Nominal Prices and Interest Rates in General Equilibrium: Endowment Shocks

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The effect of exogenous shocks in the endowment on nominal prices and interest rates is analyzed. An exchange economy is assumed, and the monetary growth rate is constant. Endowment shocks always affect the nominal price level negatively, but the direction of their effect on the nominal interest rate depends on the elasticity of the utility-of-money function. The derived equilibrium expressions for real and nominal interest rates and inflation are used to discuss the Fisher relation between real and nominal interest rates, the term structure of nominal interest rates, and the correlation between expected and unexpected inflation and stock returns.

I. Introduction

The responses of nominal prices and interest rates to random changes in the money stock were analyzed in the preceding paper. Here the same two-state rational-expectations representative-individual exchange model is adopted, except that it is assumed that random variations in nominal prices and interest rates occur as consequences of variations in the real endowment rather than in the money stock. It is shown in Section II that endowment shocks induce variations in the price level in the opposite direction, with the magnitude of the effect depending on the elasticity of the utility-of-money function. The higher the Arrow-Pratt measure of risk aversion to lotteries denominated in real balances (i.e., the lower the elasticity of utility), the less sensitive is the price level to the endowment realization, and hence the less volatile are inflation rates. The effect of endowment shocks on nominal interest rates also depends on relative risk aversion: if the measure of relative risk aversion is greater (less) than unity, the nominal interest rate is lower (higher) in the high-endowment state than in the low-endowment state.

The relation between nominal and real interest rates is considered next. Here the point of reference is the theory of Fisher. Two properties must be satisfied if Fisher's theory is to be correct. First, the expected real rate of return on nominal

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bonds must equal the real interest rate and, second, the real interest rate must be unaffected by whatever causes the changes in expected inflation. It is shown in Section III that neither property is satisfied if changes in the variables under consideration are generated by endowment shocks rather than monetary shocks.[1] With regard to the first property, nominal bonds will generally be priced to yield a positive rather than zero risk premium. This is so because the real rate of return on nominal bonds will be high when the price level is low, which in turn will occur when the endowment is high. But then the real rate of return on bonds covaries negatively with the marginal utility of consumption (because the level of consumption equals the endowment, and individuals are risk averse). Accordingly, the expected real rate of return on bonds will always exceed the real interest rate. With regard to the second property required for the validity of the Fisher analysis, shocks in the endowment will generate inversely related shocks to the real interest rate, as is widely known, so that again the contrary result presumed in the Fisher theory is incorrect.

The theory of the term structure of the nominal interest rates is discussed in Section IV. The term premia are defined there as the difference between the expected real rate of return on a portfolio of nominal bonds and the corresponding real interest rate. The signs of these term premia are shown to depend in the usual way on the covariance between the real rate of return on the bond portfolio and the marginal utility of consumption. In Section V it is shown that the term premia of nominal interest rates are of higher order than the deviation of the marginal utility of consumption from its expectation, so the expectations hypothesis applies to nominal as well as real interest rates when individuals are nearly risk neutral.

Applications of the model are briefly discussed in the last two sections. The relation implied by the model between stock returns and inflation is analyzed in Section VI. It has been documented in the empirical literature that stock returns are negatively correlated with both expected and unexpected inflation. This finding is regarded as anomalous since it contradicts the presumption that stock, being title to the yields of physical assets, should be hedged against inflation. Fama (1981) has suggested that the observed correlations are due not to some direct functional relation between inflation and stock prices but to the covariance between each of these and real activity. In the present model, fluctuations in both stock returns and inflation are explained in terms of exogenous fluctuations in the endowment, so the situation postulated by Fama is true by assumption. It is therefore

1. Thus the practice, commonly encountered in the empirical literature, of taking the average real rate of return on nominal bonds as a proxy for the real interest rate is incorrect to the extent that endowment shocks rather than monetary shocks are the cause of variations in expected inflation.

appropriate to ask whether the present model generates the observed correlations. It is shown that our model in fact does suggest that stock returns should be negatively correlated with both expected and unexpected inflation, lending support to Fama's reasoning.

The paper concludes (Sec. VII) with the observation that, of the many disparate propositions asserted by monetarists and denied by their critics, several are in fact logical consequences of one underlying assumption. The assertions (1) that the real interest rate is constant, (2) that the Fisher relation is correct, (3) that Martingale models of stock prices are valid, and (4) that the expectations hypothesis is correct are in fact logical consequences of the assumption that fluctuations in the money stock are the principal cause of variations in nominal GNP, itself a major tenet of monetarism. If, however, the critics of monetarism are correct in denying the primacy of money in determining changes in nominal GNP and related variables, so that fluctuations in the real sector are an important independent cause of fluctuations in nominal GNP and related variables, then, according to the results of this paper, they are also correct in denying each of the monetarist propositions just listed.

