)$, and the amount paid in period $t+1$ is B_{t+1}. The period-t government budget constraint is

$$(3.5) \quad \frac{B_{t+1}}{1+R} + P_t s = B_t, \ t = 0, 1, 2, \ldots.$$

The terms on the left of the equality represent the government's sources of funds, and the terms on the right denote the uses of funds to pay off the debt.[36] It is convenient to rewrite this expression in real terms, that is, in terms of $b_t \equiv B_t/P_t$. Dividing equation (3.5) by P_t, taking the Fisher equation (3.2) into account, and rearranging the terms, we obtain

$$(3.6) \quad b_{t+1} = (1+r)(b_t - s).$$

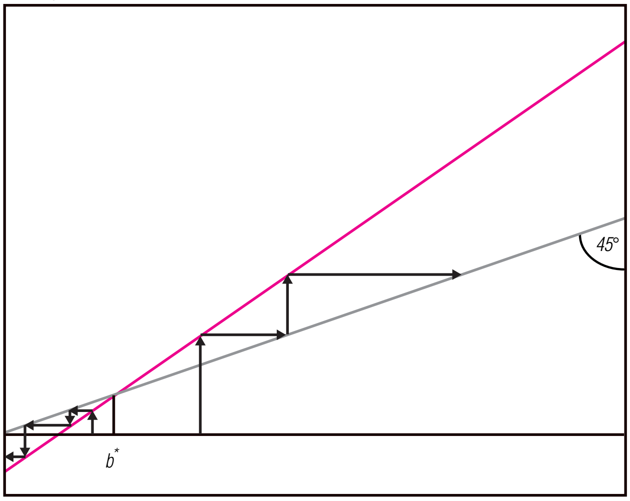
34 A cash-in-advance model displayed in Christiano, Eichenbaum, and Evans' (1998) illustrates this. Because this is a cash-in-advance model, velocity is fixed and the factors discussed in the previous paragraph are ruled out. Christiano, Eichenbaum, and Evans show that for different specifications of the monetary policy rule for selecting M_t, the model has a continuum of equilibria. If the non-Ricardian assumption were adopted in this model, the equilibrium would be pinned down. For other examples like this, see Carlstrom and Fuerst (1998).

35 For simplicity, we assume the interest rate peg was in place in period −1, too. A more rigorous treatment which does not make this assumption can be found in the appendix.

36 An alternative representation, which has some theoretical advantages, expresses the government's budget equation in terms of its total nominal liabilities, $B_t + M_{t-1}$. We work with this alternative representation in the appendix.

FIGURE 1

Debt Evolution Under Non-Ricardian Fiscal Policy



Finally, we develop the multiperiod analog of $B'=0$ in part II. Recall the logic we used there: First, $B'>0$ is not optimal, since households could increase utility by raising consumption and financing it with a reduction of B'. A negative value of B' is also not optimal, since we have removed it from the feasible set by assumption. We continue to assume that holdings of government bonds must be non-negative; that is, households only lend to the government, they do not borrow from it.

The analog of $B'=0$ in this setting is

$$\text{(3.7)} \quad \lim_{T\to\infty} \frac{B_T}{(1+R)^T} = 0.$$

We establish that household optimization implies this condition by the same reasoning used to establish $B'=0$. The limit cannot be positive, for otherwise households could increase utility by reducing their holdings of government debt. To see this, suppose the limit *is* positive. Eventually, government debt would grow at the rate of interest, that is,

$B_t = B_{t^*}(1+R)^{t-t^*}$, $t \geq t^*$ for some t^*.

At this point, the government is engaged in what is called a Ponzi scheme with households. The principal and interest on debt coming due are financed entirely and forever with newly issued debt. Under these circumstances, households can do better by saying no to the Ponzi game, consuming the principal and interest on debt coming due in one period and then never holding any more government debt. The household is better off, because the action allows a one-time increase in consumption without the need to reduce consumption at any other date. An optimizing household would not pass up an opportunity like this; therefore, household optimization implies the limit cannot be positive. But the limit cannot be negative either, because $B_t<0$ is not allowed. Equation (3.7) is called the *transversality condition*. It is convenient for us to express this condition in real terms, after substituting out for the nominal rate of interest from the Fisher equation (3.2). Using that equation, we find[37]

$$(1+R)^t = (1+r)^t \frac{P_t}{P_0}, \; t=1, 2, \ldots,$$

so that $B_T/(1+R)^T = P_0 b_T/(1+r)^T$. The transversality condition can then be written as

$$\text{(3.8)} \quad \lim_{T\to\infty} \frac{b_T}{(1+r)^T} = 0, \; b_T = \frac{B_T}{P_T}.$$

We have now stated the entire model. The household's part is given by equations (3.1), (3.2), and (3.8) and by the condition $B_t \geq 0$. The government is summarized by its policy, equation (3.4), and by its flow-budget constraint, equation (3.6). Does this economy uniquely determine the price level? To see that it does, first note that the money-demand equation and the government's policy of pegging the interest rate have the effect of pinning down real balances, but not M or P separately. Double M and P, and those equations remain satisfied. The same is true of the Fisher equation: Double P at all dates, and it continues to hold, too. So, the level of the money stock and the price level are not pinned down. It turns out that the non-Ricardian specification of government policy, together with the household's transversality condition, is sufficient to pin down the price level uniquely.

To see that the price level is uniquely determined, consider figure 1, which illustrates the government budget equation, $b' = (1+r)(b-s)$. The vertical axis measures b' and the horizontal axis measures b. (The 45-degree line is included in the figure for convenience.) The intercept for the budget equation is negative, and it cuts the 45-degree line from below. Its slope is steeper than 45 degrees because we assume $r > 0$.

37 For example, for $t = 2$,

$$(1+R)^2 = \left[(1+R)\frac{P_1}{P_2}\right]\left[(1+R)\frac{P_0}{P_1}\right]\frac{P_2}{P_0} = (1+r)^2\frac{P_2}{P_0}.$$

Figure 1 shows what happens to b over time for any initial value of b.[38] Denote the value of b where the budget equation intersects the 45-degree line by b^*,

$$(3.9)\quad b^* = \frac{1+r}{r}\, s = \sum_{t=0}^{\infty} \frac{s}{(1+r)^t} .$$

As the last equality indicates, b^* is the present value of future surpluses.

The value of b in period 0, b_0, is now an endogenous variable. Although the nominal debt, B_0, is predetermined at date 0, the price level is not. Consider three possibilities: Suppose $0 \le b_0 < b^*$. Figure 1 indicates that b quickly spirals into the negative zone, violating the non-negativity constraint on the household's holdings of debt.[39] Next consider $b_0 > b^*$. In this case, figure 1 indicates the debt diverges to plus infinity. To see how the debt's growth rate evolves, divide equation (3.6) by b_t:

$$\frac{b_{t+1}}{b_t} = (1+r)\left(1 - \frac{s}{b_t}\right).$$

As b_t grows, s becomes relatively small, and the growth rate of b_t eventually converges to $1+r$. At this point, the debt becomes so large that s is, by comparison, insignificant. The government is now running a Ponzi scheme. For the reasons we have given above, it is not in the household's interest to participate in this scheme (technically, the household's transversality condition, equation [3.8], is violated). Since households will not hold this debt, we conclude that all $b_0 > b^*$ do not correspond to equilibria.

This leaves only $b_0 = b^*$ to consider. Since the level of real debt is fixed in this case, the transversality condition is now trivially satisfied. Thus, only $P_0 = B_0 / b^*$ is consistent with equilibrium. We conclude this version of the FTPL is an internally consistent theory of the price level.

Is the FTPL Fragile?

The assumptions underlying economists' theories are, at best, only approximations. We don't think of them as being *exactly* true. Therefore, we trust theories more if their central implications do not change when we alter the assumptions a little. But, if key implications evaporate with small changes—particularly changes that are arguably in the direction of greater empirical plausibility—then there is reason for concern. In this case, we say a theory is *fragile*.

Here, we describe one concern about the fragility of the FTPL, based on Canzoneri, Cumby, and Diba (1998). We show that small, plausible perturbations of non-Ricardian policy collapse the FTPL's ability to pin down the price level.[40] Consider the following alternative to the canonical non-Ricardian policy of setting s to a constant. Suppose $s = \varepsilon b$, where $0 < \varepsilon \le 1$. With this policy, $b' = (1+r)(1-\varepsilon)b$, or

$$\frac{b_t}{(1+r)^t} = (1-\varepsilon)^t \frac{B_0}{P_0},$$

so that the transversality condition is satisfied for all $P_0 > 0$. Clearly, this is a Ricardian policy; the FTPL does not pin down the price level. Now, this policy may appear to be a significant perturbation of the policy $s_t = s$. Perhaps so, but it has close cousins in which the perturbation appears to be much smaller.

Consider the following alternative to the canonical non-Ricardian policy:

$$(3.10)\quad s_t = \begin{cases} -\dfrac{\xi}{1+r} + \dfrac{1+r-\gamma}{1+r}\, b_t & b_t > \bar{b}, \\ s & b_t \le \bar{b} \end{cases}$$

where

$$0 \le \gamma < 1,$$

$$\frac{1+r}{r}\, s < \bar{b} < \frac{\xi}{1-\gamma} .$$

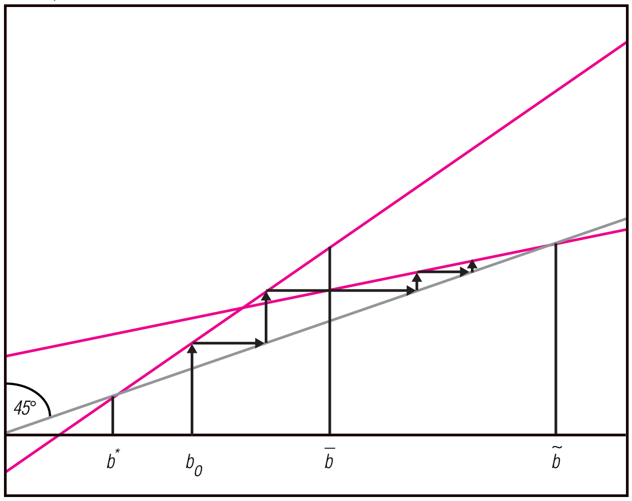
In this case, as long as real debt remains below some upper bound, $\bar{b}$, then the policy is the constant-surplus policy that we have been analyzing. But, as soon as b_t exceeds $\bar{b}$, fiscal policy adjusts to bring the debt back in line.

38 To see this, specify an initial value for b on the horizontal axis. Proceed vertically to the budget line, move horizontally to the 45-degree line, move vertically to the budget line, and so on.

39 Implicitly, we have ruled out the possibility that a negative b implies a negative P. This cannot be, since P_0 is positive for $0 \le b_0 < b^*$. The Fisher equation (3.2) then pins down the price path for P_t and cannot produce a negative P_t if $P_0 > 0$.

40 Woodford (1998a) argues there may be a local sense in which the FTPL's ability to pin down the price level survives the sort of perturbations we consider here. He also discusses versions of the model with learning which may survive these perturbations.

