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Nominal Prices and Interest Rates in General Equilibrium: Money Shocks

The effects of exogenous shocks in the monetary growth rate on nominal prices and interest rates are analyzed. A representative-individual rational-expectations framework is adopted, and a demand for money is generated by entering money in the utility function. The effects of monetary shocks on inflation and interest rates are shown to depend on the autocorrelation of the monetary growth rate: for example, under serial independence the inflation rate equals the monetary growth rate and the nominal interest rate is nonrandom, while under positive (negative) autocorrelation the nominal interest rate is affected positively (negatively) by positive growth rate shocks.

I. Introduction

The outstanding problem of neoclassical monetary theory has been to determine the effect of changes in the money stock on nominal prices and interest rates. The classic result is that, in the absence of money illusion, contracts fixed in nominal terms, and other such impedimenta, money is neutral. Thus a doubling of the money stock results in a doubling of the nominal price level and in no change in nominal interest rates (except at most for a temporary "liquidity effect" while the price level is rising to its new level; during the adjustment period the nominal interest rate might temporarily drop). This neutrality result has conventionally been proved in a static framework. In intertemporal models it is necessary to distinguish between static neutrality and dynamic neutrality: if a model displays dynamic neutrality, fluctuations in the money stock immediately induce a proportional change in the nominal price level, and no change in nominal interest rates or any real variable. This property is different from and stronger than static neutrality, which, in dynamic models, states only that if the money stock at all dates is doubled so will be the price level at all dates.

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The validity of the dynamic neutrality property is readily demonstrated under the essential proviso that the new money is put in circulation in the form of an interest payment on preexisting money. In that case the fact that the variations in the own rate of return on money just offset the variations in the inflation rate means that the real opportunity cost of holding money is invariant to fluctuations in the nominal money stock. Thus monetary changes are identical to currency reforms, under which units of the old currency are exchangeable for new units at a fixed rate. But the analogy between monetary policy and currency reforms is highly dubious: in fact, the own rate of return on money is fixed by law at a constant rate (until recently, zero), so that the real opportunity cost of holding money fluctuates with the rate of inflation. It is far from clear what the relation between money and prices will be if new money is put into circulation by some means other than an interest payment on old money.

In this paper we describe the effect of exogenous fluctuations in the monetary growth rate on the inflation rate and nominal interest rate when money is non-interest-bearing. It is assumed that the monetary growth rate is autocorrelated, and the effects of monetary shocks will be seen to depend essentially on the sign of the autocorrelation. The following paper is concerned with the effect on nominal prices and interest rates of real rather than money shocks: to this end, the money stock is assumed there to follow a constant growth rate rule, but the endowment of the consumption good is taken to be Markov, rather than constant as here.

II. Optimization of the Representative Individual

We assume that the representative trader maximizes

$$E \sum_{t} \beta^{t}[U(x_t) + V(M_t/p_t)]$$

subject to a budget constraint.[1] Here x_t is consumption of the real good, which we call X, at date t. M_t is the nominal money stock, p_t is the

1. The assumption that utility is additively separable over time, while restrictive, is customary in models of this sort. The specification of additive separability in consumption and holdings of real balances at the same date is adopted because its relaxation involves complications which have no intuitive interpretation. The reason appears to be that, although it is natural to specify that the marginal utility of real balances rises with the level of consumption (because of the usual transactions demand argument), it is difficult to understand why the marginal utility of consumption should rise with real balances. However, one cannot have one without the other, second partial derivatives being what they are. Additive separability also has the convenient property that the analysis of real securities prices and interest rates reported in LeRoy and LaCivita (1981) and LeRoy (1982*b*, 1983) carries over to the present monetary model without modification.

nominal price level (so that M_t/p_t is real balances), β is a discount factor ($0 < \beta < 1$), E is the expectations operator, and U and V are smoothly differentiable and strictly concave functions.[2] Additionally, V obeys the Inada conditions $V'(0) = \infty$ and $V'(\infty) = 0$. The budget constraint requires that the dollar value of the financial assets the individual owns as trading at date t begins, W_t^i, equal the dollar value of consumption in excess of the endowment plus the dollar value of all securities held after trading has concluded. Here i is the state prevailing at date t; i will equal h or l as the monetary growth rate is high or low, as defined below. The financial assets consist of money, 1-period nominal bonds, and 1-period real bonds. A nominal bond costs \$1.00 at date t and yields $1 + n_t^i$ dollars for certain at date $t + 1$. A real (or index-linked) bond, by contrast, costs one unit of X at date t and yields $1 + r_t^i$ units of X for certain at date $t + 1$. Except in the degenerate case of certainty about the price level at the next date, nominal and real bonds are significantly different financial assets since the real rate of return on the nominal (real) bond is random (nonrandom), just as the nominal rate of return on the real (nominal) bond is random (nonrandom).

The disposition of financial wealth into consumption in excess of the endowment and asset holdings is described by the budget constraint

$$W_t^i = p_t^i(x_t^i - \bar{x}) + M_t^i + B_t^i + p_t^i b_t^i, \tag{1}$$

where B_t^i is the number of nominal bonds the individual holds after trading, b_t^i is the number of real bonds held after trading (note that quantity variables measured in money terms are indicated by capital letters throughout, while lowercase letters denote prices and quantity variables measured in units of the consumption good), $\bar{x}$ is the nonrandom endowment, and M_t^i is the nominal money stock held between dates t and $t + 1$ if the state at t is i. In turn, next-period nominal wealth W_{t+1}^j is related to the quantity of financial assets held this period according to

$$W_{t+1}^j = M_t^i + (\overline{M_{t+1} - M_t})^j + B_t^i(1 + n_t^i) + p_{t+1}^j b_t^i(1 + r_t^i). \tag{2}$$

Here j denotes the state prevailing at $t + 1$, and the term $(\overline{M_{t+1} - M_t})^j$ indicates a lump-sum transfer of new currency from the government, the amount of which depends on the state at date $t + 1$.