II. Derivation and Characterization of Equilibrium

The representative trader is assumed to maximize the same utility function as in the preceding paper, subject to the same budget constraint. Accordingly, the first-order conditions are

$$U_t^i = \beta[\pi(h|i)U_{t+1}^h + \pi(l|i)U_{t+1}^l](1 + r_t^i) \tag{1}$$

$$U_t^i = V_t^i + \beta\left[\pi(h|i)\frac{U_{t+1}^h p_t^i}{p_{t+1}^h} + \pi(l|i)\frac{U_{t+1}^l p_t^i}{p_{t+1}^l}\right] \tag{2}$$

$$\frac{V_t^i}{U_t^i} = \frac{n_t^i}{1 + n_t^i}, \quad i = h, l, \tag{3}$$

coinciding with (4), (5), and (6) of the preceding paper.

To generate equilibrium nominal prices and interest rates, we set $x^h = \bar{x}^h$, $x^l = \bar{x}^l$, where $\bar{x}^h > \bar{x}^l$, and $B_t^i = b_t^i = 0$. Also, it is assumed that the money stock increases at rate θ: $M_{t+1} = M_t(1 + \theta)$, with θ satisfying $\beta < 1 + \theta$. As before, we take the real money stock to be the same in each high-endowment state and in each low-endowment state: $M_t^i/p_t^i \equiv m^i$, $i = h, l$. Finally, define μ by $\mu \equiv m^l/m^h$, also as in the preceding paper. The conditional inflation rates can be expressed as

$$\begin{aligned} \frac{p_{t+1}^h}{p_t^h} &= \frac{p_{t+1}^l}{p_t^l} = (1 + \theta) \\ \frac{p_{t+1}^h}{p_t^l} &= (1 + \theta)\mu, \quad \frac{p_{t+1}^l}{p_t^h} = (1 + \theta)/\mu. \end{aligned} \tag{4}$$

Assuming for simplicity that the high-endowment and low-endowment states are equally (unconditionally) probable, we can replace $\pi(h|h)$ and $\pi(l|l)$ by π. For reference below, we note that the conditional expected rates of depreciation of money are given by

$$\begin{aligned} E^h(p_t/p_{t+1}) &\equiv E(p_t/p_{t+1})|_{i=h} \\ &= \pi\, p_t^h/p_{t+1}^h + (1 - \pi)\, p_t^h/p_{t+1}^l, \end{aligned}$$

and similarly for $E^l(p_t/p_{t+1})$, so from (4) it is immediate that

$$\begin{aligned} E^h(p_t/p_{t+1}) &= (1 + \theta)^{-1}[\pi + (1 - \pi)\mu] \\ E^l(p_t/p_{t+1}) &= (1 + \theta)^{-1}[(1 - \pi)/\mu + \pi]. \end{aligned} \tag{5}$$

To obtain two equations for the equilibrium real money stocks we now have only to substitute (4) in (2), yielding

$$\begin{aligned} \frac{V^h}{U^h} &= 1 - \beta[\pi + (1 - \pi)\lambda\mu]/(1 + \theta) \\ \frac{V^l}{U^l} &= 1 - \beta[(1 - \pi)/\lambda\mu + \pi]/(1 + \theta). \end{aligned} \tag{6}$$

Here λ is defined as U^l/U^h, where the marginal utilities are evaluated at the endowment. This ratio gives the relevant measure of risk aversion toward lotteries denominated in units of consumption (LeRoy and LaCivita 1981). To complete the derivation of competitive equilibrium, we assume that the utility of real balances is of the constant-elasticity form

$$V(M/p) = \begin{cases} \dfrac{(M/p)^{1-\gamma}}{1 - \gamma}, & \gamma \neq 1 \\ \ln(M/p), & \gamma = 1. \end{cases} \tag{7}$$

Here γ is the Arrow-Pratt measure of relative risk aversion, so that $1 - \gamma$ is the elasticity of utility. Using (7), (6) may be written

$$\frac{(m^h)^{-\gamma}}{U^h} = 1 - \beta[\pi + (1 - \pi)\lambda\mu]/(1 + \theta) \tag{8}$$

$$\frac{(m^l)^{-\gamma}}{U^l} = 1 - \beta[(1 - \pi)/\lambda\mu + \pi]/(1 + \theta). \tag{9}$$

Since (8) determines m^h as an increasing function of μ and (9) determines m^l as a decreasing function of μ, we can define a function $\psi(\mu)$ by

$$\psi(\mu) \equiv \frac{m^l}{m^h} = \left(\frac{\lambda\{1 - \beta[(1 - \pi)/\lambda\mu + \pi]/(1 + \theta)\}}{\{1 - \beta[\pi + (1 - \pi)\lambda\mu]/(1 + \theta)\}}\right)^{-1/\gamma}, \tag{10}$$

so that equilibrium consists of a fixed point for ψ. Evidently ψ is defined for any μ for which both terms in braces are positive. Given the assumption that $\beta < 1 + \theta$ (the role of which is the same as in the preceding paper), $\mu = \lambda^{-1}$ is such a point. Thus the domain of ψ consists of a nonempty interval $(\bar{\mu}, \bar{\bar{\mu}})$, with $\bar{\mu} < \lambda^{-1} < \bar{\bar{\mu}}$, such that $\lim_{\bar{\mu}\to\mu^+} \psi(\mu) = \infty$ and $\lim_{\mu\to\bar{\bar{\mu}}^-} \psi(\mu) = 0$. Because ψ is decreasing, equilibrium exists and is unique.