FIGURE 2
Debt Evolution Under Perturbed Fiscal Policy



Although the algebraic representation of this policy may seem forbidding, it is easy to analyze it with the help of figure 2. For $b_t > \bar{b}$, real debt evolves according to $b_{t+1} = \gamma b_t + \alpha$.

If real debt followed this equation forever, it would eventually converge to $\tilde{b} = \xi/(1-\gamma)$. This equation, as well as equation (3.6), is graphed in figure 2.

Figure 2 simulates the evolution of real debt under the fiscal policy in equation (3.10). The simulation is initiated with the indicated value of b_0. Real debt initially follows the steep line with slope $1+r>1$ until it passes $\bar{b}$, at which point it follows the flatter line with slope $\gamma<1$. All paths with $b_0 \geq b^*$ are consistent with the transversality condition because they converge to a finite value, either b^* or $\tilde{b}$. With the given change in policy, the FTPL cannot pin down the price level.

This perturbation of the non-Ricardian policy seems realistic. At low levels of debt, fiscal policy is exogenous, as it is in the canonical non-Ricardian policy. If the debt gets out of line, then fiscal policy adjusts to bring it under control. This rings true in light of the U.S. experience in the 1980s and 1990s and the provisions of the Maastricht Treaty, which limit the real debts of European Union member countries.

The FTPL with Stochastic Fiscal Policy

Thus far, we have illustrated non-Ricardian fiscal policy with $s_t = s$, a constant. But the essence of non-Ricardian fiscal policy is simply that s_t is *not* calibrated to satisfy the intertemporal budget equation for all prices; it is compatible with a much larger class of specifications for s_t than $s_t = s$. Here, we study non-Ricardian policies in which surpluses, s_t, are random. We use this specification to make three points.

First, Barro's (1979) famous policy of absorbing fiscal shocks by raising taxes in the future can be represented as non-Ricardian fiscal policy.[41] This is an important example, partly because it clarifies the definition of non-Ricardian policy as it is used in the FTPL. Clarification is necessary because one might mistakenly attach other meanings to the term "non-Ricardian," based on economists' everyday usage of the term "Ricardian."

Second, unless policy takes the form advocated by Barro, fiscal shocks cause the inflation rate to fluctuate randomly about its average. The average value of inflation is determined by the value of the monetary authority's interest rate peg.

Third, we describe an important result from Woodford (1996, 1998a). He shows that under the FTPL, instability in fiscal policy *must* affect the price level, no matter how committed the monetary authority is to price stability.[42] We call this striking result "Woodford's *really* unpleasant arithmetic," to contrast it with Sargent and Wallace's famous title. Woodford's arithmetic is even tougher than that of Sargent and Wallace, who argue that if the central bank is weak, then the fiscal authority can push it into producing price instability. However, Sargent and Wallace's pessimistic ("unpleasant") conclusion is balanced by their optimism that, if the central bank just hangs tough, the problem of price stability will be solved. From the perspective of the FTPL, Woodford argues that no matter how tough the central bank is, it still cannot stabilize the price level.[43]

41 It seems obvious that the Barro policy can also be represented as Ricardian, so we do not discuss it here.

42 Implicitly, we have in mind non-Ricardian policies other than those advocated by Barro (1979).

43 Recall, however, our second point, that the central bank *can* control the average inflation rate.

Random Fiscal Policy

Suppose the surplus obeys the first-order autoregressive representation,

$$(3.11)\quad s_{t+1}=(1-\rho)s+\rho s_t+\varepsilon_{t+1}\,.$$

In this equation, ε_{t+1} is an independently and identically distributed white-noise process independent of s_{t-j}, $j\geq 0$. A positive realization of ε_t induces a change in the date-t government surplus and in the expected value of future government surpluses. Let ψ_j denote this effect at date $t+j$ for $j\geq 0$:

$$(3.12)\quad \psi_j\varepsilon_t=E_t s_{t+j}-E_{t-1}s_{t+j},\ j\geq 0,\ \psi_0\equiv 1.$$

E_t denotes the expectation operator, conditional on information available at date t ($E_t s_t=s_t$). When the surplus has the time-series representation, equation (3.11), then $\psi_j=\rho^j$. Of course, equation (3.12) applies more generally, even when s_t does not have the time-series representation given in equation (3.11). The present value of ε_t's impact on current and expected future surpluses can be defined as

$$\psi\left(\frac{1}{1+r}\right)\varepsilon_t\equiv\varepsilon_t+\frac{\psi_1}{1+r}\varepsilon_t+\frac{\psi_2}{(1+r)^2}\varepsilon_t+\ldots.$$

(Note here that $\psi(\cdot)$ is a function.) In the case of equation (3.11), this is

$$\psi\left(\frac{1}{1+r}\right)=\frac{1+r}{1+r-\rho}.$$

When $\rho=0$, so that s_t is independently and identically distributed, then the present-value term is just unity. In this case, the effect of an innovation in the surplus is limited to the current surplus only. As ρ increases above zero, then the present-value term increases to take into account the future effects of innovation. Negative values of ρ cause the present-value terms to fall as innovations in the current surplus generate expected reductions in the future surplus.

It is interesting to compare the fiscal policy considered in equation (3.11) with that advocated by Barro (1979). He argues that a negative shock to government finances (due, for instance, to war) should be met by a large increase in debt, coupled with a constant increase in the labor tax rate that is sufficient to pay off the interest and principal on that debt over time. In particular, he advocates fiscal policies of the form

$$\psi\left(\frac{1}{1+r}\right)=0\,.$$

For example,[44]

$$\psi_0=1,\ \psi_1=-(1+r),\ \text{or,}$$

$$\psi_0=1,\ \psi_i=-\left(\frac{1+r}{2}\right)^i,\ i\geq 1;$$

that is,

$$s_t=s+\varepsilon_t-(1+r)\varepsilon_{t-1},\ \text{or,}$$

$$s_t=s+\varepsilon_t-\sum_{i=1}^{\infty}\left(\frac{1+r}{2}\right)^i\varepsilon_{t-i}\,.$$

These examples head off misunderstandings about the definition of "non-Ricardian" policy. In everyday discussion, the word "Ricardian" is used in a variety of senses. For instance, economists may refer to a policy as Ricardian when a current tax cut is financed by increases in future taxes that are large enough in present value to match the current cut.[45] It is clear from the preceding discussion that this type of policy can be part of a non-Ricardian regime.

Under the fiscal policy just discussed, the price level is insulated from fiscal shocks. Shocks to the real primary surplus are financed by appropriate movements in the opposite direction later. In the next section, we will show that when $\psi\neq 0$, surplus shocks are at least partially financed by movements in the price level.

44 See, for example, Woodford (1998a), footnote 18.

45 Hayashi (1989) is one example.

Inflation with Random Fiscal Policy

We continue to assume that policy pegs $R_t = R$, so that the seignorage component of s_t is the constant value given in equation (3.3). As a result, the random nature of s_t in equation (3.11) reflects randomness in fiscal policy. The Fisher equation still holds, although it must be adjusted to take into account uncertainty,

$$1+r = (1+R)E_t \frac{P_t}{P_{t+1}},$$

where E_t is the conditional expectation, given information available at time t. This expression shows the central bank controls the expected rate of deflation through its choice of R. This translates into control over the average rate of deflation by the fact $E[E_t(P_t/P_{t+1})] = E(P_t/P_{t+1})$. Imposing the suitably adjusted version of the household's transversality condition, equation (3.8), on the government's flow-budget equation, the intertemporal budget equation becomes[46]

$$(3.13)\quad \frac{B_t}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{(1+r)^j} = s\left(\frac{1+r}{r}\right)\left(\frac{1-\rho}{1+r-\rho}\right) + \left(\frac{1+r}{1+r-\rho}\right) s_t .$$

We now have a completely specified theory of the price level and inflation. One way to understand it is to use the model to simulate a sequence of prices for a given realization of primary surpluses. Suppose we have a time series, $s_0, s_1, \ldots, s_T$, from equation (3.11) and an initial level of nominal debt, B_0.[47] P_0 is computed by evaluating equation (3.13) at $t = 0$. B_1 is then computed from the government's flow-budget equation, $B_{t+1} = (1+R)(B_t - P_t s_t)$, for $t = 0$. A sequence, $P_0, P_1, \ldots, P_T$, is obtained by performing these calculations in sequence for $t = 0, 1, 2, \ldots, T$.

The interest rate peg guarantees that the expected rate of inflation (actually, deflation) is constant in these simulations. As a result, the rate of inflation itself will be approximately uncorrelated over time, an artifact of the constant interest rate peg. If the interest rate rule were instead dependent on past interest rates and/or past inflation, then persistence would presumably appear in the model's inflation process.[48]

One can gain further insight into equation (3.13) by subtracting $E_{t-1}B_t/P_t$:

$$(3.14)\quad \frac{B_t}{P_t} - E_{t-1}\left(\frac{B_t}{P_t}\right) = \left(\frac{1+r}{1+r-\rho}\right)(s_t - \mathrm{E}_{t-1} s_t)$$

$$= \psi\left(\frac{1}{1+r}\right)\varepsilon_t .$$

This says that a date-t shock in the primary surplus induces a contemporaneous change in the real value of the debt equal to the present value of the shock.[49] Since B_t is predetermined at time t, the change is brought about entirely by a change in the price level.[50]

46 To see how this is derived, consider first the expression to the right of the first equality in equation (3.13). Note from the Fisher equation (3.2):

$$\frac{1}{P_{t+j}(1+R_{t+j})} = E_{t+j}\left[\frac{1}{(1+r)P_{t+j+1}}\right], \text{ all } t, j \geq 0.$$

Then, the government's flow-budget constraint can be written

$$\frac{B_{t+j}}{P_{t+j}} = s_{t+j} + \frac{B_{t+j+1}}{P_{t+j}(1+R_{t+j})} = s_{t+j} + \frac{1}{1+r} = E_{t+j}\frac{B_{t+j+1}}{P_{t+j+1}},$$

or, after applying the law of iterated mathematical expectations,

$$E_t \frac{B_{t+j}}{P_{t+j}} = E_t s_{t+j} + \frac{1}{1+r} E_t \frac{B_{t+j+1}}{P_{t+j+1}} \;(^{**}).$$

Substitute this, for $j = 1$, into the period-t flow-budget constraint of the government:

$$\frac{B_t}{P_t} = s_t + \frac{1}{1+r}\frac{B_{t+1}}{P_t(1+R)},$$

$$= s_t + \frac{1}{1+r} + E_t\frac{B_{t+1}}{P_{t+1}}.$$

Applying (**) repeatedly to this expression, for $j = 1, 2, \ldots$ results in the expression to the right of the first equality in equation (3.13), if we apply the transversality condition, $\lim_{T\to\infty} E_0 b_T/(1+r)^T = 0$.