We assume that the economy is in a stochastic steady-state equilibrium.[3] Because (1) money illusion is excluded, (2) utilities are addi-

2. Alternatively, one could allow random endowments ($\bar{x}^h \neq \bar{x}^l$) but restrict U to be linear. That case would be identical to the model studied here since under linearity random variations in the endowment do not affect the marginal utility of consumption, and therefore do not induce substitution among financial assets.

3. For a brief discussion of the equilibrium concept used in this paper, see the passage on recursive equilibrium in LeRoy (1982*a*), and consult the papers cited there for fuller

tively separable over time, and (3) probabilities are Markov, equilibrium levels of all real quantity variables and relative prices can be written as functions of the current value of the monetary growth rate alone, and therefore do not depend on the date or the lagged values of the monetary growth rate (nominal prices and values, however, necessarily have whatever trend is assumed for the money stock, so as to preserve the stationarity of the probability distribution of real balances).[4] Consequently, it is permissible to assume that there exist two utility-of-real-wealth functions $Z^h(W_t^h/p_t^h)$ and $Z^l(W_t^l/p_t^l)$, one each for the high and low states, such that the representative individual's maximization may be written in the dynamic programming form

$$Z^i(W_t^i/p_t^i) = \max \{U(x) + V(M/p_t^i) + \beta[\pi(h|i)Z^h(W_{t+1}^h/p_{t+1}^h) + \pi(l|i)Z^l(W_{t+1}^l/p_{t+1}^l)]\}. \tag{3}$$

Here $\pi(j|i)$ denotes the probability that the state will be j at $t + 1$ given that it is i at t $(i, j = h, l)$. To obtain the first-order conditions, let us maximize (3) over x, M, B, b, W_{t+1}^h, and W_{t+1}^l subject to the three budget constraints (1) and (2) $(j = h, l)$.[5] The first-order conditions are

$$U_t^i = \beta[\pi(h|i)U_{t+1}^h + \pi(l|i)U_{t+1}^l](1 + r_t^i), \tag{4}$$

$$U_t^i = V_t^i + \beta\left[\pi(h|i)\frac{U_{t+1}^h p_t^i}{p_{t+1}^h} + \pi(l|i)\frac{U_{t+1}^l p_t^i}{p_{t+1}^l}\right], \tag{5}$$

and

$$\frac{V_t^i}{U_t^i} = \frac{n_t^i}{1 + n_t^i}, \tag{6}$$

where U_t^i and V_t^i are the derivatives of U and V in state i at date t.

These conditions are readily interpreted. Equation (4) sets the inverse of the real interest rate (plus unity) equal to the expectation of the marginal rate of substitution between consumption at successive dates.

treatment. See Brock (1975) for derivation of conditions sufficient to rule out the existence of non-steady-state paths in deterministic monetary models similar to this one.

4. A more rigorous procedure would be to define the state as a function of both the level of the money stock and its rate of change over the period just ended. Thus the Z functions would have two arguments rather than one. The resulting equilibrium conditions would then be such that nominal prices and interest rates would be homogeneous of degrees one and zero, respectively, in the nominal money stock. Thus the solution procedure adopted here, which assumes these properties without proof, would be shown to be valid. The more rigorous derivation would follow exactly the procedure of Mehra and Prescott (1983). Mehra and Prescott's model is nonmonetary, but it has homogeneity properties similar to those encountered here. Here the homogeneity property is a consequence of the absence of money illusion, whereas in Mehra and Prescott it follows from the assumption of constant relative risk aversion, plus the specification that the probability distribution of the endowment is Markov in its rate of change.

5. Use the result that the derivatives of the utility-of-real-wealth functions equal the marginal utilities of consumption, from the envelope theorem.

Since this relation between real interest rates and the marginal utilities of consumption is familiar, further interpretation here is unnecessary. Real interest rates will not be considered further until the companion paper.

To understand (5), suppose that an individual were considering whether to consume an additional unit of real output or instead hold real balances for one period and spend the increment at the next date. In the former case, he gains current utility of U_t^i. If he holds one unit of real balances his utility gain is V_t^i now plus the discounted expected utility of the additional consumption made possible in the next period. If the high money stock state occurs at date $t + 1$, the unit of real balances becomes p_t^i/p_{t+1}^h units of real balances due to the change in the price level. When these real balances are spent, the discounted utility is therefore $\beta U_{t+1}^h p_t^i/p_{t+1}^h$. From this, (5) is immediate.

Equation (6) expresses the marginal rate of substitution between real balances and consumption as a function of the nominal interest rate. The implication that the demand for real balances is directly related neither to inflation (nor, had we included them, to many-period interest rates) nor to any consideration of risk is controversial. To appreciate that this conclusion depends on a fairly general feature of our model (and not, e.g., on the specific assumption that money is an argument of the utility function), we justify (6) by means of an arbitrage argument. Suppose that an individual is initially at an optimum, and that he modifies his portfolio by adding \$1.00 to his holdings of money. This costs him \$1.00 now, will generate a \$1.00 return next period, and increases current utility by V_t^i/p_t^i. Additionally, let the trader go short $\$1/(1 + n_t^i)$ in nominal bonds. This generates a current revenue of $\$1/(1 + n_t^i)$, but it will cost him \$1.00 next period to cover his position. Finally, assume that the individual decreases his consumption now by $\$n_t^i/(1 + n_t^i)$. From inspection of table 1 it is evident that these changes involve a total cost of zero both currently and in the next period. But since the individual can conduct either the outlined arbitrage or its reverse, the net utility gain must also be zero. Equation (6) results.