We now show that μ^* is less than unity. To do so it is sufficient to demonstrate either that $\bar{\bar{\mu}}$ (the upper bound of the domain of ψ) is less than unity or that $\psi(1) < 1$.[2] From (9), m^l is unambiguously defined for $\mu = 1$ (recall that $\lambda > 1$). If the right-hand side of (8) is negative, then $\bar{\bar{\mu}} < 1$, as required. If the right-hand side of (8) is positive, then the term in braces in the numerator of (10) is greater than the corresponding term in the denominator. From this, $\psi(1) < 1$ is immediate.

The fact that μ^* is less than unity implies that the price level at date t varies negatively with the endowment realization (given the price level at the preceding date). To understand why this should be so, suppose instead that prices rise nonrandomly at the same rate as that assumed for the money stock. In that case endowment variations will be associated with variations in the marginal utility of current consumption (because individuals are risk averse), but not in the marginal service yield of real balances because by assumption real balances are constant over time (there will also be variations in the discounted expected utility of next-period consumption, but it can be shown that these are not sufficient to offset the variation in the marginal utility of current consumption). Thus the equilibrium condition cannot be satisfied. Individuals have a positive (negative) excess demand for real balances when the endowment realization is high (low). If, however, the price level varies in the opposite direction from the endowment, then the marginal service yield on money can do so as well, tending to reduce the excess demand. If the movement of prices is of the appropriate magnitude, equilibrium will be restored. The more responsive the marginal service yield is to real balances (i.e., the higher the Arrow-Pratt measure of relative risk aversion) the smaller is the variation in the price level required to clear markets.

The fact that $\mu^* < 1$, together with equations (4) relating the various conditional inflation rates to μ^*, allows us to correlate the inflation rate over the interval from t to $t + 1$ to the realization of the endowment at t (fig. 1*a*), to the realization of the endowment at $t + 1$ (fig. 1*b*), and to the rate of change of the endowment (fig. 1*c*). To understand these, suppose that the endowment is low at date t. Then the inflation rate

2. This assumes either that $\bar{\bar{\mu}} < 1$ or that unity is in the domain of ψ; the other possibility, that $1 < \bar{\mu}$ (the lower bound of the domain of ψ) is impossible since λ^{-1} was shown to be in the domain of ψ.