To obtain the expression to the right of the second equality in (3.13), first solve equation (3.11) to find

$$E_t \frac{s_{t+j}}{(1+r)^j} = s\left[\left(\frac{1}{1+r}\right)^j - \left(\frac{\rho}{1+r}\right)^j\right] + \left(\frac{\rho}{1+r}\right)^j s_t,$$

for $j = 0, 1, 2, \ldots$ Then substitute this into equation (3.13) and apply the geometric sum formula.

47 It should be obvious how this procedure could be adapted to accommodate any other time-series representation for s_t.

48 Loyo (1999) emphasizes this in his discussion of the persistent rise in inflation observed in Brazil in the 1980s.

49 The analysis of price determination under the FTPL is similar to the analysis of consumption in the permanent-income hypothesis. See Christiano (1987).

50 Divide both sides of equation (3.14) by B_t and take into account that $E_{t-1}B_t = B_t$ to set

$$\frac{1}{P_t} - E_{t-1}\left(\frac{1}{P_t}\right) = \frac{1}{B_t}\Psi\left(\frac{1}{1+r}\right)\varepsilon_t .$$

Fiscal policies like equation (3.14) underscore the fact that movements in the price level are an alternative to Barro's way of financing shocks to the primary surplus. A jump in the price level acts as a capital levy on holders of government bonds, which helps to finance government spending just as surely as the sort of taxes included in the primary surplus. We describe an environment with this type of efficient fiscal policy in part IV.

*Woodford's **Really** Unpleasant Arithmetic*

Woodford's argument—that instability in fiscal policy must affect the price level—is a simple proof by contradiction. Suppose the monetary authority could perfectly stabilize inflation and the price level. This implies $P_{t+1} = P_t$, so that the nominal rate of interest is fixed and equal to the real rate. This, in turn, implies that seignorage, s^m, is zero and, as a result, $s_t = s_t^f$. Now, suppose fiscal policy is stochastic, with $\psi\,[1/(1+r)] \neq 0$. According to equation (3.14), P_t responds to innovations in s_t. But this contradicts our assumption that P_t is constant. It follows that with shocks to fiscal policy, it may not be feasible for the monetary authority to insulate the price level from those shocks.

Bear in mind that the monetary authority *can* control the expected rate of inflation in the FTPL. For Woodford's *really* unpleasant arithmetic to be truly unpleasant, shocks to the realized price level must have socially inefficient consequences. This is not the case in many economic environments, where only the expected inflation rate matters (see, for example, Chari, Christiano, and Kehoe [1991]). Shocks to the realized price level are costly in environments with nominal rigidities and in environments with heterogeneous agents.[51]

The FTPL and the Control of Average Inflation

The previous section described how the monetary authority can control the average rate of inflation by pegging the nominal interest rate to an appropriate value.[52] In policy discussions about inflation, it is sometimes suggested that inflation can be controlled more effectively with an interest rate rule that responds aggressively to inflation. In this section, we show how such a monetary policy could, in fact, lead to disaster if fiscal policy were non-Ricardian.

Suppose the monetary authority adjusts the interest rate according to the rule

$$1+R_t = \alpha_0 + \alpha_1 \pi_t, \;\; \pi_t = P_t / P_{t-1}.$$

The monetary authority implements this rule by adjusting the money supply so that money demand is satisfied at the targeted rate of interest. An "aggressive" interest rate rule is one in which α_1 is large. For example, Taylor (1993) has argued that α_1 should be around 1.5. This means that if inflation rises 1 percentage point, then the central bank raises the nominal interest rate by 1.5 percentage points. According to conventional wisdom, an aggressive interest rate rule such as this is a good way to keep inflation under control. As we shall see, this is not necessarily true if policy is non-Ricardian.

We suppose the rest of the economy corresponds to the example in "Is the Price Level Overdetermined in the FTPL?" (page 11). As in that model economy, we assume there is no uncertainty, since it is not essential to the analysis here. Combining the interest rate rule with the Fisher equation (3.2), we obtain the following expression, which must hold in equilibrium:

$$\pi_{t+1} = \frac{\alpha_0}{1+r} + \frac{\alpha_1}{1+r}\pi_t .$$

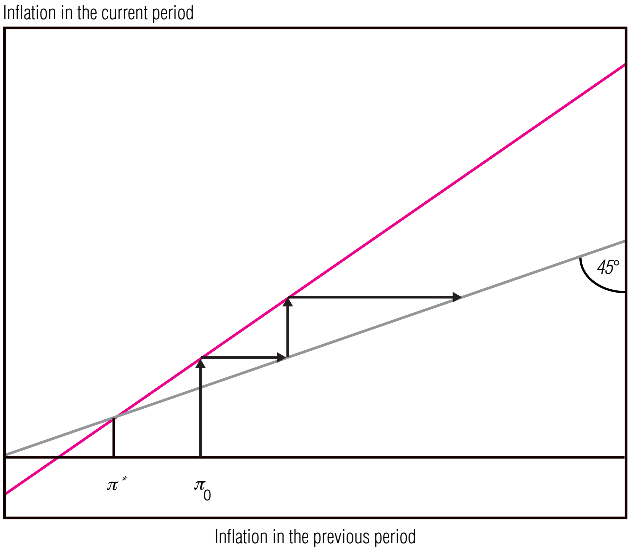
Consider an aggressive interest rate rule with $\alpha_1/(1+r) > 1$. The relationship between π_{t+1} and π_t is illustrated in figure 3. There is a particular inflation rate, π^*, such that if $\pi_t = \pi^*$, then $\pi_{t+1} = \pi^*$. However, if the initial inflation rate is greater than π^*, then π_t grows without

51 See Woodford (1996) for an environment with endogenous production and sticky prices. With sticky prices, shocks to the aggregate price level distort the allocation of resources across the production of different goods. They also distort aggregate output.

52 See "Inflation with Random Fiscal Policy" on page 17.

FIGURE 3

Evolution of Inflation under Aggressive Interest Rate Rule and Non-Ricardian Policy



bound. This possibility is shown in figure 3, in which inflation starts at π_0 in period 0 and then explodes.

As in our previous model economy, the initial price level is determined by fiscal policy according to the intertemporal budget equation (3.9). Technically, s is no longer constant because variations in inflation cause seignorage to vary over time, too. However, we assume that seignorage revenues are small enough to ignore, so that s comprises only s^f. We continue to assume that s^f is constant.[53] The price level in period 0 is determined by $P_0 = B_0/b^*$, where B_0 is the initial nominal debt and b^* is defined in equation (3.9).

With P_0 determined by the intertemporal budget equation and P_{-1} determined by history, π_0 is uniquely pinned down. However, there is no way to rule out the possibility that this value of π_0 lies to the right of π^*, in which case inflation explodes.[54]

One way to gain insight into the mechanics of this exploding inflation is to focus on the government's budget constraint, equation (3.5). From that equation, we see that a higher nominal interest rate leads to a more rapid increase in the nominal debt, B_{t+1}. Assuming the outlook for the fiscal primary surplus does not change, the real value of the debt remains constant. With the nominal debt growing more quickly and its real value constant, inflation must rise.

The central bank's monetary policy responds to the rise in inflation by driving the interest rate up even further, leading to an additional increase in inflation. This circular, self-reinforcing process produces the explosion in inflation.

The possibility just outlined, whereby an aggressive interest rate rule leads to exploding inflation, may seem peculiar. Loyo (1999) refers to it as a "tight money paradox." According to the model, if the central bank, instead of being aggressive, adopts a more accommodating stance by choosing a value of α_1 substantially less than unity, then exploding inflation cannot occur. In the previous section, with $\alpha_1=0$, inflation fluctuated around a constant value. It is easy to confirm, using the logic of figure 3, that the same is true for $0<\frac{\alpha_1}{1+r}<1$. Relative to a simple monetarist perspective, it is certainly a paradox that adopting an aggressive stance against inflation by increasing α_1 could convert stable inflation into an exploding inflation.[55] However, we have just seen that it can occur in a coherent economic model. Moreover, Loyo argues the model captures the driving forces behind Brazil's inflation take-off in the early 1980s. Although we are skeptical that tough monetary policy caused Brazil's high inflation, the hypothesis certainly does seem intriguing.

Woodford (1998b, pp. 399–400) uses the exploding-inflation scenario to understand the nature of fiscal policy in the United States over the past two decades. He observes that econometric estimates of the Federal Reserve's policy rule in the 1980s and 1990s place α_1 substantially above unity (see Clarida, Gali, and Gertler [1998]), and there is no evidence of instability in U.S. inflation. He concludes that policy in the United States during this time must not have been non-Ricardian.

53 Here we are assuming the economy is in the "cashless limit" discussed by Woodford (1998 a,b,c) and defined in footnote 30 of the present paper.

54 In this situation, both fiscal policy and monetary policy are active in the sense defined by Leeper (1991). Our analysis is consistent with Leeper's, which concludes that for almost all values of fiscal policy (s^f), there is no stationary equilibrium inflation rate.

55 Tight money paradoxes also exist in environments with Ricardian fiscal policy. For example, Sargent and Wallace (1981) showed tight monetary policy may lead to an immediate rise in inflation in such an environment. See Kocherlakota and Phelan (1999) for a discussion of a tight money paradox in the FTPL.

IV. Fiscal Theory and the Optimal Degree of Price Instability

The FTPL literature has drawn attention to the possibility that some price instability may be desirable when unavoidable shocks to the government budget constraint occur (Sims [1999], Woodford [1998a]). When there is nominal government debt, unanticipated shocks to the price level act as capital levies on bondholders. The idea is that it is efficient to absorb unanticipated shocks with capital levies rather than by changing distortionary taxes.

We illustrate these observations in the simplest possible model.[56] Relative to the one-period model of part II, this model incorporates two essential complications. First, we must take into account the distortionary effects on the bond-accumulation decision that may arise from price-level instability. For this reason, we adopt a two-period model. The bond-accumulation decision is taken in the first period, and the government-spending shock and price-level uncertainty occur in the second period. Second, the model must capture the notion that taxes are distortionary. Accordingly, we assume the labor supply is endogenous and taxes are raised using a proportional tax on labor income.

The model is an example of the FTPL because government policy—the choice of labor tax rates—is non-Ricardian. We illustrate how FTPL advocates study the optimal degree of price stability by examining the "best" equilibrium of such a model (see, for example, Sims [1999] and Woodford [1998a]). The literature on optimal fiscal and monetary policy (see, for example, Lucas and Stokey [1983]) calls this equilibrium the *Ramsey equilibrium.*

First, we describe the model. To simplify the analysis, the model does not include money; as a consequence, the model again illustrates price determination in an economy with no government-provided money. Next, we characterize the best (that is, the Ramsey) equilibrium in this economy. We then present a numerical example to illustrate the role of price instability in bringing about efficient resource allocation in the model. We assess the results in a summary section.