TABLE 1 **Justification for Equation (6)**

	Current Utility	Current Cost	Next-Period Cost
1. Long \$1 in money	$\frac{V_t^i}{p_t^i}$	1	-1
2. Short \$1 in nominal bonds	0	$-\frac{1}{1 + n_t^i}$	1
3. Consume $\frac{\$n_t^i}{1 + n_t^i}$ less	$-\frac{U_t^i}{p_t^i}\frac{n_t^i}{(1 + n_t^i)}$	$-\frac{n_t^i}{1 + n_t^i}$	0
Total cost	. . .	0	0

This argument makes it apparent why inflation risk does not directly affect the individual's demand for real balances: 1-period nominal bonds have exactly the same risk exposure as money. Whether or not the individual chooses actually to use nominal bonds to offset the inflation risk in his holdings of real balances, the fact that he can do so means that the price of nominal bonds captures all the user costs of holding real balances. Hence the nominal interest rate is the only price that is set against the utility gain from holding real balances. Additionally, the arbitrage construction clearly indicates that the argument has nothing to do with the fact that a demand for money was generated by including money in the utility function. For suppose instead that we had generated a demand for money by adopting a cash-in-advance constraint along the lines of Clower (1967) and Lucas (1980). Then the entry in table 1 for the utility gain resulting from the increase in real balances held would be the discounted value of the expected utility gain from being able to consume \$1.00 more if the cash constraint should be binding in the next period. Aside from replacing V_t^i by this expected utility term, the argument just presented would be unaffected.

However, the argument that the demand for money depends only on the 1-period nominal interest rate does have one major limitation. The arbitrage argument depends essentially on the assumption that a \$1.00 bond becomes identical to \$1.00 in money next period; that is, that the cost of cashing in the matured 1-period bond is zero. In such models as Santomero's (1974), this property is not satisfied: an individual can convert his savings deposit to cash only by paying a transactions cost. In models of this sort the demand for money will generally depend on more than one interest rate.

III. Equilibrium

To derive equilibrium expressions for prices and interest rates, we first impose clearing of the markets for goods, bonds, and money. Since we are dealing with a representative individual model with nonrandom real endowments, the market-clearing conditions for goods and bonds are $x_t^i = \bar{x}$ and $B_t^i = b_t^i = 0$, while we will clear the market for money by equating the quantity of money individuals wish to hold with that carried over from the preceding period plus the new money provided by the government.

We will assume that if the state at t is high (low), the monetary growth rate from $t - 1$ to t is θ^h (θ^l), with $\theta^h \geqslant \theta^l$. Further, it is assumed that

$$\beta^{-1}(1 + \theta^l) > 1, \tag{7}$$

so that the monetary growth rate in both states either is positive or, if negative, is smaller in absolute value than the rate of time preference. This restriction will be seen below to assure the existence of stationary equilibrium (although it is stronger than necessary) and to simplify its analysis.

Under the adopted equilibrium concept, the equilibrium real money stock will depend on the state but not the date:

$$M_t^i/p_t^i = m^i, \qquad i = h, l \quad \text{for all } t, \tag{8}$$

where m^i is the real money stock in state i at any date. Using the equilibrium condition (8) and the assumed money supply rule, conditional 1-period inflation rates are expressible as

$$\begin{aligned} \frac{p_{t+1}^h}{p_t^h} &= 1 + \theta^h, \quad \frac{p_{t+1}^l}{p_t^l} = 1 + \theta^l \\ \frac{p_{t+1}^h}{p_t^l} &= \frac{(1 + \theta^h)m^l}{m^h}, \quad \frac{p_{t+1}^l}{p_t^h} = \frac{(1 + \theta^l)m^h}{m^l}. \end{aligned} \tag{9}$$

If these expressions are used to derive the equilibrium distribution of real balances from the first-order condition (5), the equations for equilibrium m^h and m^l are seen to be

$$V'(m^h) = U'(\bar{x})\left[1 - \beta\left(\frac{\pi(h|h)}{1 + \theta^h} + \frac{\pi(l|h)m^l}{(1 + \theta^l)m^h}\right)\right] \tag{10}$$

$$V'(m^l) = U'(\bar{x})\left[1 - \beta\left(\frac{\pi(h|l)m^h}{(1 + \theta^h)m^l} + \frac{\pi(l|l)}{1 + \theta^l}\right)\right]. \tag{11}$$

Finally, to economize on parameters it will be assumed that $\pi(h|h) = \pi(l|l) = \pi$, so (10) and (11) become

$$V'(m^h) = U'(\bar{x})\left[1 - \beta\left(\frac{\pi}{1 + \theta^h} + \frac{(1 - \pi)m^l}{(1 + \theta^l)m^h}\right)\right] \tag{12}$$