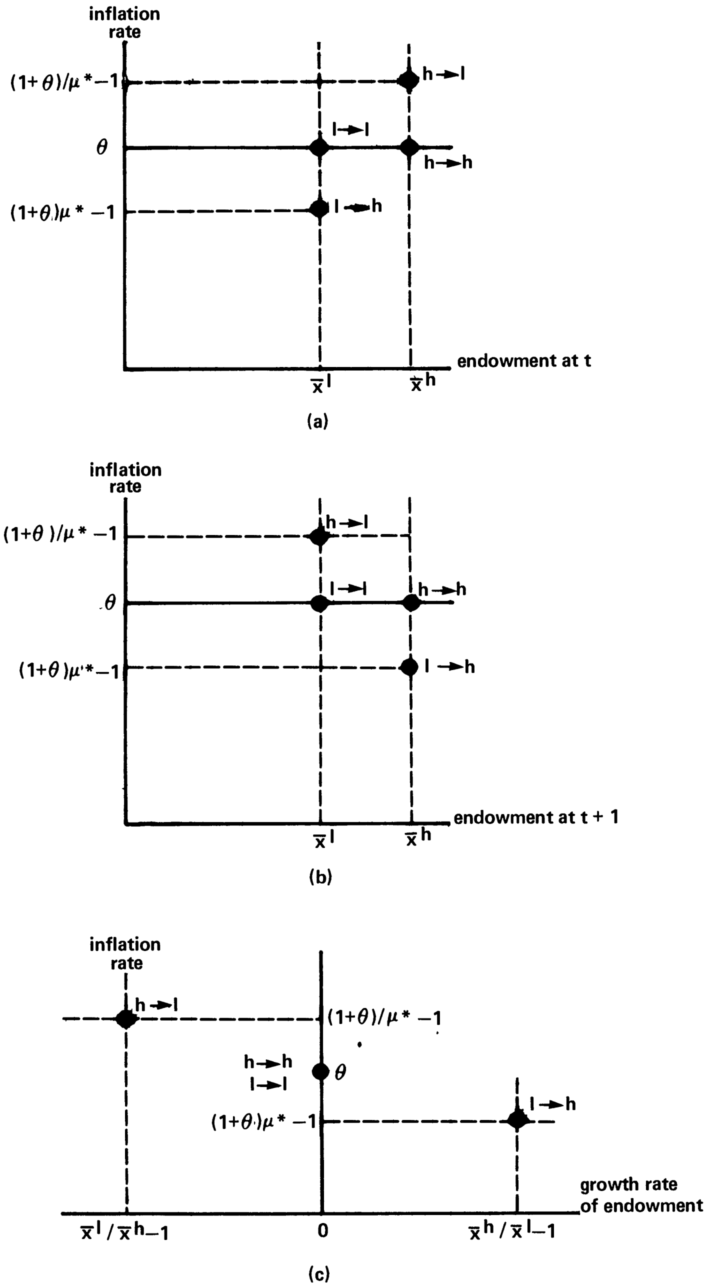

FIG. 1.—Inflation rate from t to $t + 1$

from t to $t + 1$ will be either θ if the endowment at $t + 1$ is also low, or $(1 + \theta)\mu^* - 1 < \theta$ if the endowment at $t + 1$ is high. Similarly, the set of possible inflation rates beginning in the high-endowment state is θ and $(1 + \theta)/\mu^* - 1 > \theta$. Plotting these in figure 1a, the positive correlation between the endowment at t and the inflation rate from t to $t + 1$ is obvious. The interpretation of the other figures proceeds similarly.

It is interesting that the qualitative characterization of the effect of endowment shocks on the inflation rate does not depend on whether the autocorrelation of the endowment is positive, zero, or negative. In this respect the present endowment shocks model differs from the money shocks model of the preceding paper, where it was shown that the qualitative effect of shocks in the monetary growth rate depends essentially on whether that variable is positively or negatively autocorrelated. The reason is that the current realization of the monetary growth rate is a sunk cost, important only insofar as it affects the conditional distribution of the next-period monetary growth rate. It is obvious that the information content of a past high monetary growth rate depends essentially on whether the monetary growth rate is positively or negatively autocorrelated. The endowment shock, however, is not a sunk cost since it directly determines current consumption and therefore the marginal rate of substitution between current and future consumption. This is so whether the endowment realization is positively or negatively autocorrelated.

Equilibrium 1-period nominal interest rates are given by

$$\begin{aligned} 1 + n^h &= \beta^{-1}(1 + \theta)[\pi + (1 - \pi)\lambda\mu^*]^{-1} \\ 1 + n^l &= \beta^{-1}(1 + \theta)[\pi + (1 - \pi)/\lambda\mu^*]^{-1}. \end{aligned} \tag{11}$$

To derive these, write (2) as

$$\frac{U_t^i}{U_t^i - V_t^i} = \frac{U_t^i}{\beta[\pi(h|i)U_{t+1}^h p_t^i/p_{t+1}^h + \pi(l|i)U_{t+1}^l p_t^i/p_{t+1}^l]},$$

and (3) as

$$\frac{U_t^i}{U_t^i - V_t^i} = 1 + n_t^i, \quad i = h, l.$$

Equating the right-hand sides, we have

$$U_t^i = (1 + n_t^i)\beta[\pi(h|i)U_{t+1}^h p_t^i/p_{t+1}^h + \pi(l|i)U_{t+1}^l p_t^i/p_{t+1}^l]. \tag{12}$$

Substituting (4) for the conditional inflation rates, (11) results. From (11), $n^h \lesseqgtr n^l$ as $\mu^* \gtreqless \lambda^{-1}$. From inspection of figure 2, it is clear that $\mu^* \gtreqless \lambda^{-1}$ as $\psi(\lambda^{-1}) \gtreqless \lambda^{-1}$. From (10), $\psi(\lambda^{-1}) = \lambda^{-1/\gamma}$, and $\lambda^{-1/\gamma} \gtreqless \lambda^{-1}$ as $\gamma \gtreqless 1$. Therefore