The Model

The economy comprises firms, households, and a government. Households and firms interact in competitive markets. The government must finance an exogenously given level of expenditures by levying a distortionary tax rate on labor and possibly by issuing debt. There is no uncertainty in the first period. However, there is uncertainty in the second period's level of government spending. Spending could be high or low, with probability 1/2 each, with the uncertainty being resolved at the beginning of the second period. Consistent with the non-Ricardian assumption, the government commits to its policies before the first period. Trade occurs by barter, and there is no money in the model.[57]

Firms have access to a linear production technology,

$$y = n,\ \ y^h = n^h,\ \ y^l = n^l,$$

where y and n denote output and labor, respectively, in the first period, and y^i, n^i denote output and labor in the second period, $i = h, l$. The superscript h or l indicates the second period when government spending is high or low, respectively. The linearity in the production function guarantees the real wage is always unity in equilibrium; henceforth, we simply impose this result and do not refer to firms any more.

Preferences of households over consumption and labor during the two periods take the form

$$(4.1)\quad U(x) = c - \frac{1}{2}n^2 + \frac{1}{2}\beta\left\{\left[c^h - \frac{1}{2}(n^h)^2\right] + \left[c^l - \frac{1}{2}(n^l)^2\right]\right\},\ 0 \le \beta \le 1,$$

where c denotes consumption in the first period and β is the discount rate, with $\beta = (1+r)^{-1}$. Similarly, c^i denotes consumption in the second period, conditional on the realization of government spending, $i = h, l$. β is the discount rate of the household, and the fraction "1/2"

56 The example illustrates the results on the desirability of tax smoothing and volatile prices reported in Chari, Christiano, and Kehoe (1991).

57 We could imagine there is "inside money" and trade is accomplished through an efficient exchange of IOUs.

preceding β corresponds to the probability of the h or l state of the world. Finally,

(4.2) $x = (c, c^h, c^l, n, n^h, n^l)$.

The linear-quadratic structure of preferences is chosen to ensure a simple analysis. The household's period-1 budget constraint is

(4.3) $$\frac{B'}{1+R} + Pc \leq B + P(1-\tau)n,$$

where P is the period-1 price level, B is the nominal bonds the household inherits from the past, and R is the nominal rate of interest. Also, τ denotes the tax rate on labor and B' denotes bonds acquired from the government. The household's budget constraint in the second period, conditional on the realization of uncertainty, is

(4.4) $$P^h c^h \leq B' + P^h(1-\tau^h)n^h,$$

$$P^l c^l \leq B' + P^l(1-\tau^l)n^l.$$

Again, superscripts indicate the realization of the exogenous government-spending shock. There is no government-supplied money in this economy.

The household maximizes utility by its choice of non-negative values for B', c, c^h, c^l, n, n^h, and n^l. It must respect the budget constraints just specified, and it takes prices and the interest rate as given and beyond its control. The Euler equations associated with the household's optimal choice of labor and bonds are

(4.5) $$n = 1-\tau,\ n^h = 1-\tau^h,\ n^l = 1-\tau^l,$$

$$\frac{1}{(1+R)P} = \frac{1}{2}\beta\left(\frac{1}{P^h} + \frac{1}{P^l}\right).$$

The last of these equations tell us that the expected gross real rate of return on bonds must be $1/\beta$. That this is true, independent of the intertemporal pattern of consumption, reflects our assumption that utility is linear in consumption.

The government's budget constraints in the first and second periods are given by

(4.6) $$\frac{B'}{1+R} + P\tau n \geq B + Pg$$

$$P^h\tau^h n^h \geq B' + P^h g^h$$

$$P^l\tau^l n^l \geq B' + P^l g^l.$$

Here, g denotes government consumption in the first period, and g^i denotes period-2 government consumption, $i = h, l$. In the equations of our model, R appears everywhere as $(1+R)/P^h$, $(1+R)/P^l$, or $B'/(1+R)$.[58] Thus, we cannot pin down R, P^h, P^l, and B' separately. For this reason, we adopt the normalization $R=0$ from here on. Government policy is a vector of three numbers, π, where

$$\pi = (\tau, \tau^h, \tau^l).$$

This is a non-Ricardian policy because there is no set of values for π that will satisfy the government's intertemporal budget equation (see below) for all prices.

Combining the government and household budget equations, we obtain the resource constraints:

(4.7) $$c + g \leq n,\ c^h + g^h \leq n^h,\ c^l + g^l \leq n^l.$$

There are 10 variables to be determined in equilibrium: P, P^h, P^l, B', c, c^h, c^l, n, n^h, and n^l. They are determined by the three household budget constraints ([4.3] and [4.4]), evaluated with a strict equality; the four household Euler equations ([4.5]); and the three resource constraints ([4.7]). These 10 equations, together with the requirement $P, P^h, P^l > 0$, characterize the equilibrium (if one exists!) associated with a given government policy. The mapping from π to these variables is single valued. We denote the function relating the last six variables to π by $x(\pi)$, where x is defined in equation (4.2).

58 The statement is obviously true in the case of the household Euler equation in (4.5). To see that it is also true of equations (4.3),(4.4), and (4.6), replace B' with $\widetilde{B} = B'/(1+R)$ and divide the period-2 budget constraints by $1+R$.

The Ramsey Equilibrium

The Ramsey equilibrium is associated with the policies, π, that solve the problem

$$\max_{\pi} U[x(\pi)],$$

subject to the requirement that prices be strictly positive, $B' \geq 0$, and the elements in x be non-negative.[59] The Ramsey equilibrium is easy to compute in this model economy.

After substituting out for the endogenous variables in terms of π in equations (4.7) and (4.5), the utility function is represented by

$$(4.8) \quad U[x(\pi)] = -\tau^2 - \frac{1}{2}\beta\left[(\tau^b)^2 + (\tau^l)^2\right] + \kappa,$$

where κ is a constant.[60] To complete the statement of the Ramsey problem, we need a simple representation of the restrictions placed on π by the positive-price requirement. Before we do this, we must confront a technical issue.

It is well known in the literature on Ramsey equilibria that it is efficient to renege on the initial nominal debt, B, by selecting policies that produce an infinite first-period price level. Allowing this would plunge us into exotic mathematical issues, distracting us from the central focus of the example: the desirability of letting prices in the second period react to the realization of government spending in that period. With this in mind, we simply fix the period-1 price level at $P=1$. Since the nominal debt, B, is given from the past, it follows that we have fixed the initial real debt. It is important to emphasize, however, that we do not fix the second-period price levels.[61]

The restriction on π implied by $P=1$ is easy to determine by expressing the government's first-period intertemporal budget equation in terms of π. Combine the household's intertemporal Euler equation (4.5) with the government's budget constraints (4.6),

$$(4.9) \quad B \leq \tau(1-\tau) - g + \frac{1}{2}\beta\left[\tau^b(1-\tau^b) - g^b + \tau^l(1-\tau^l) - g^l\right],$$

where we have imposed $P=1$. The restrictions on second-period prices come from the intertemporal government budget equations that obtain

$$(4.10) \quad \tau^b(1-\tau^b) - g^b \geq 0, \; \tau^l(1-\tau^l) - g^l \geq 0$$

in those periods. The Ramsey problem, modified to incorporate the restriction $P=1$ is set up in Lagrangian form:

$$\max_{\tau,\, \tau^b, \tau^l} \; -\tau^2 - \frac{1}{2}\beta\left[(\tau^b)^2 + (\tau^l)^2\right]$$

$$+ \lambda\left\{\tau(1-\tau) - g + \frac{1}{2}\beta\left[\tau^b(1-\tau^b) - g^b + \tau^l(1-\tau^l) - g^l\right] - B\right\}$$

$$+ \mu^b\left[\tau^b(1-\tau^b) - g^b\right] + \mu^l\left[\tau^l(1-\tau^l) - g^l\right],$$

where λ, μ^b, and $\mu^l \geq 0$ are Lagrange multipliers.[62] The necessary and sufficient conditions associated with the maximum are the inequality constraints on the multipliers, λ, μ^b, and $\mu^l \geq 0$; the inequality constraints in equations (4.9) and (4.10); the "complementary slackness" conditions,

$$(4.11) \quad 0 = \lambda\left\{\tau(1-\tau) - g + \frac{1}{2}\beta\left[\tau^b(1-\tau^b) - g^b + \tau^l(1-\tau^l) - g^l\right] - B\right\},$$

$$0 = \mu^b\left[\tau^b(1-\tau^b) - g^b\right],$$

$$0 = \mu^l\left[\tau^l(1-\tau^l) - g^l\right];$$

59 See Bizer and Judd (1989), Chari, Christiano, and Kehoe (1990), Judd (1989), and Lucas and Stokey (1983), among others.

60 Here, $\kappa = 2\left[\frac{1}{2} - g + \frac{1}{2}\beta\left(1 - g^b - g^l\right)\right]$.

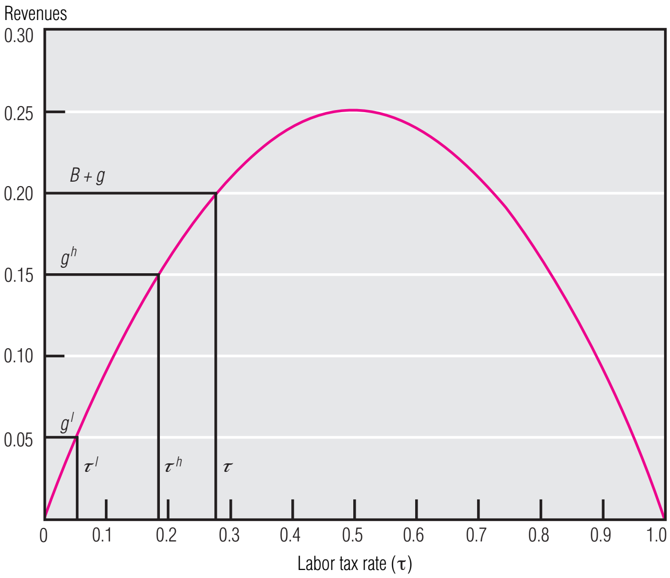
To see how we obtain this expression, note that $c-(1/2)n^2$ is $y-g-(1/2)n^2$ after using the resource constraint, $y=c+g$. Imposing $n=1-\tau$, then, yields that $c-(1/2)n^2$ is $(1/2)(1-\tau^2)-g$.

61 Chari, Christiano, and Kehoe (1991) confront the same problem, which they deal with by setting the initial debt to zero. In our context, that creates a problem because it leaves us with no ability to pin down P.

62 Our model differs from Sims' (1999) model in two respects. First, ours has only two periods, while Sims' has an infinite horizon. (It is trivial to extend our model to the infinite horizon.) Second, we model agents at the level of preferences and technology, while Sims adopts a reduced-form representation analogous to the one in Barro (1979). Our reduced-form utility function coincides with Sims', but our budget constraint does not. Sims models taxes as lump-sum in the budget constraint, whereas we take into account the distortionary effects of taxation. For example, Sims would have τ in the budget constraint, rather than $\tau(1-\tau)$, as we do. The conclusions of the analysis are not sensitive to these differences.