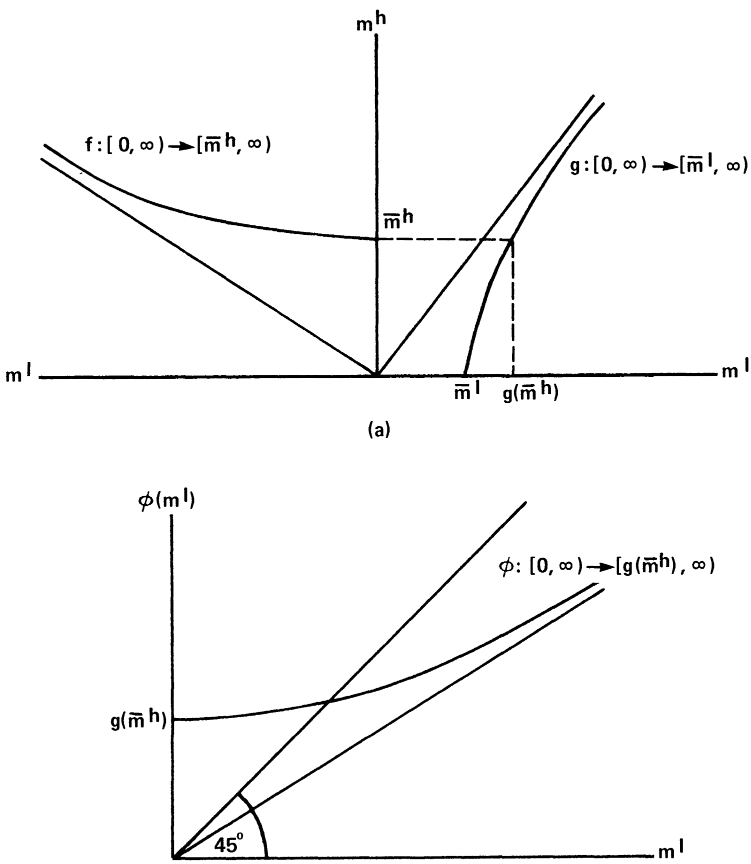
$$V'(m^l) = U'(\bar{x})\left[1 - \beta\left(\frac{(1 - \pi)m^h}{(1 + \theta^h)m^l} + \frac{\pi}{1 + \theta^l}\right)\right]. \tag{13}$$

Given m^h and m^l, n^h and n^l are immediate from (6).

IV. Existence and Uniqueness

In nonmonetary representative-individual exchange models with smooth preferences, existence and uniqueness of equilibrium are immediate; the vector of equilibrium prices is just the gradient of the representative individual's utility function evaluated at the endowment vector. But in a monetary economy the "endowment" of real balances is endogenous since it depends on the nominal price level, so this argument does not apply.

To demonstrate existence, observe that (12) implicitly defines m^h as an increasing function of m^l. Denote this function f. The function f has domain $[0, \infty)$ and range $[\bar{m}^h, \infty)$, where $\bar{m}^h = \tilde{V}'\{U'(\bar{x})[1 - \beta\pi/(1 + \theta^h)]\}$ and $\tilde{V}'$ is the inverse function of V'. Similarly, (13) implicitly defines m^l as an increasing function g of m^h. The domain of g is $[0, \infty)$ and its range is $[\bar{m}^l, \infty)$, where $\bar{m}^l = \tilde{V}'\{U'(\bar{x})[1 - \beta\pi/(1 + \theta^l)]\}$. Finally, define $\phi: [0, \infty) \rightarrow [g(\bar{m}^h), \infty)$ as the composite function $\phi(x) = g(f(x))$. Evidently equilibrium consists of a fixed point for ϕ. Functions f and g are diagrammed in figure 1a, while ϕ is shown in figure 1b.



(b)

FIG. 1.—Existence of equilibrium

We need to show that ϕ crosses the 45° line. Since zero is mapped onto $g(\bar{m}^h) > 0$ and ϕ is continuous, it is sufficient to show that for high m^l, $\phi'(m^l)$ is bounded below unity. We have

$$f' = -\frac{U'(\bar{x})\beta(1-\pi)}{(1+\theta^l)\{m^h V_h'' + V_h' - U'(\bar{x})[1-\beta\pi/(1+\theta^h)]\}}$$

$$\cong \frac{\beta(1-\pi)}{(1+\theta^l)[1-\beta\pi/(1+\theta^h)]}$$

$$g' = -\frac{U'(\bar{x})\beta(1-\pi)}{(1+\theta^h)\{m^l V_l'' + V_l' - U'(\bar{x})[1-\beta\pi/(1+\theta^l)]\}}$$

$$\cong \frac{\beta(1-\pi)}{(1+\theta^h)[1-\beta\pi/(1+\theta^l)]},$$

where $V_h' \equiv V'(m^h)$, and similarly for V_h'', V_l', and V_l''. The approximate equalities are consequences of the fact that for m^l and m^h large, $V' \cong V'' \cong 0$. Thus $\phi' = f'g'$ is given approximately by

$$\phi' \cong \frac{\beta^2(1-\pi)^2}{(1+\theta^h-\beta\pi)(1+\theta^l-\beta\pi)}$$

when the argument of ϕ is large. To see that $\phi' < 1$ asymptotically, observe that assumption (7) (that $\beta < 1 + \theta^l$) implies

$$1 + \theta^l - \beta\pi > \beta(1-\pi). \tag{14}$$

A fortiori,

$$1 + \theta^h - \beta\pi > \beta(1-\pi). \tag{15}$$

Multiplying (14) by (15), $\phi' < 1$ is the consequence.

To understand the meaning of this restriction on the parameters, let us abstract from uncertainty by setting $\theta^h = \theta^l = \theta$. If the restriction were violated, so that θ were negative and large in absolute value, then at a stationary equilibrium the price level would be decreasing faster than individuals' rate of time preference. But then stationary equilibrium could not exist since at any proposed solution to the individual's problem involving consumption constant over time and inflation equal to the rate of monetary expansion, agents could always increase utility by consuming less now, acquiring money, waiting while the real value of the money increased due to deflation, and consuming later.[6]

6. The necessity for the condition $\beta < 1 + \theta$ for the existence of steady-state equilibrium in the deterministic case was noted by Brock (1975), among others. Note that none of this rules out the existence of nonstationary equilibria if $\beta > 1 + \theta$.