$$n^h \lesseqgtr n^l \text{ as } \gamma \gtreqless 1. \tag{13}$$

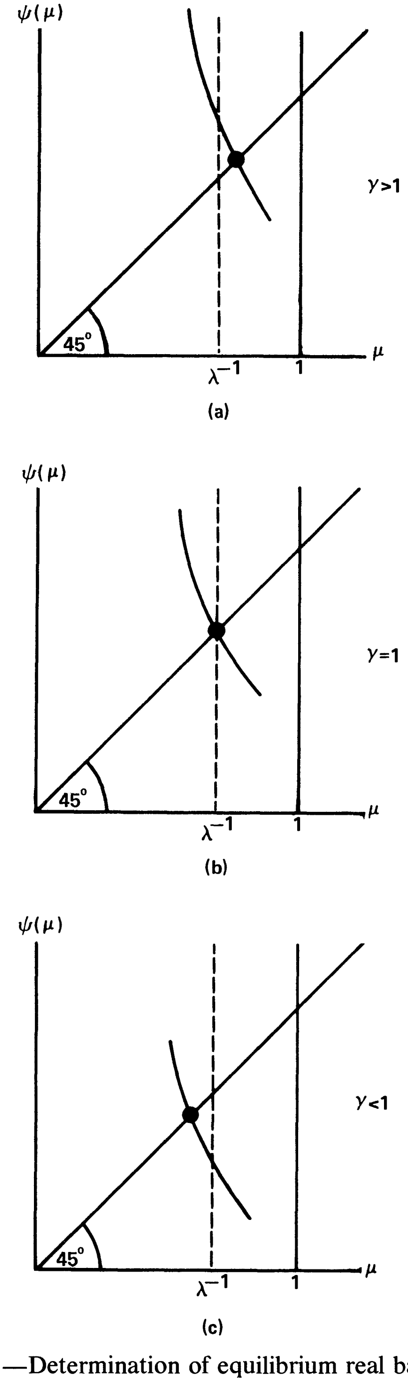

FIG. 2.—Determination of equilibrium real balances

To understand why the sign of the effect of the endowment realization on the nominal interest rate depends on the Arrow-Pratt measure of relative risk aversion, it is helpful to consider the borderline case of logarithmic utility ($\gamma = 1$). To see why in this case nominal interest rates are nonrandom, observe that the current value of next-period consumption, measured in units of current utility, made possible by a small increase in real balances held today is affected by (1) the differential inflation tax and (2) the fact that a unit of consumption has less utility when the endowment is high than when it is low, due to risk aversion toward lotteries denominated in the consumption good. The first of these factors makes it unattractive (attractive) to defer consumption via increased holdings of real balances when the current endowment state is high (low), while just the opposite is the case for the second. Under logarithmic utility the behavior of nominal prices is such that the two factors just listed exactly offset one another, so that the value of expected next-period consumption measured in units of current utility is the same whether the current endowment state is high or low. This, in turn, can be an equilibrium only if the current marginal service yield on real balances, measured in units of current utility, is also the same in either state, since otherwise individuals would shift between consumption and real balances. But this marginal rate of substitution between real balances and consumption controls the nominal interest rate, which is therefore the same in either state.

The same conclusion can be generated from a different angle. If utility depends on the logarithm of real balances, then the marginal utility of real balances varies with the inverse of the level held. But the level of real balances held varies with the inverse of the marginal utility of consumption, by the argument of the preceding paragraph. It follows that the marginal utility of consumption is exactly proportional to the marginal utility of real balances, so that the marginal rate of substitution between consumption and real balances is nonrandom. Again we have that the nominal interest rate is nonrandom.

III. The Fisher Relation

Let us define the real risk premium $F(i)$ on a nominal bond as the difference between the expected real rate of return on that bond and the real interest rate:

$$F(i) \equiv (1 + n^i)E^i(p_t/p_{t+1}) - (1 + r^i), \; i = h, l. \tag{14}$$

As before, i is the state prevailing at the time the bond is bought. This "Fisher premium" will in general depend on the endowment state, as the notation indicates. According to the Fisher theory, $F(i) = 0$, $i = h, l$, and it is also assumed that the real interest rate is unaffected by whatever causes changes in expected inflation. Therefore the nominal interest rate will covary one for one (to a linear approximation) with changes in expected inflation. In this section it is shown that neither property is satisfied when price changes are caused by endowment shocks. That changes in the endowment affect the real interest rate as well as the nominal interest rate and expected inflation rate is well known. Thus we have only to show that $F(i) \neq 0$. In fact we can prove that $F(i) > 0$ for $i = h, l$ for any admissible parameter values.

To prove that $F(h) > 0$—the proof that $F(l) > 0$ is similar—we recall the expressions for the real and nominal interest rates. We have $1 + r^h = \beta^{-1}[\pi + (1 - \pi)\lambda]^{-1}$ and $1 + n^h = \beta^{-1}(1 + \theta)[\pi + (1 - \pi)\lambda\mu^*]^{-1}$, from (1) and (11). Substituting these in (14) and using (5) to eliminate the expression for the conditional expected rate of depreciation of money, we obtain

$$\begin{aligned} F(h) = \beta^{-1}\{&(1 + \theta)[1 + (1 - \pi)(\lambda\mu^* - 1)]^{-1} \\ &\times (1 + \theta)^{-1}[1 + (1 - \pi)(\mu^* - 1)] \\ &- [1 + (1 - \pi)(\lambda - 1)]^{-1}\}. \end{aligned} \tag{15}$$

Equation (15) can be simplified by multiplying through by the two terms which appear as inverses, leading to

$$\begin{aligned} F(h) = \phi\{&[1 + (1 - \pi)(\lambda - 1)][1 + (1 - \pi)(\mu^* - 1)] \\ &- [1 + (1 - \pi)(\lambda\mu^* - 1)]\}, \end{aligned} \tag{16}$$

where ϕ is a positive-valued function of the parameters. Upon further simplification, (16) becomes finally $F(h) = -\phi\pi(1 - \pi)(\lambda - 1)(\mu^* - 1)$. Since we have $\mu^* < 1$ for all $\lambda > 1$, it is proved that $F(h) > 0$.