FIGURE 4

Revenues from Labor Tax



and the three first-order conditions corresponding to τ, τ^h, τ^l. After rearranging, these are

$$(4.12)\quad \lambda = \frac{2\tau}{1-2\tau},$$

$$\mu^h = \beta\left[\frac{\tau^h}{1-2\tau^h} - \frac{\tau}{1-2\tau}\right],$$

$$\mu^l = \beta\left[\frac{\tau^l}{1-2\tau^l} - \frac{\tau}{1-2\tau}\right].$$

We solve the (constrained) Ramsey problem by finding multipliers, λ, μ^h, μ^l, and policies, τ, τ^h,τ^l, that satisfy these conditions.

Once the Ramsey policies have been identified, n, n^h, and n^l are obtained from equation (4.5) and c, c^h, and c^l from equation (4.7). Then, B', P^h, and P^l are obtained by solving equation (4.6). Several qualitative features of the solution are immediately apparent. First, the weak inequality in equation (4.9) is satisfied as a strict equality.[63] This is not surprising—otherwise, taxes would be higher than necessary and, given the form of preferences, this would be counterproductive. Also, because the period-0 intertemporal budget equation is satisfied as a strict equality, it would have been optimal to inflate away the initial debt by setting $P=\infty$, had we not imposed the requirement the government pay off B with $P=1$.[64] Second, ignoring the requirements of the non-negativity constraints on prices in the second period, the optimal outcome is $\tau=\tau^h=\tau^l$. To see this, note that the first-order conditions in this case are equation (4.12) with $\mu^h \equiv \mu^l \equiv 0$. Inspecting the second two equations in (4.6), it is obvious that $P^h > P^l$ as long as $g^h > g^l$. Third, in practice, the constant tax rate policy is not necessarily feasible, since it may conflict with the positive-price requirement. In this case, however, the price fluctuations across states of nature are even more extreme.

Suppose, for example, the constant tax rate policy is inconsistent with the first of the two inequalities in equation (4.10). Then, $\mu^h > 0$, $\tau^h > \tau$, and, by equation (4.11), $\tau^h(1-\tau^h)-g^h=0$. The latter implies the government inflates away the debt completely in state h, with $P^h=\infty$. To ensure that households still have an incentive to accumulate debt in the first period, equation (4.5) indicates P^l must satisfy $P^l = (\beta/2)P(1+R)=\beta/2$ in this case. That is, the real rate of return on debt into state l must be high.

A Numerical Example

This section studies a numerical example to illustrate the properties of P^h and P^l in the Ramsey equilibrium. A natural benchmark to consider is the no-debt equilibrium: τ is selected so that $B'=0$, and τ^h and τ^l are selected so the constraints in equation (4.10) are satisfied as exact equalities. With this as a benchmark, we evaluate the Ramsey equilibrium in which $B'>0$ and consider P^h and P^l.

To see how taxes are determined in the benchmark equilibrium, consider figure 4, which graphs $\tau n=\tau(1-\tau)$ for $\tau \in (0,1)$. We have a single-peaked Laffer curve in our model economy. The horizontal lines indicate the

63 Here is a proof by contradiction. Suppose the weak inequality in equation (4.9) were a strict inequality. Then, by the first expression in equation (4.11) and in equation (4.12), we have $\lambda=0$ and $\tau=0$. The strict inequality in equation (4.9) implies that at least one of the weak inequalities in (4.10) is strict. That implies, by (4.11), the associated multiplier is zero. Equation (4.12) implies the associated tax rate is zero, but this contradicts the non-negativity of the primary surplus in that period.

64 In that case, the constraint would have been equation (4.11) without the term $-B$.

TABLE 1

Two Equilibria

Variable	Benchmark, no-debt equilibrium (B'=0)	Ramsey equilibrium
P	1	1
P^h	—	64.67
P^l	—	0.49
n	0.72	0.82
n^h	0.82	0.82
n^l	0.95	0.82
c	0.52	0.62
c^h	0.67	0.67
c^l	0.90	0.77
τ	0.28	0.19
τ^h	0.18	0.19
τ^l	0.05	0.19
$g+B$	0.20	0.20
g^h	0.15	0.15
g^l	0.05	0.05
B'	0	0.05
Utility	–0.0941	–0.0674

revenue requirements in the first and second periods. We assume the first-period revenue requirement, $B+g$, is 0.20. The second-period revenue requirement is $g^h = 0.15$ when government spending is high and $g^l = 0.05$ when government spending is low. The benchmark equilibrium requires that τ, τ^h, and τ^l be set as indicated on the horizontal axis. In particular, $\tau = 0.28$, $\tau^h = 0.18$, and $\tau^l = 0.05$, after rounding. The value of equation (4.8) in this equilibrium is –0.0941, ignoring κ and setting $\beta = 0.97$. Tax rates are very uneven over time and over states of nature.

Now consider the Ramsey tax rates. We proceed under the conjecture (subsequently verified) that they are optimally chosen to be a constant, τ^*, across dates and states. We use the fact, established in the previous section, that the constraint, equation (4.9), is binding. Two constant tax rates solve equation (4.9) evaluated as a strict equality. Given preferences, equation (4.8), we go with the lower one, $\tau^* = 0.19$, after rounding. To verify this solves the Ramsey problem, we must confirm that equation (4.10) is satisfied. Indeed it is, with $\tau^*(1-\tau^*) - g^h = 0.0008$ and $\tau^*(1-\tau^*) - g^l = 0.10$.

Solving the first expression in equation (4.6), we find that $B' = 0.05$. In addition, we find from the second two expressions in equation (4.6) that $P^h = 64.67$ and $P^l = 0.49$. Essentially, the government reneges on the debt in period h and pays an attractive 100 percent rate of return in state l. Finally, the utility of this equilibrium is –0.0674. These results, plus the consumption and labor allocations, are summarized in table 1. By issuing debt, it is possible to stabilize employment and consumption across dates. By issuing the debt in nominal terms and allowing the price level to fluctuate, it is possible to make the payoff on that debt state-contingent in real terms.

Summary

We have described a model in which an efficient fiscal program issues nominal debt and then allows the price level to fluctuate. Although we demonstrated this finding in an economy with no government-provided money, this feature of our model plays no fundamental role in the result. The same result was obtained by Chari, Christiano, and Kehoe (1991) and by Woodford (1998a) using models with money.

In our model, the equilibrium is equivalent to one in which the government issues debt whose payoff is denominated in real terms in the first period, and where the payoff is explicitly contingent on the realization of government spending in the second period.[65] From this perspective, the natural question is, why not use the state-contingent-debt strategy, rather than going to the trouble of issuing nominal debt and allowing the state contingency to arise because of fluctuations in the price level?

65 Lucas and Stokey (1983) emphasize the desirability of this type of debt.

To address this question, we must invoke considerations that are not included in the model. One advantage of the nominal-debt strategy is that it is likely to have lower costs of administration and information acquisition, because the appropriate response of the real payoff on the debt to shocks is achieved automatically as a by-product of price fluctuations generated in the market-clearing process (Sims [1999] and Woodford [1998a]).

This may provide an overly optimistic view of the nominal-debt strategy. For example, if there are sticky prices, then fluctuations in the price level could distort resource allocations. In addition, price volatility may interfere with private contracts by inducing reallocations of wealth among private agents. Presumably, a version of the Ramsey problem that incorporates those costs would still exhibit price fluctuations, though they would likely be smaller.[66] Designing a fiscal system that properly balances benefits and costs would presumably be very difficult, reducing the cost advantages of the nominal-debt strategy we allude to above.

There is another reason to question the advantages of both the nominal and real state-contingent-debt strategies. Unless the government has substantial ability to commit to its policies, either strategy could backfire, a possibility that can be seen in the example. It is efficient in the first period for the government to inflate away the debt. But when time moves forward one period, the second period becomes the first period. When that time arrives, it is again in the government's interest to inflate away the debt! Households that understand this in the first period may well choose not to accumulate debt in the first place.[67]

Now, this case was excluded in our analysis because of the assumption that policy is non-Ricardian: The policy is just a sequence of numbers (tax rates) through time, and the possibility of adjusting them ex post is ruled out. Is this a realistic assumption? Does it assume that governments have more commitment power than they actually have? The literature on Ramsey policy has generally concluded the answer is yes, and has moved on to equilibrium concepts that do not presume as much commitment power.[68]

In principle, one can make the case that the degree of commitment needed for the policy to work is not implausibly large. This might be so if the required price fluctuations occurred automatically, in a way that legislatures have difficulty interfering with. For example, Judd (1989) suggests that price movements in the U.S. economy correspond roughly to the requirements of an efficient fiscal program. He notes that good shocks to the government budget constraint, such as technology shocks, tend to produce a negative shock to the price level, generating transfers to holders of government bonds. Similarly, bad shocks, like a jump in government spending due to war or natural disaster, tend to drive the price level up, taxing government bond holders.

Our point is *not* that the degree of commitment required for the volatile price strategy is necessarily too great. Our point is only that commitment is a fundamental assumption of the volatile price strategy. In the absence of commitment, the strategy is likely to backfire.

66 It would be interesting to investigate this question in quantitative models. There is a possibility that the efficient degree of volatility in prices would be reduced to zero if price volatility introduced distortions. Chari, Christiano, and Kehoe (1991) argue that, in principle, there are many ways to achieve state contingency in fiscal policy. If there were costs to using the price level for this, then the efficient thing to do would be to use another way. Only if there were costs associated with all ways of achieving state contingency in fiscal policy would some volatility in prices be desirable.

67 Sims (1999) considers a proposal that Mexico adopt the U.S. dollar as its national currency. He criticizes the proposal on the grounds that, with the Mexican national debt denominated in a foreign currency, the Mexican government loses the fiscal benefits of the policy described in the text. That is, it would not be able to periodically renege on and subsidize holders of its debt through fluctuations in the Mexican price level. Our point here is that giving up this option may not be very costly to Mexico, if the Mexican government lacks credibility. Indeed, giving up the option may be a good thing. In the absence of credibility, attempts to use the option may lead to the disastrous situation in which everyone refuses to buy Mexican government debt.

68 Research on optimal policy that presumes a lack of commitment includes Chari and Kehoe (1990) and Stokey (1991). In contrast, Lucas (1990) argues forcefully in favor of implementing Ramsey-optimal capital taxation, a theme continued in Atkeson, Chari, and Kehoe (1999).

V. Conclusion

What insights does the FTPL provide into the two questions about price stability we posed at the beginning of this review? That is, how can price stability be achieved? And, how much price stability is desirable? Conventional wisdom holds that if there is no doubt about the central bank's commitment to low and stable inflation, then low and stable inflation is exactly what will happen.[69] According to the FTPL, however, this overstates the central bank's power. Still, it remains an open question just how severe the limitations on central banks' powers are. These limitations may not be very great for modern, developed economies. In the FTPL models that we have studied, the central bank can determine the *average* rate of inflation. However, it cannot perfectly control the variance of inflation because it cannot eliminate the impact of shocks to fiscal policy on the price level. But in a modern Western economy, the stock of outstanding nominal government liabilities is quite large—Judd's (1989) estimate for the United States puts it at one year's GDP. Therefore, a relatively small change in the price level can absorb a fairly large fiscal policy shock.[70] In practice, then, the conventional answer to the first question may be roughly the right one, even under the FTPL.