Uniqueness is immediate. Let us define $\mu = m^l/m^h$. Then the equilibrium conditions may be written

$$V'(m^h) = U'(\bar{x})\left\{1 - \beta\left[\frac{\pi}{1 + \theta^h} + \frac{(1 - \pi)\mu}{1 + \theta^l}\right]\right\} \tag{16}$$

$$V'(m^l) = U'(\bar{x})\left\{1 - \beta\left[\frac{(1 - \pi)}{(1 + \theta^h)\mu} + \frac{\pi}{1 + \theta^l}\right]\right\}. \tag{17}$$

Now solve (16) for m^h and (17) for m^l, and divide the latter by the former. Denoting the resulting function ψ, we have

$$\psi(\mu) \equiv \frac{m^l}{m^h} = \frac{\tilde{V}'\left\{U'(\bar{x})\left[1 - \beta\left(\frac{(1 - \pi)}{(1 + \theta^h)\mu} + \frac{\pi}{1 + \theta^l}\right)\right]\right\}}{\tilde{V}'\left\{U'(\bar{x})\left[1 - \beta\left(\frac{\pi}{1 + \theta^h} + \frac{(1 - \pi)\mu}{1 + \theta^l}\right)\right]\right\}}.$$

Since equilibrium consists of a fixed point for ψ, if there existed more than one equilibrium it would be the case that $\psi'(\mu) = 1$ for some value of μ. But an increase in μ increases m^h (from [16]), and decreases m^l (from [17]), so $\psi'(\mu) < 0$.

V. Characterization of Equilibrium

In order to describe equilibrium prices and interest rates it is necessary to know more about the function $\psi(\mu)$. We have

FACT 1: The domain of ψ may be taken as an open interval $(\bar{\mu}, \bar{\bar{\mu}})$ which includes unity. Also we have $\lim_{x \to \bar{\mu}^+} \psi(x) = \infty$ and $\lim_{x \to \mu^-} \psi(x) = 0$.

To verify these facts, note that ψ is defined only when both terms in brackets in (16) and (17) are positive. By inspection the set of values of μ for which this is true is an interval. By successively setting each of the terms in brackets to zero we may therefore derive $\bar{\mu}$ and $\bar{\bar{\mu}}$ that bound the domain of ψ. We have

$$\bar{\mu} = \frac{1 - \pi}{[\beta^{-1} - \pi/(1 + \theta^l)](1 + \theta^h)}$$

$$\bar{\bar{\mu}} = \frac{(1 + \theta^l)[\beta^{-1} - \pi/(1 + \theta^h)]}{1 - \pi}.$$

To assure ourselves that the interval $(\bar{\mu}, \bar{\bar{\mu}})$ is not empty, we may verify that $\bar{\mu} < \bar{\bar{\mu}}$ is equivalent to $\beta^2 (1 - \pi)^2 < (1 + \theta^h - \beta\pi)(1 + \theta^l - \beta\pi)$. But this is just the condition for existence of equilibrium.

To prove that $\bar{\mu} < 1$, assume the contrary, solve the resulting inequality for $1 + \theta^l$, use the assumption $1 + \theta^l > \beta$ to eliminate θ^l, and simplify to obtain $\beta > 1 + \theta^h$, contradicting the assumed restriction on

θ^h. The proof that $1 < \bar{\bar{\mu}}$ is similar.[7] Finally, $\lim_{x \to \mu^+} \psi(x) = \infty$ is immediate, since as x approaches $\bar{\mu}$, the term in brackets in (17) approaches zero. But this implies that m^l goes to infinity. The justification for $\lim_{x \to \bar{\bar{\mu}}^-} \psi(x) = 0$ is similar.

FACT 2: In equilibrium, $m^h \lesseqgtr m^l$ as $\pi \gtreqless 1/2$.

To see this, first observe that since unity is in the domain of ψ, $\psi(1)$ is well defined. Further, $\psi(1) \gtreqless 1$ as $\pi \gtreqless 1/2$, as is directly proved from (16) and (17). But if $\psi(1) \gtreqless 1$, then because ψ is downward sloping and because we know that there exists a unique fixed point μ^* in the domain of ψ, it must follow that $\mu^* \gtreqless 1$ (fig. 2). To interpret this result, suppose that the monetary growth rate over the past period was high. If $\pi > 1/2$, agents extrapolate the past realized monetary growth rate and therefore expect unusually high monetary growth over the coming period. Assuming that high monetary growth translates into high inflation (this is verified as fact 4 below), agents are led to economize on real balances, cutting back to that point at which the higher anticipated depreciation of money is offset by its higher service yield. If $\pi < 1/2$, however, the unusually high current monetary growth rate leads agents to anticipate unusually low monetary growth over the coming period, resulting in $m^h > m^l$. Dynamic neutrality occurs in the intermediate case of $\pi = 1/2$, since then monetary growth rates are serially independent and agents are therefore led to bid the real money stock to the same level at every date irrespective of the realized monetary growth rate.

FACT 3: $n^h \gtreqless n^l$ as $\pi \gtreqless 1/2$.

From (6), the interest rate is negatively related to the real money stock, so fact 3 follows directly from fact 2. One may interpret fact 3 either from a Keynesian viewpoint (nominal interest rates are high because the [real] money stock is low) or a Fisherian viewpoint (nominal interest rates are high because expected inflation is high). Both interpretations are superficial at best since they attach a causal interpretation to the relation between two endogenous variables.

FACT 4: The inflation rate is higher in the high monetary growth rate state than in the low monetary growth rate state.

This statement is ambiguous as it stands because the inflation rate depends on the state at the preceding date as well as the current state. It is taken to mean that both of the possible values taken on by the inflation rate in the high growth rate state are higher than either of the values it takes on in the low growth rate state. It seems intuitively

7. If instead of $\beta^{-1}(1 + \theta^l) > 1$, we had assumed only the weaker inequality $\beta^2(1 - \pi)^2 < (1 + \theta^h - \beta\pi)(1 + \theta^l - \beta\pi)$, which allows θ^l to violate $\beta^{-1}(1 + \theta^l) > 1$ provided that θ^h is sufficiently high, a stationary equilibrium would still exist but it would no longer generally be true that $\bar{\mu} < 1 < \bar{\bar{\mu}}$. This would complicate the proof that equilibrium $\mu^* \gtreqless 1$ as $\pi \gtreqless 1/2$, although the result itself remains valid.