IV. Multiperiod Nominal Interest Rates

So far our attention has been confined to 1-period interest rates. But k-period (discount) interest rates are derived by a simple extension of the treatment of 1-period rates just presented.[3] If the k-period interest rate prevailing at date t in state i is defined as ${}^kn_t^i$, the k-period analogue to (12) is

$$U_t^i = (1 + {}^kn_t^i)^k\beta^k[\pi_k(h|i)U_{t+k}^h p_t^i/p_{t+k}^h + \pi_k(l|i)U_{t+k}^l p_t^i/p_{t+k}^l], \tag{17}$$

3. In the model of the preceding paper k-period nominal interest rates are less easily derived than here (although the development is still tractable). The reason is that under money shocks, the price level at $t + k$ can take on any of $k + 1$ values (conditional on the state up to t) according to the realizations of the monetary growth process from $t + 1$ to $t + k$, whereas in the present model it can take on only two values according to the endowment realization at $t + k$.

which says that at an optimum the trader must be indifferent between consuming a little more at t and investing in k-period nominal bonds, with the matured bonds being sold and the proceeds consumed at $t + k$. Here $\pi_k(h|i)$, the probability that the state at $t + k$ is h given that the state at t is i, is given by

$$\begin{aligned} \pi_k(h|h) &= \pi_k(l|l) = \frac{1 + (2\pi - 1)^k}{2} \\ \pi_k(l|h) &= \pi_k(h|l) = \frac{1 - (2\pi - 1)^k}{2}. \end{aligned} \tag{18}$$

The expressions corresponding to (4) for the k-period conditional inflation rates are

$$\begin{aligned} &\frac{p^h_{t+k}}{p^h_t} = \frac{p^l_{t+k}}{p^l_t} = (1 + \theta)^k \\ &\frac{p^h_{t+k}}{p^l_t} = (1 + \theta)^k\mu^*, \ \frac{p^l_{t+k}}{p^h_t} = (1 + \theta)^k/\mu^*. \end{aligned} \tag{19}$$

Substituting (18) and (19) in (17) and solving, we obtain

$$\begin{aligned} (1 + {}^kn^h)^k &= 2(1 + \theta)^k\beta^{-k}\{1 + (2\pi - 1)^k + [1 - (2\pi - 1)^k]\lambda\mu^*\}^{-1} \\ (1 + {}^kn^l)^k &= 2(1 + \theta)^k\beta^{-k}\{1 + (2\pi - 1)^k + [1 - (2\pi - 1)^k]/\lambda\mu^*\}^{-1}. \end{aligned} \tag{20}$$

Equipped with expressions (20) for k-period nominal interest rates, we can extend previous results on the term structure of real interest rates (LeRoy 1982*b*, 1983) to nominal rates. As in the earlier work, the principal analytical vehicle is the term premium. In order to apply theoretical results on risk premia to the term premia, it is necessary to define the term premia with reference to yields generated by two different portfolio strategies at a common date. For example, suppose that at date t the investor is concerned about the value of his investment at date $t + j$, and assume initially that he is considering holding a bond with maturity longer than j periods. The relevant term premia may be defined by subtracting the value of j-period real bonds held to maturity from the expected real return on an i-period nominal bond held until $t + j$:

$$T^h(i, j) = E^h\left[\frac{(1 + {}^in^h_t)^i \, p^h_t}{(1 + {}^{i-j}n_{t+j})^{i-j}p_{t+j}}\right] - (1 + {}^jr^h_t)^j, \ i > j. \tag{21}$$

Here the current state is taken to be high; an expression for the term premium in the low state is derived by replacing h by l. Expression (21) coincides with the corresponding expression for real interest rates (LeRoy 1982*b*. eq. [4]) except that here the nominal value of the i-

period bonds at $t + j$ must be corrected for inflation before being compared with the value of the real bonds. Note that in (21), ${}^{i-j}n_{t+j}$ and p_{t+j} are random variables, being functions of the endowment realization at $t + j$.