Regarding the second question, Sims (1999) has stressed the potential benefits of price volatility.[71] Variations in the price level in response to fiscal shocks have the effect of taxing and subsidizing holders of nominal government liabilities. Under certain circumstances, this can enhance the overall efficiency of government fiscal and monetary policy. But this result also raises questions, because it is obtained in an enviro-
ment with few of the frictions observed in actual economies that make price volatility costly. Whether the result would survive the introduction of a realistic set of frictions—and a realistic set of alternative methods for dealing with fiscal shocks—is unclear at this time.

69 See Sargent and Wallace (1981), last paragraph.

70 Sims (1999) also stresses this point.

71 Woodford (1998a) has made a similar suggestion. The result has also been obtained in Chari, Christiano, and Kehoe (1991).

VI. Appendix: The Logical Coherence of Fiscal Theory

An important concern regarding the FTPL has to do with its internal logical consistency. When the FTPL uses the intertemporal government budget equation to pin down the price level, is that price level consistent with the one determined by the rest of the economy? In some cases, the answer is no.[72] Do these cases warrant the conclusion that the FTPL is not logically coherent? We think not, as enough interesting examples can be constructed in which the fiscal theory *is* logically coherent. One example is given in the body of this review. The point is also illustrated in several articles of a special issue of *Economic Theory* in 1994. In this appendix, we present another example.

The model we work with is the cash/credit-good model of Lucas and Stokey (1983). We examine a range of parameter values, including the empirically plausible ones, according to estimates reported in Chari, Christiano, and Kehoe (1991). We skip detailed proofs in certain places, though never without providing the intuition for the argument. Readers who wish to see an extensive and rigorous treatment of the properties of the equilibria of this model should consult Woodford (1994). This appendix presents an extended example to illustrate his Propositions 2 and 10 at the level of an advanced undergraduate or first-year graduate economics course.

We first consider the case in which monetary policy is characterized by a constant money-growth rate. We show the model has a unique equilibrium when the non-Ricardian assumption is adopted. We then consider the case in which the monetary authority pegs the interest rate. Like the example in the text, the model has a unique equilibrium when the non-Ricardian assumption is adopted. When that assumption is *not* adopted, the model fails to exhibit a unique equilibrium. In this case, the model reproduces the classic Sargent and Wallace (1975) result: The price level is indeterminate. From a technical standpoint, the non-Ricardian assumption is a device that can eliminate the price-level indeterminacy associated with interest rate pegging that Sargent and Wallace analyze.[73]

The first section below describes the agents of the model and defines equilibrium. The following section addresses the case in which monetary policy is characterized by constant money growth. The final section addresses the case of interest rate pegging.

The Lucas–Stokey Cash/Credit-Good Model

Households

The Household Problem and Constraints

The model abstracts from differences among households by assuming they are all identical. In addition, households are assumed to live infinitely long. This assumption can be interpreted, following Barro (1974), as reflecting that each household actually lives a finite amount of time but cares in a particular way for its offspring.

The preferences of the representative household are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),\ u(c)=\log(c),\ 0<\beta<1,$$

$$c=\left[(1-\sigma)c_1^{\upsilon}+\sigma c_2^{\upsilon}\right]^{\frac{1}{\upsilon}},$$

where $0<\sigma<1$ and c_t denotes consumption services. In our analysis, we restrict υ to the range $0<\upsilon<1$. Chari, Christiano, and Kehoe (1991) argue this is the empirically relevant case; based on postwar U.S. data, their point estimates are $\sigma=0.57$ and $\upsilon=0.83$.

Consumption services are generated by the acquisition of two market-produced goods, as indicated. The first, c_{1t}, is called a "cash good" and the other, c_{2t}, is a "credit good." To purchase the cash good, households need to set aside cash in advance.

To make the notion of "in advance" precise, the model adopts a particular timing. Each period is divided into two parts. In the first part (the "morning"), the household participates in an asset market, and in the second part (the "afternoon") the household participates in a goods market. The cash that households need to purchase c_{1t} in the afternoon of a given day must be set aside at the end of asset-market

72 A simple example, in which the traditional quantity theory holds, is presented in the body of this review. In the model of Obstfeld and Rogoff (1983), there is a countable set of equilibria. The Ricardian assumption does not sit comfortably with that model, because the likelihood of the fiscal and monetary authorities choosing a fiscal policy consistent with one of those equilibria seems slim. Buiter (1999) provides another example.

73 This is how Kocherlakota and Phelan (1999) interpret the FTPL.

trading the same morning. They hold these balances idle until the morning of the following day, when actual payment is due. Credit goods work differently. For these goods, the household has no need to accumulate cash in advance. The household simply pays for the goods with cash in the next morning's asset market.

Do not be misled by the labels used to identify these goods. It is not that one can be bought "on credit" in the traditional sense, and the other cannot. Both goods are paid in cash the morning after the purchase. No credit is offered by the seller in either case. The difference is simply that in the case of the cash good, the household must forfeit interest: To buy the cash good, the household must carry idle cash in its pocket throughout the afternoon. From the point of view of the seller, the goods are completely the same. The terms of the transaction are identical—cash only, to be delivered the morning after the sale.

The distinction between cash and credit goods may at first seem artificial. In fact, it is a clever device for capturing the idea that transactions in some goods are more cash-intensive than in others. It will produce a demand for money, one that is a function of the interest rate.

We assume the marginal rate of transformation in production is unity between the two goods. Therefore, the price of the two goods, P_t, is identical in equilibrium. Moreover, in any equilibrium it must be that $R_t \geq 0$ and $P_t > 0$. Market clearing is impossible if either of these two conditions fails to be satisfied.

Let A_t denote the household's financial assets at the end of asset-market trading. In the first period, $t=0$, this is simply a given number, A_0. The household can allocate A_t as follows:

(A1) $$M_t^d + \frac{B_{t+1}^d}{1+R_t} + T_t \leq A_t,\ t=0,\ 1,\ 2,\ \ldots,$$

where M_t^d denotes money balances; B_{t+1}^d denotes government debt, which costs $B_{t+1}^d/(1+R_t)$ today and pays off B_{t+1}^d in the next period's asset market; and T_t denotes lump-sum taxes. The household does not set M_t^d to zero because it must set aside cash in advance, $P_t\, c_{1t} \leq M_t^d$, if it wishes to consume cash goods. Assets at the beginning of the next period are

(A2) $$A_{t+1} = M_t^d + P_t\,(y - c_{1t} - c_{2t}) + B_{t+1}^d\,.$$

Here, M_t^d is the cash balance carried into the previous period's goods market; $P_t y$ denotes the receipts from the sale of y in the previous period's goods market; $P_t\,(c_{1t} + c_{2t})$ represents the bill of goods purchased in the previous period's goods market; and B_{t+1}^d is the receipts from government debt purchased in the previous period's asset market.

It is useful here to follow Woodford's (1994) suggestion to write equations (A1) and (A2) in a slightly different form. "Spending" is defined as

$$S_t = P_t\, c_{1t} + \frac{P_t\, c_{2,t}}{1+R_t} + \left(1 - \frac{1}{1+R_t}\right)\left(M_t^d - P_t\, c_{1t}\right).$$

In this measure of spending, excess holdings of money balances, above what are needed for the cash-in-advance constraint, have a positive price if $R_t > 0$. The relative "prices" of c_{1t} and c_{2t} accurately reflect that the former involves sacrificing interest earnings. "Income," I_t, is defined as

$$I_t = \frac{P_t y}{1+R_t} - T_t\,.$$

Divide both sides of equation (A2) by $(1+R_t)$ and substitute out for $B_{t+1}^d/(1+R_t)$, using equation (A1) to obtain

(A3) $$A_{t+1} \leq (1+R_t)(A_t + I_t - S_t).$$

The accumulation of household assets obeys the usual simple equation one finds in a non-monetary, single-good model economy.

A lower-bound constraint must be placed on A_t to ensure the household has a bounded consumption set. We impose the assumption that the current value of assets must eventually be non-negative,

(A4) $$\lim_{T\to\infty} q_T\, A_T \geq 0,$$

where

$$q_T = \frac{1}{(1+R_0)\ (1+R_1)\ldots\ (1+R_{T-1})},\ q_0 = 1.$$

It is easy to verify that equations (A3) and (A4) are equivalent to the usual single present-value budget constraint for consumption, S_t, and income, I_t.[74]

We suppose that at each date the household chooses $c_{1t+j}, c_{2t+j} \geq 0, M^d_{t+j}\, B^d_{t+1+j} \geq 0$, to maximize its utility, subject to the restrictions just described, and takes A_t, R_{t+j}, P_{t+j}, $j \geq 0$, as given and beyond its control.[75]

Necessary and Sufficient Conditions for Household Optimization

The household first-order conditions are

$$\text{(A5)} \quad \frac{u_{2,t}}{P_t} = \beta \frac{u_{1,t+1}}{P_{t+1}}$$

and

$$\text{(A6)} \quad \frac{u_{1,t}}{u_{2,t}} = 1+R_t.$$

Here, $u_{i,t}$ denotes the partial derivative of utility with respect to c_{it}, $i=1, 2$. To understand why the first of these Euler equations is implied by household optimization, consider the following argument: Suppose the household reduces its purchases of credit goods in the period-t goods market by one dollar and applies that dollar to additional cash-good consumption in period $t+1$. Credit-good consumption today drops by $1/P_t$, which translates into an immediate decrease in utility of $u_{2,t}/P_t$. This reduction of expenditures frees one dollar in the asset market in the next period, which can be applied toward the cash-in-advance constraint for purchasing $1/P_{t+1}$ units of the cash good in next period's goods market. The utility benefit from the standpoint of period t is $\beta u_{1,\ t+1}/P_{t+1}$. If the gain exceeded the cost, the household could not be optimizing, or we would have found a change in its plan that would improve utility.

Similarly, if the gain were less than the cost, the household could raise utility by increasing credit-good consumption in period t and reducing cash-good consumption in period $t+1$. Optimization requires that neither of these strategies raises utility, and this is why the first Euler equation above (A5) is an implication of household optimization.

The second Euler equation (A6) is also implied by household optimization, established by an argument similar to the one in the previous paragraph. The argument exploits the trade-off between cash and credit goods within the same period. The household can increase current-period cash-good consumption by reducing its acquisition of government debt. This reduces its cash receipts in the next period's asset market, thus reducing the cash available for credit-good consumption today. This Euler equation makes considerable sense: When R is high, it implies that u_1 is relatively high, so that c_1 is relatively low. This makes sense because high R raises the cost to households of purchasing c_1.

There is also a condition associated with the cash-in-advance constraint, which we write as

$$\text{(A7)} \quad R_t\left(P_t c_{1t} - M^d_t\right) = 0.$$

As noted above, only the case $R_t \geq 0$ must be considered. Since $P_t c_{1t} - M^d_t$ cannot be negative, equation (A7) is a mathematically concise way of stating that if $R_t > 0$, it follows that $P_t c_{1t} = M^d_t$, while if $R_t = 0$, then all we know is $P_t c_{1t} \geq M^d_t$. From the point of view of the analysis below, the key is that when $R_t > 0$, the cash-in-advance constraint holds as a strict equality. This makes sense: When the interest rate is positive, it is inconsistent with optimization to carry cash in the afternoon that is not absolutely necessary.