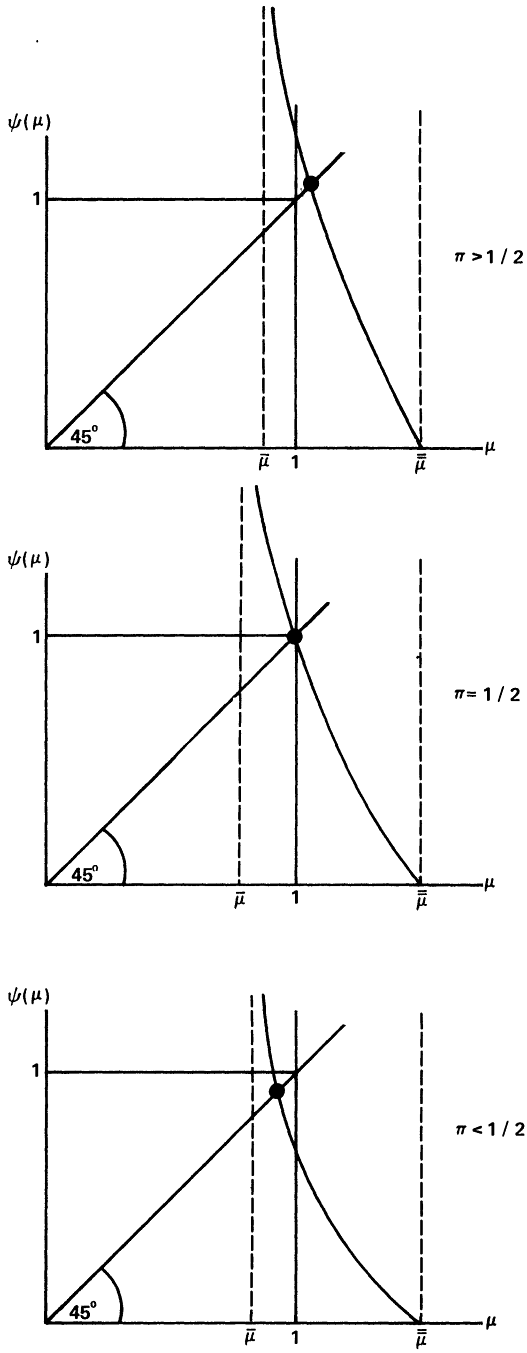


FIG. 2.—Characterization of real money stocks

TABLE 2 Inflation Rate

Current Monetary Growth Rate	Next-Period Monetary Growth Rate: High	Next-Period Monetary Growth Rate: Low
High	θ^h	$(1 + \theta^l)/\mu^* - 1$
Low	$(1 + \theta^h)\mu^* - 1$	θ^l

plausible that inflation rates should covary positively with monetary growth rates, but since the result does not always obtain in overlapping-generations models (LeRoy and Raymon 1983), it is noteworthy that it can be proved here.

First, the definitions of the inflation rate are recalled in table 2. In view of the fact that $\mu^* \gtreqless 1$ as $\pi \gtreqless 1/2$, the monetary growth rates and inflation rates are as shown in figure 3. It is evident from the diagram that the required property is satisfied if $\pi \geqslant 1/2$, so it is necessary to consider only $\pi < 1/2$. The inequality we need to prove, $p^h_{t+1}/p^l_t > p^l_{t+1}/p^h_t$, is equivalent to $1/[(1 + \theta^h)\mu^*] < \mu^*/(1 + \theta^l)$. To show this when $\pi < 1/2$, suppose the contrary. But then the term in brackets of (16) is unambiguously greater than the corresponding term of (17), since $\pi/(1 + \theta^h) < \pi/(1 + \theta^l)$ and $(1 - \pi)\mu^*/(1 + \theta^l) < (1 - \pi)/[\mu^*(1 + \theta^h)]$, so $\psi(\mu^*) > 1$. Since $\mu^* = \psi(\mu^*) < 1$ when $\pi < 1/2$, the assumption that μ^* is the equilibrium is contradicted.

VI. An Apologia[8]

The device of generating a demand for money by including money as an argument in agents' utility functions is in bad odor among monetary theorists. Now, the most direct way to argue against putting money in the utility function would be to point out concrete instances in which we have been misled by those who have done so. Given that many of our most valuable insights into neoclassical monetary theory have been based either explicitly or implicitly on money in the utility function (MIUF) models—Patinkin (1965), Friedman (1969), Brock (1975), for example—there would seem to be no shortage of potential targets. But the critics have almost never taken this line. Rather, the argument is methodological: including money in the utility function constitutes "implicit theorizing" (Wallace 1980). Because, it is argued, the motivation for holding money is buried in an exogenously assumed utility function, it becomes impossible completely to account for monetary

8. McCallum (1982) has provided a balanced and intelligent critique of overlapping-generations models and defense of money in the utility function which, however, emphasizes points somewhat different from those raised here. See also Danthine and Donaldson (1983).