Similarly, the investor might transfer purchasing power from t to $t + j$ by buying an i-period bond ($i < j$) and rolling it over at $t + i$. In that case the relevant term premium is defined as the difference between the certain real return on a j-period real bond held to maturity and the expected real return on an i-period nominal bond followed by a $(j - i)$-period nominal bond:

$$T^h(i, j) = (1 + {}^jr_t^h)^j - (1 + {}^in_t^h)^i \, E^h[(1 + {}^{j-i}n_{t+i})^{j-i} p_t^h / p_{t+j}], \; i < j. \tag{22}$$

A brute force approach now would be to substitute (20) for nominal interest rates and (19) for inflation rates into (21) and (22) and characterize the term premia as functions of λ and μ^*. Since this exercise is similar to (but considerably more tedious than) that conducted in the analysis of the "Fisher premia" in the preceding section, we leave it to the reader with an appetite for elementary algebra. As before, strong qualitative characterizations of the term premia are available from a direct application of the consumption CAPM. We restrict our attention to $i > j$. We know that T, being a risk premium, is (positive, negative, zero) if the term in brackets in (21) (covaries positively with, covaries negatively with, is independent of) the endowment at $t + j$, assuming $\lambda > 1$. If $\gamma \geqslant 1$, $T^h(i, j)$ and $T^l(i, j)$ are unambiguously positive. This is so because in that case if the state at $t + j$ is high the nominal interest rate at which the i-period bond is discounted at $t + j$ is less than or equal to the corresponding rate if the endowment at $t + j$ is low (${}^{i-j}n_{t+j}^h \leqslant {}^{i-j}n_{t+j}^l$). This follows from (13) if $i - j = 1$; the proof for $i - j > 1$, being similar, is omitted. Hence the nominal rate of return at $t + j$ on an i-period bond covaries positively with the state at $t + j$. But the nominal price level at $t + j$ covaries negatively with the endowment whether or not $\gamma \geqslant 0$ so, a fortiori, the real return on an i-period bond covaries positively with the endowment. Therefore holding a nominal i-period bond is unambiguously a positive-beta investment at $t + j$ and will be priced to yield a positive term premium. If $\gamma < 1$, the nominal return on an i-period bond sold at $t + j$ covaries negatively with the endowment. Whether the real endowment does so as well, and therefore whether the term premium turns negative, obviously depends on whether or not this effect is offset by the behavior of the inflation rate.

A similar analysis can be conducted if $i < j$, but note that in that case the results depend on whether $j - i$ is odd or even if the endowment is negatively autocorrelated, for reasons discussed in LeRoy (1982*b*).

V. The Expectations Hypothesis

The expectations hypothesis says, loosely, that the expected rates of return from bond portfolios of different maturities are equal; equivalently, that all term premia are zero. It is known that the expectations hypothesis as applied to real bonds is exactly satisfied in an exchange economy only if traders are risk-neutral (Cox, Ingersoll, and Ross 1981; LeRoy 1982*b*). In that case the expectations hypothesis is trivial since rates of return on bond portfolios are nonrandom. Further, under nonzero risk aversion the expectations hypothesis cannot be true, from an application of Jensen's inequality. However, it is also known that under near risk neutrality (i.e., if individuals' utility functions are nearly but not exactly linear, or if there is nearly no risk in the system ($\bar{x}^h \cong \bar{x}^l$)) then the expectations hypothesis is satisfied as an approximation (this proposition is incorrectly denied in LeRoy 1982*a*, p. 199; 1982*b*, proposition 4; see LeRoy (1983) for a correct statement). It is shown here that exactly the same approximation is valid for the term premia of nominal interest rates under endowment shocks.[4]

We sketch only the proof for $i > j$; the case of $i < j$ is similar. From the consumption CAPM, it is known that the term premia can be expressed as

$$T^k(i,j) = -\psi \operatorname{cov}^k \left[\frac{(1 + {}^i n_t^k)^i p_t^k}{(1 + {}^{i-j}n_{t+j})^{i-j} p_{t+j}}, U'_{t+j} \right],$$

where k is the current state and ψ is a positive constant. Now, U'_{t+j} can be written as $U'_{t+j} = E(U'_{t+j}) + s0(\lambda - 1)$,[5] and similarly for the j-period holding yield on an i-period bond. Here s is a random variable defined on the state at $t + j$. Obviously we have

$$T^k(i,j) = 0(\lambda - 1)^2 \tag{23}$$

or equivalently,

$$\lim_{\lambda \to 1^+} \frac{T^k(i,j)}{\lambda - 1} = 0.$$

In other words, it is not only true that the term premia vanish as risk aversion goes to zero (given that the term premia equal zero under risk neutrality, this is a trivial consequence of continuity), but also that the term premia vanish more quickly than risk aversion. Thus the same

4. Under monetary shocks the expectations hypothesis holds exactly without the restriction of near risk neutrality. In that case the restriction to near risk neutrality is not needed because the marginal utility of consumption is nonrandom.

5. If $f(x)$ is a real-valued function of a real variable, then $f(x) = 0(x)$ means here that $\lim_{x \to 0} \sup |[f(x)]/x|$ is nonzero and finite.

approximation which obtained for real bonds applies also to nominal bonds.