In addition to equations (A5)–(A7), the transversality condition is also implied by household optimization:

$$\text{(A8)} \quad \lim_{T\to\infty} q_T A_T = 0.$$

The intuition for this condition is straightforward. To see that the limit cannot be positive, suppose, on the contrary, that it is. In this case, A_t grows faster than the interest rate. It would then be feasible for households to increase spending

74 By recursive substitution, equation (A3) implies

$$q^T A_T \leq A_0 + \sum_{t=0}^{T-1} q_t (I_t - S_t).$$

Driving $T \to \infty$, we obtain

$$\sum_{t=0}^{\infty} q_t S_t \leq A_0 + \sum_{t=0}^{\infty} q_t I_t.$$

This shows that equations (A3) and (A4) imply the standard single-equation budget constraint. To establish the reverse, simply show that if $\{S_t, I_t\}$, $t=0, 1,\ldots$ satisfy the budget constraint, then they also satisfy (A3) and (A4). That the present value of income is finite will be a feature of equilibrium. Otherwise, demand would be unbounded and no equilibrium could exist.

75 It is easy to verify that in any equilibrium, it must be that $R_t \geq 0$ and $P_t > 0$. Market clearing is impossible if either of these conditions is not satisfied.

in one date without reducing it in another. If this extra spending were financed by a loan, the power of compound interest would cause the resulting debt to spiral upward at a rate equal to the interest rate. However, with total assets rising at an even greater rate, the household's net asset position would remain consistent with equation (A4). The increase in consumption financed in this way raises utility because of nonsatiation, and so we have a contradiction. Thus, optimization implies the above expression cannot be positive, but it also cannot be negative because of the restriction of equation (A4).

For purposes of our analysis, it is convenient to write the transversality condition in a different form. Combining equations (A5) and (A6), we find $u_{1,t} = \beta(1+R_t)u_{1,t+1}P_t/P_{t+1}$. Substituting this into the expression for q_t, we find

$$\text{(A9)} \quad q_t = \left(\frac{P_0}{u_{1,0}}\right)\frac{\beta^t u_{1,t}}{P_t},\ t = 0,\ 1,\ 2,\ ....$$

After multiplying both sides of equation (A8) by the positive constant, $u_{1,0}/P_0$, the transversality condition reduces to

$$\text{(A10)} \quad \lim_{T\to\infty} \beta^T u_{1,T}\frac{A_T}{P_T} = 0.$$

Equations (A5)–(A10) are not just necessary for optimization, they are also sufficient. This is easily established with a suitably adjusted version of the proof to Stokey, Lucas, and Prescott's theorem 4.15 (1989).

Government

The government purchases no goods, it only participates in the asset market. Its sources of funds in the asset market are new debt issues, tax revenues, and newly created money, $M_t^s - M_{t-1}^s$. It uses these funds to pay its outstanding debt obligations, B_t^s. Equating sources and uses of funds gives us

$$\frac{B_{t+1}^s}{1+R_t} + T_t + M_t^s - M_{t-1}^s = B_t^s.$$

At time t, the government takes M_{t-1}^s and B_t^s as given, $t = 0,\ 1,....$ At date 0, $M_{-1}^s + B_0^s = A_0$. Government policy is a sequence of B_{t+1}^s, T_t, and M_t^s that satisfies this flow-budget constraint, which can also be written as

$$\frac{A_{t+1}^s}{1+R_t} + T_t + \frac{R_t}{1+R_t}M_t = A_t^s.$$

Here, A_t^s measures total nominal assets, $A_t^s = B_t^s + M_{t-1}^s$, and $A_0^s = A_0$.

Recursively substituting this expression forward, we find that for each fixed T,

$$\text{(A11)} \quad q_T A_T^s + \sum_{t=0}^{T-1} q_t\left[T_t + \frac{R_t}{1+R_t}M_t\right] = A_0.$$

The presence of $R_tM_t/(1+R_t)$ reflects the interest costs the government saves when it issues money rather than bonds. The government's intertemporal budget equation is represented by the above expression, with $q_T\ A_T^s$ absent and T–1 replaced by ∞:

$$\text{(A12)} \quad \sum_{t=0}^{\infty} q_t\left[T_t + \frac{R_t}{1+R_t}M_t\right] = A_0.$$

The only restriction we have placed on government policy is that the flow-budget constraint is satisfied for all possible values of prices, $\{q_t,\ P_t,\ R_t,\ t \geq 0\}$. That is, we require that equation (A11) hold. But no assumption has been made that equation (A12) holds for all possible prices. Government policy is said to be Ricardian if (A12) holds for all possible prices, and it is non-Ricardian if (A12) holds only at equilibrium prices. (We shall see that, at equilibrium prices, [A12] must be satisfied regardless of whether government policy is Ricardian or non-Ricardian. This follows from equation [A8] and the fact that, in equilibrium, $A_t^s = A_t$.)

Equation (A11) converges to equation (A12) if and only if

$$\text{(A13)} \quad q_T A_T^s \to 0.$$

We can equivalently define a Ricardian policy as one that enforces equation (A13) at all possible prices and a non-Ricardian policy as one that does not.

Firms

Firms in this economy are simple. They buy y from households and transform it into cash and credit goods. Given the assumed linearity of the production technology, the resource constraint has the form

$$\text{(A14)} \quad c_{1t} + c_{2t} = y.$$

Equilibrium

A general equilibrium for this economy is a sequence of prices and interest rates, P_t and R_t; a sequence of consumptions, c_{1t}, c_{2t}; and a sequence of money supplies and bonds, M_{t+1} and B_{t+1}, such that households optimize, the government flow-budget constraint is satisfied, and markets clear. Bond-market clearing requires

$$B^s_{t+1} = B^d_{t+1} = 0,$$

and money-market clearing requires

$$M^s_t = M^d_t = M_t,$$

say, for $t \geq 0$. These conditions imply that $A_{t+1} = M^d_t + B^d_{t+1} = A^s_{t+1}$. Goods-market clearing corresponds to the resource constraint, equation (A14).

A feature of equilibrium that will be useful in the analysis is

$$\text{(A15)} \quad 1+R_t = \frac{1-\sigma}{\sigma}\frac{1}{w_t^{1-\upsilon}} \geq 1,\; w_t \equiv \frac{c_{1t}}{c_{2t}},$$

which we obtain from equation (A6) and our parametric form for the utility function. When $R_t > 1$, we can rewrite this to obtain the model's "money-demand" function. The binding cash-in-advance constraint, $c_{1t} = m_t$, and the resource constraint imply $w_t = m_t/(y - m_t)$ where

$$m_t = \frac{m_t}{P_t}$$

Solving equation (A15) for m_t yields

$$\text{(A16)} \quad m_t = \frac{y}{1+\left[\frac{\sigma}{1-\sigma}(1+R_t)\right]^{\frac{1}{1-\upsilon}}}.$$

Constant Money Growth

Here, we consider the set of equilibria associated with a fixed money growth rate policy. We show there is one equilibrium in which inflation is constant and equal to money growth. There is also a continuum of equilibria with explosive inflation.

We suppose the government sets $B^s_{t+1} = 0$ for all $t \geq 0$ by paying off the entire stock of debt in the first period. In addition, it sets $M^s_t = \mu M^s_{t-1}$ for $t = 0, 1, \ldots,$ where $\mu \geq 1$. Money growth is accomplished by means of lump-sum tax transfers. In particular,

$$T_0 = B^s_0 - (\mu - 1)M^s_{-1},$$

$$T_t = -(\mu - 1)M^s_{t-1},\; t \geq 1.$$

It is straightforward to verify that, with this specification of policy, there are many price sequences that satisfy equation (A10). Technically, it does not fit into our formal definition of a Ricardian policy, because (A10) is not satisfied for all prices. Under this policy, A_t comprises only the money supply. Thus, equation (A10) would be violated if the price level fell sufficiently rapidly. Still, for practical purposes we will think of this as a Ricardian policy.

It is useful to rewrite the household's dynamic Euler equation by multiplying both sides of equation (A5) by M_t and using $M_{t+1} = \mu M_t$ to obtain

$$\text{(A17)} \quad u_{2,t}m_t = \frac{\beta}{\mu}u_{1,t+1}m_{t+1}.$$

A sequence of prices and quantities represents an equilibrium if and only if equations (A7)–(A14) and $P_t > 0$, R_t, c_{1t}, $c_{2t} \geq 0$ are satisfied.

A Characterization Result for Equilibria

We now simplify the equilibrium conditions to obtain a useful set of sufficient conditions for equilibria in which the cash-in-advance constraint binds. In this case, equation (A14) allows us to express equation (A17) as a difference equation in m_t and m_{t+1} alone. Because the cash-in-advance constraint binds, w_t in equation (A15) can be written as

$$w_t = \frac{m_t}{y - m_t}.$$

Using this notation, equation (A17) can be expressed as a difference equation in w_t and w_{t+1},

$$\text{(A18)} \quad a(w_t) = b(w_{t+1}),$$

where

$$\text{(A19)} \quad a(w) = \frac{\sigma w}{(1-\sigma)w^{\upsilon} + \sigma},$$

$$a'(w) = \frac{\sigma}{[(1-\sigma)w^{\upsilon} + \sigma]^2}[(1-\sigma)(1-\upsilon)w^{\upsilon} + \sigma]$$

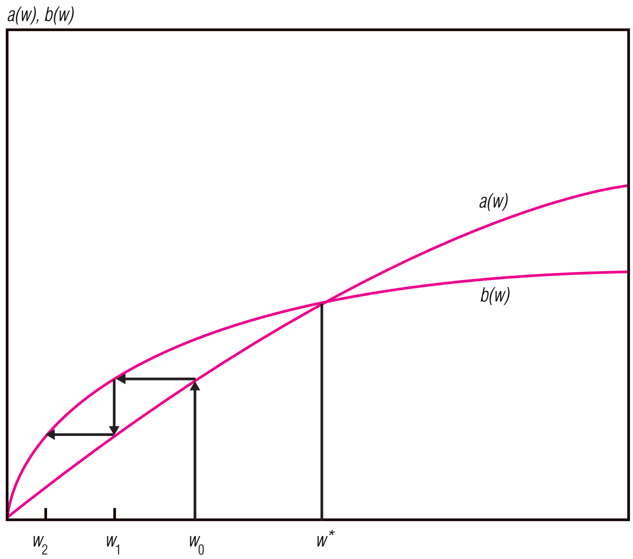
and

$$\text{(A20)} \quad b(w) = \frac{\beta(1-\sigma)}{\mu}\frac{w^{\upsilon}}{(1-\sigma)w^{\upsilon} + \sigma},$$

$$b'(w) = \frac{\beta(1-\sigma)}{\mu}\frac{w^{\upsilon-1}\upsilon\sigma}{\left[(1-\sigma)w^{\upsilon} + \sigma\right]^2}.$$

FIGURE A

Equilibrium in the Cash/Credit-Good Model



Here, a' and b' represent the derivatives of a and b, respectively, with respect to w. The transversality condition reduces, in the present notation, to

$$\text{(A21)} \quad \lim_{T\to\infty} \beta^T b(w_T) = 0.$$

We are now in a position to state our characterization result.