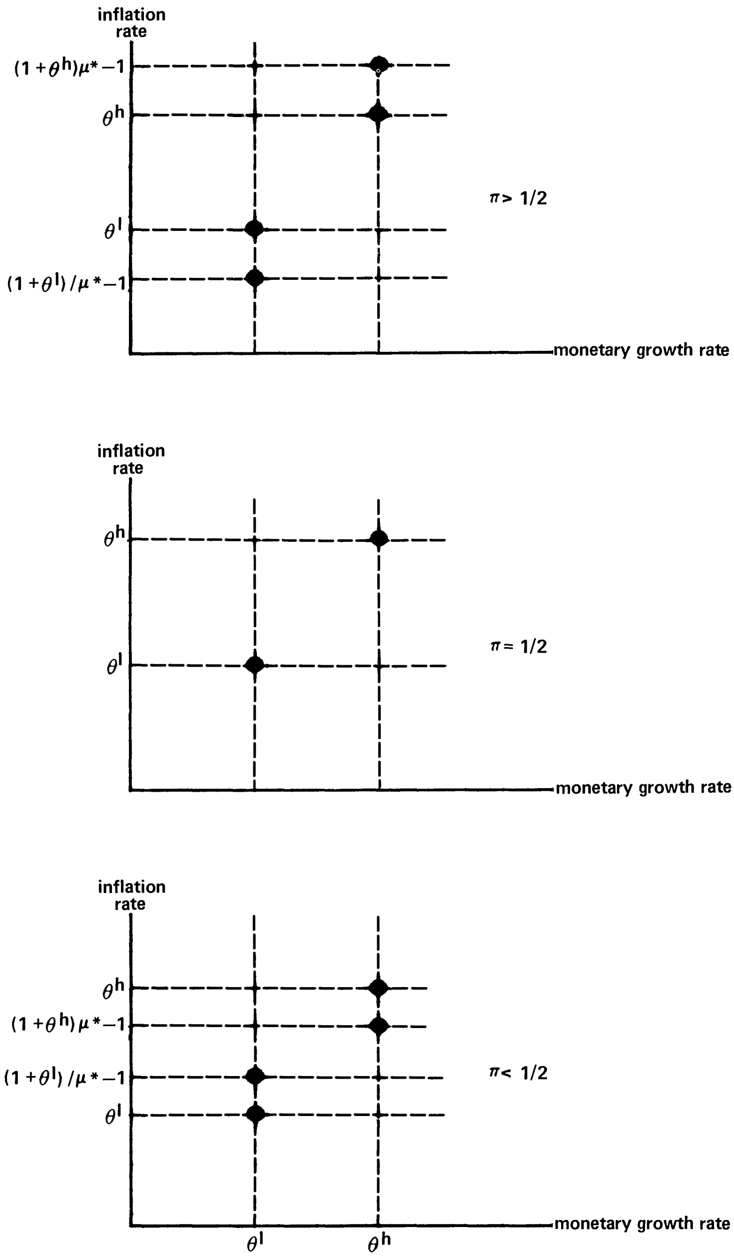


FIG. 3.—Money and inflation

phenomena in the terms economists are accustomed to take as primitive: preferences among goods which (unlike money) are actually consumed, and technology.

It cannot be denied that there is merit in this view, and that it counts against money in the utility function. But there are extenuating circumstances. It is not always easy to draw the line separating implicit from explicit theorizing. For example, Lucas (1980) generated a demand for money by imposing a cash-in-advance constraint on traders. At first glance, imposition of an arbitrary requirement that consumption be limited by cash on hand appears to be no less ad hoc than putting money in the utility function. But when Lucas appended a story about worker-consumer pairs and goods of different colors, it became clear that the cash-in-advance constraint can in fact be derived from bona fide economic primitives. Then the complaint might be that Lucas's model is artificial or restrictive, but there is no question of implicit theorizing. Now, the question is this: Would Lucas have been open to the charge of implicit theorizing had he neglected to provide the background interpretation? The actual model would have been the same either way.

It is easy to multiply instances in which what appears to be implicit theorizing goes unchallenged. Students of adjustment-cost investment models or learning-by-doing models include the time derivative of the capital input or of the rate of output, or the integral of the latter, as an argument in the production function. Plainly such a production function is not a technological primitive but implicitly represents the outcome of some sort of optimization. Similarly, analysts who assume the nonexistence of some markets without explicit reference to, for example, costs of enforcing contracts are conducting implicit theorizing. In what way are these different from putting money in the utility function? The point is simply that we should not be too rigid or hasty in labeling incomplete analysis or exposition as implicit theorizing.

It is fair to say that as yet we have no fully satisfactory paradigm for modeling money. Consequently, we are not in a position to give an acceptable account of what distinguishes money from other assets. There have been two responses to this situation (besides that of indefinitely postponing doing applied work on money). One is to assume that nothing distinguishes money from other assets. This at least has the merit of avoiding implicit theorizing. Overlapping-generations models of money are of this class, and the most striking conclusions reached by overlapping-generations theorists depend essentially on the assumption that money is basically no different from other assets. Wallace's (1981) argument that the method of financing a government deficit is irrelevant is the most obvious example: if money is the same as bonds, it cannot matter which the government uses to reconcile taxes and expenditures. In the international context, the corollary of the principle that money is no different from other assets is the principle that national moneys are no different from one another. Kareken and Wallace (1981), using an overlapping-generations model, pursued this idea to its logical conclusion. If national moneys are all equivalent as stores of wealth, agents will treat moneys as perfect substitutes (as long as they are priced to yield the same rate of return). Kareken and Wallace showed that, as a derivable consequence of this proposition, a fixed exchange rate regime is indefinitely sustainable. Further, it does not matter what that exchange rate is. Readers of Kareken and Wallace's paper would be surprised to learn that Americans hold many more dollars than French francs, while the reverse is true for Frenchmen; these facts are not easy to explain if national moneys are intrinsically useless and therefore essentially identical. An equally puzzling fact from the vantage of the Kareken-Wallace framework is that in floating exchange rate regimes, exchange rates do in fact fluctuate. Finally, the breakdown of the Bretton Woods system of fixed exchange rates must appear entirely mysterious. These examples suggest that avoiding implicit theorizing provides no guarantee that our models will not produce results at odds with the simplest facts of monetary life.

A second response to the fact that we are without a completely adequate way to model money is, of course, to enter real balances as an argument in the utility function. By doing so we sidestep the fundamental problem of explaining the role of money. However, we also at least remind ourselves that money does in fact render a service different from that of other assets, even though we are not able or willing to characterize that service explicitly. Comparison of results derived under MIUF with, for example, those under overlapping generations serves to inform us which overlapping-generations results depend essentially on the assumption that money does not render any distinctive service, and which are likely to be more general. That the Wallace irrelevance results do not obtain if money is different from bonds is intuitively obvious (in any case, it is easy to verify this formally using an MIUF model very similar to that presented above, but modified to allow for government taxation and transfers). It is equally immediate that the Kareken-Wallace results on exchange rates would not obtain if, for example, Americans were assumed to derive utility from their holdings of dollars and Frenchmen from their holdings of French francs. Of course, it would be incorrect to draw from these examples the conclusion that MIUF models are superior to their rivals. The relation between MIUF and overlapping-generations or cash-in-advance models is symmetric: each has blind spots, and each should be used to check results derived from the others. The fact that putting money in the utility function constitutes implicit theorizing is a real disadvantage, and one not shared by overlapping generations; our point is not to deny this, but only to assert that other approaches have equally important disadvantages.

The outstanding advantage of putting money in the utility function is that one can use a representative-individual framework. This is a major analytical convenience: in representative-individual models existence and uniqueness of equilibrium are immediate, and comparative dynamics is almost equally easy. Of course, this convenience is purchased at some sacrifice of logic: if all agents really were identical, no transactions would occur, so a (token) money would not be held. This point is well taken but should not be used to dismiss MIUF models out of hand. The question is not whether the logical underpinnings of MIUF are impeccable, but whether concrete results derived under MIUF are likely to be useful. We have already argued that where MIUF and overlapping-generations results differ, the former have an important role to play as a check on the latter.

But to a surprising extent, the results derived under MIUF appear to be similar to those obtained from models which do not require implicit theorizing. As an example of this similarity, compare Bewley (1980) (see also Bewley [1981]) and Brock (1975). Bewley considered an exchange economy composed of heterogeneous (in terms of preferences and endowments) infinitely lived risk-averse agents. These traders can smooth their consumption streams by accumulating and decumulating money as their endowment realizations are high or low. Money balances are augmented by interest payments from the government, where the interest rate is constant. Since the taxes imposed to finance the interest payments are just sufficient to cover these interest payments, the aggregate money stock is constant. Bewley showed that if the interest rate on money is less than agents' rates of time discount, then equilibrium nominal prices exist. However, the resulting allocation is not Pareto optimal. The reason is that, because of the assumption on the interest rate, agents can insure themselves against low endowment realizations only at a positive opportunity cost. But the social cost of insuring against individual misfortunes is near zero, assuming a large number of individuals. The wedge between private and social cost implied by the positive spread between the rate of return on money and individuals' rates of time discount is what accounts for the fact that the equilibrium allocation is Pareto dominated.

So far Bewley's analysis amounts to a precise exposition of Friedman's (1969) analysis of the optimal quantity of money, as Bewley noted. Bewley's major original contribution was to show that if one modified the model by setting the interest rate equal to agents' rate of time discount (these are now assumed equal over individuals), so as to do away with the source of the suboptimal allocation, then equilibrium does not exist, except in special cases. This is so because if agents can accumulate money at zero opportunity cost, they will attempt to acquire infinite holdings of money to insure against indefinitely long runs of bad luck.

Let us now take the same question Bewley answered and ask it of Brock's MIUF model. Brock assumed that individuals are identical, with utility depending on real balances held as well as consumption. In Brock's specification, unlike that of Bewley, money is non-interest-bearing, but Brock assumed that the rate of monetary expansion may be nonzero. This difference is merely one of convention, as Bewley noted ("Remark," p. 176). Now, Brock showed that steady-state equilibrium exists only if the rate of monetary expansion is positive or, if negative, is lower in absolute value than agents' rate of time preference.[9] But Brock's restriction on the rate of monetary expansion is exactly equivalent to Bewley's restriction on the rate of interest on money, as is easy to verify using the convention outlined in Bewley's remark. The reason for nonexistence of equilibrium in Brock's setup is the same as in Bewley's model: in any steady-state equilibrium in which the real opportunity cost of holding money is zero, individuals are induced to accumulate infinite real balances.[10] It is worth emphasizing that the point here is not to assert that an analysis of the optimal quantity of money can be performed just as well in an MIUF framework as in any other. This is hardly the case: the very idea of a Pareto-optimal allocation becomes ambiguous if money is assumed to generate utility. Nonetheless, the fact remains that an inquiry into the question of the optimal quantity of money in Brock's framework provides esentially the same answer as in Bewley's, and comparison of the two papers shows that the development is far more accessible in the former case.

The question arises whether a similar correspondence exists between the results reported here and in the companion paper and those derived in an otherwise identical model that does not specify MIUF. It does: the behavior of the inflation rate derived here under MIUF is very similar (but, as noted above, not identical) to that occurring in an overlapping-generations model [LeRoy and Raymon 1983]). Given the similarity of the results and the much greater simplicity of their derivation under MIUF, the suggestion is that MIUF models have a role to play as a heuristic. That is, even if one does not take MIUF results at face value, still an analysis provided at very low cost on MIUF models

9. Since Brock's model is deterministic and Bewley restricted his attention only to equilibrium price sequences bounded away from zero and infinity so as to exclude bubbles, nonstochastic steady-state equilibrium is the analogue in Brock's framework of Bewley's equilibrium concept.

10. This assumes that individuals are nonsatiable in real balances. However, it can be shown that steady-state equilibrium does not exist in Brock's model even if the utility-of-money function incorporates a bliss point.

may suggest results which would then be explored using other methods.

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