Proposition A1: Suppose that $w_t \geq 0$, $t = 0, 1, 2, \ldots$, satisfies equations (A18), (A21), and (A15). Then, w_t corresponds to an equilibrium.

Proof: Write

$$m_t = y\,\frac{w_t}{1+w_t},\; P_t = \frac{M_t}{m_t}$$

$$R_t = \frac{\sigma}{1-\sigma}\,\frac{1}{w_t^{1-\upsilon}},\; c_{2t} = \frac{y}{1+w_t}, c_{1t} = c_{2t}w_t,$$

and verify that all equilibrium conditions are satisfied at these prices and quantities. QED.

Multiple Equilibria with Ricardian Constant Money Growth

We use the characterization result to show there is a continuum of equilibria in this economy when the money growth rate, μ, is constant and greater than unity. From equation (A18), there is exactly one equilibrium with $w_t = w^*$ for all t,

$$\text{(A22)} \quad w^* = \left[\frac{1-\sigma}{\sigma}\,\frac{\beta}{\mu}\right]^{\frac{1}{1-\upsilon}}.$$

It is easily verified that this satisfies the conditions of the characterization result. For example, substituting w^* into equation (A15) yields a positive interest rate with $1+R = \mu/\beta$. This is greater than unity by our assumptions on μ and β. Because real balances are constant in this equilibrium, the rate of inflation is equal to μ.

The intuition underlying equation (A22) is straightforward: The relative quantity of cash goods consumed in the equilibrium (that is, w^*) is increasing in $1-\sigma$, which is the relative weight in utility on these goods. It is decreasing in the money growth rate, μ, because increases in μ raise the nominal rate of interest, in turn increasing the cost of the cash good. Finally, consider $\upsilon \to 1$. This is easiest to interpret when $\sigma = 1/2$. In this case, the two consumption goods are perfect substitutes. Consequently, if the cash good is more expensive than the credit good, as is the case when $\mu \geq 1$, zero cash goods will be consumed, and $w^* = 0$ as $\upsilon \to 1$.

There are other equilibria in which inflation exceeds μ. To show this, we first study the properties of the functions $a(w)$ and $b(w)$.

According to equation (A19), $a(0) = 0$ and $a'(0) = 1$. Also, $a'(w) > 0$ for all $w \geq 0$. At the same time, equation (A20) indicates that $b(0) = 0$, $b'(w) \to \infty$ as $w \to 0$, and $b'(w) > 0$ for $w > 0$. These observations establish that $a(w)$ and $b(w)$ coincide at $w = 0$, with b rising more steeply than a for small values of w.

From the discussion leading up to equation (A22), we know there is a unique value of $w > 0$—namely, w^* in equation (A22)—where $a(w) = b(w)$. Since the two functions are continuous for $0 < w < w^*$, it follows that $b(w) > a(w)$ for w in this interval, and that a is steeper than b at $w = w^*$. The latter observation can be confirmed by direct differentiation, which yields

$$\frac{a'(w^*)}{b'(w^*)} = \left(\frac{1-\sigma}{\sigma}\right)^{\frac{1}{1-\upsilon}}\left(\frac{1-\upsilon}{\upsilon}\right)\left(\frac{\beta}{\mu}\right)^{\frac{\upsilon}{1-\upsilon}} + \frac{1}{\upsilon} > 1.$$

The strict inequality reflects that the expression immediately after the equality is positive and that $1/\upsilon > 1$ because $0 < \upsilon < 1$.

Our results on the a and b functions are summarized in figure A. Note how b rises above a and then crosses once. Eventually, the two curves are parallel, since $a'(w)$ and $b'(w)$

both converge to zero as $w \to \infty$. We can use this figure to study the set of equilibria for the model.

Consider an arbitrarily selected $w_0 < w^*$. To determine the value of w_1 implied by equation (A18), draw a vertical line up to $a(w_0)$. Then, identify w_1 such that $b(w_1)$ equals $a(w_0)$. This can be found by following a horizontal line to the left of $a(w_0)$ until it intersects b. The properties of these curves guarantee that such an intersection will occur for a positive value of w. With w_1 in hand, compute w_2 in the same way, and so on.

It should be clear that the sequence of w_t computed in this way converges to 0. Along this path, $b(w_t) \geq 0$ and $b(w_t) \to 0$ as $t \to \infty$. Because b is bounded above along the path, equation (A21) is satisfied. Because w_t declines monotonically, $R > 0$ at w^* and $w_t > 0$ for a given t, equation (A15) implies that $R_t > 0$ for each t. This establishes that the sequence just computed constitutes an equilibrium.[76] The same argument can be applied for each $0 < w_t < w^*$; in each of these equilibria there is a hyperinflation as $w_t \to 0$.[77]

Unique Equilibrium with Non-Ricardian, Constant Money Growth

The previous section showed that with a particular Ricardian policy, constant money growth results in a continuum of equilibria. Here is a particular non-Ricardian policy:

$$T_t = P_t s - \frac{R_t}{1+R_t} M_t ,$$

where s is a positive constant. It is easy to verify that the set of equilibria under this policy is a strict subset of the set of equilibria analyzed in previous section. Thus, we conclude that with constant money growth, a non-Ricardian policy does not lead to an overdetermined price level.

Fixed Interest Rate Policies

This section considers two representations of policy in which the government pegs the nominal rate of interest to a constant value, $R > 0$. In the first representation, fiscal policy is Ricardian and there exists a continuum of equilibria. In the second, policy is non-Ricardian and the equilibrium, if it exists, is unique.

The fixed value of R pins down m (see equation [A16]), c_1, and c_2:

$$c_1 = m, \; c_2 = y - c_1 .$$

As a consequence, the marginal utility of the cash good is constant, so that

$$\frac{P_{t+1}}{P_t} = \beta(1+R)$$

for all t. Consider two specifications of policy,

$$\text{(A23)} \quad T_t = -\frac{R}{1+R} m P_t + \varepsilon A_t$$

and

$$T_t = -\frac{R}{1+R} m P_t + d P_t ,$$

where d is a non-negative constant and $0 < \varepsilon \leq 1$. As we will show, the first policy is Ricardian, while the second is not. These policies may initially appear strange, but the motivation behind them will soon become clear. To determine whether a policy is Ricardian requires us to determine whether equation (A13) holds for all possible prices or only for equilibrium prices.

To investigate this further, it is convenient to write the flow-budget constraint in real terms,

$$\beta a_{t+1} + \tau_t + \frac{R}{1+R} m = a_t .$$

Here, $a_{t+1} = A_{t+1}/P_{t+1}$ and $\tau_t = T_t/P_t$. Substituting the first specification of policy in equation (A23) into the flow-budget constraint gives us

$$\text{(A24)} \quad a_{t+1} = \frac{1-\varepsilon}{\beta} a_t .$$

We seek to understand how $\tilde{a}_t \equiv \beta^t a_t$ evolves as $t \to \infty$. By substituting from equation (A9), we have

$$\text{(A25)} \quad q_T A_T = P_0 \beta^T \frac{A_T}{P_T} = P_0 \tilde{a}_T .$$

Recall that a policy in which $q_T A_T \to 0$ for all

76 Recall, in constructing equation (A18) we assumed the cash-in-advance constraint is binding. This assumption has been verified for $w_0 < w^*$.

77 Our results would not be significantly affected if we allowed labor to be endogenous. Introducing labor as a third argument in the utility function has the effect of adding an extra Euler equation, $-u_3/u_2 = f'(l)$, where $f'(l)$ denotes the marginal product of labor, l, and u_3 denotes the marginal utility of labor. Feasibility restricts l to some subspace, $l \in D$ (for example D might be the unit interval). Also, $y = f(l)$ denotes the production function. Combining the new Euler equation with the resource constraint produces a function, $l = F(w)$, where F has a nice analytical characterization with standard preferences and technology. To find an equilibrium, one would still start by looking for values of w_t that solve the difference equation, $A(w_t) = B(w_{t+1})$. One would then have to verify $F(w) \in D$, in addition to the other conditions listed in the characterization result, to verify that the values of w_t represent an equilibrium.

possible prices corresponds to a Ricardian policy, and one in which this occurs only for equilibrium prices is a non-Ricardian policy. Multiplying both sides of equation (A24) by β^{t+1}, we find

$$\tilde{a}_{t+1} = (1-\varepsilon)\tilde{a}_t = (1-\varepsilon)^t \tilde{a}_0,$$

or, using equation (A25),

$$q_T A_T = (1-\varepsilon)^T A_0 \rightarrow 0.$$

This establishes that the first policy in equation (A23) is Ricardian. The government's policy prevents the debt from exploding too fast, regardless of what happens. As a result, the intertemporal budget equation provides no useful restriction for pinning down prices.

Now consider the second policy in equation (A23). For this policy, total real assets evolve according to

$$a_{t+1} = \frac{1}{\beta}(a_t - d).$$

The policy makes the evolution of total assets exogenous, while letting the private economy determine the breakdown of real assets between money and bonds to be consistent with the interest rate peg. Solve for a_t and then multiply by β^t,

$$\beta^t a_t = a_0 - \frac{d}{1-\beta} + \frac{d}{1-\beta}\beta^t,$$

so that

$$\beta^t a_t \rightarrow \frac{A_0}{P_0} - \frac{d}{1-\beta},$$

where $a_0 = A_0/P_0$. This is a non-Ricardian policy because $\beta^t a_t \rightarrow 0$ for only one value of P_0—the one that satisfies

$$\frac{A_0}{P_0} = \frac{d}{1-\beta}.$$

We can now summarize our results for the interest rate peg. If it is accompanied by a Ricardian policy, the price level is not pinned down by the intertemporal budget equation, nor by the rest of the model. The model pins down only M_t/P_t and P_{t+1}/P_t, but not the numerator and denominator terms. Under the non-Ricardian policy, the intertemporal budget equation supplies the extra equation needed. Once again, the price level is not overdetermined under the non-Ricardian policy.

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Erratum

Figures were reported incorrectly in "Population Aging and Fiscal Policy in Europe and the United States," by Jagadeesh Gokhale and Bernd Raffelhüschen, in *Economic Review*, vol. 35, no. 4 (1999 Quarter 4). The first column on page 14 should have read (corrections italicized):

> By 2015, more than *a third* of the people living in these three countries will be 60 or older.? In Italy, four out of every *nine* persons will be 60 or older by 2035! In Sweden, Austria, and Germany, two out of every *five* persons will be elderly by our criterion. In comparison, the U.S. population will be much younger, with only one of every *three* persons falling into the elderly category.