The result just proved can be employed in the demonstration of various "efficient-markets" results, which in fact are approximately valid only to the extent that the expectations hypothesis is approximately valid. For example, suppose one defines e_t as the real return on a 2-period nominal bond purchased at t and sold at $t + 1$ less the real 1-period interest rate. It is often held that in an efficient market such an excess return should be orthogonal to any variable in the public's information set. We prove the validity of this assertion under the expectations hypothesis. For concreteness, it is proved that the regression of e_t on n_t, the 1-period nominal interest rate, has a population coefficient of (approximately) zero under the expectations hypothesis.

The regression coefficient δ is given by

$$\delta = \frac{\text{cov}\,(e_t, n_t)}{\text{var}\,(n_t)}. \tag{24}$$

Now, any real rate of return can tautologously be broken down into the sum of a real interest rate, a risk premium, and an unexpected term (this decomposition is tautologous because the risk premium is definitionally equal to the difference between the expected rate of return and the interest rate). For $i = 2$ and $j = 1$, the relevant risk premium is just $T_t(2, 1)$, where $T_t(2, 1) = T^h(2, 1)$ or $T^l(2, 1)$, depending on whether the state at t is h or l. Here $T_t(2, 1)$ is as defined in (21). Hence $e_t = T_t(2, 1) + \eta_t$, where η_t is the unexpected component of the rate of return. But η_t is uncorrelated with n_t, so (24) becomes

$$\delta = \frac{\text{cov}\,(T_t(2, 1), n_t)}{\text{var}\,(n_t)}.$$

Since under the expectations hypothesis we have $T_t(2, 1) = 0(\lambda - 1)^2$ and $n_t = 0(\lambda - 1)$, it is true that

$$\delta = \frac{0(\lambda - 1)^3}{0(\lambda - 1)^2} = 0(\lambda - 1).$$

If $\hat{\delta}$ is the finite-sample estimate of δ, then $\lim_{\lambda \to 1^+} \text{plim}\,(\hat{\delta}) = 0$. Hence, given a large sample, the "efficient-markets" regression of e_t on r_t will produce an approximately zero regression coefficient (and an approximately zero R^2) to the extent that risk aversion is vanishingly small.

VI. Stock Returns, Real Activity, and Inflation

It has been documented that observed stock returns are negatively correlated with both the expected and unexpected components of inflation (see Fama [1981] and the papers cited there; also see Geske and Roll [1982]). This phenomenon is held to be counterintuitive: stock prices, being the return on a real asset, should be hedged against inflation, implying that nominal stock returns should vary positively with inflation rather than negatively. Fama offered the explanation that the observed correlations are a statistical artifact due to failure to correct for variations in a third variable: real activity. If real activity is positively correlated with stock returns (as is plausible) and negatively correlated with inflation (as, according to Fama, follows from "a combination of money demand theory and the quantity theory of money" [p. 545]), then the observed negative simple correlations are generated. The model envisioned by Fama, in which variations in stock returns and in the inflation measures are ascribed to exogenous fluctuations in real income, conforms exactly to that developed in this paper. Thus it is of interest to inquire how stock returns will be correlated with expected and unexpected inflation in the present model. It turns out that for reasonable parameter values stock returns in fact appear to be negatively correlated with both expected and unexpected inflation.[6] Thus to the extent that the present model is thought to give a plausible account of stock returns and inflation, Fama's argument is supported.

Data for the rate of return on stock and the nominal rate of inflation can take on at most four values in the present model, as the endowment goes from high to high, high to low, low to high, and low to low. Let us characterize stock returns and inflation in each of these cases. If the endowment is positively autocorrelated, the (real) rate of return on stock will be highest when the endowment goes from low to high, and lowest when the endowment goes from high to low. The rate of return on stock when the endowment goes either from high to high or low to low, being between these extremes, may be characterized as "medium."[7] From figure 1*a*, the inflation rate expected at t is unambiguously higher when the endowment at t is high than when it is low. The correlation between real stock returns and expected inflation is therefore as shown in figure 3. Similarly, that component of the inflation rate from t to $t + 1$ which was unexpected at t is positive when the endowment at $t + 1$ is low and negative when the endowment at $t + 1$ is high. If the realizations of unexpected inflation are (unjustifiably) assumed not to depend on the endowment at the preceding date, the scatter between stock returns and unexpected inflation would be as in figure 4. In both figures 3 and 4 a negative correlation obtains.

6. The analysis to be presented suggests rather than proves this conclusion since in the interests of simplicity certain shortcuts are adopted without formal justification.

7. In the model of LeRoy and LaCivita (1981) the two real rates of return when the state remains unchanged at either high or low will be equal only if the Arrow-Pratt measure of relative risk aversion equals unity. Thus the labeling of both these rates of return as "medium" is imprecise. In LeRoy-LaCivita's model it can be shown that if relative risk aversion is greater (less) than unity, the real rate of return is higher (lower) when the endowment goes from low to low than high to high.

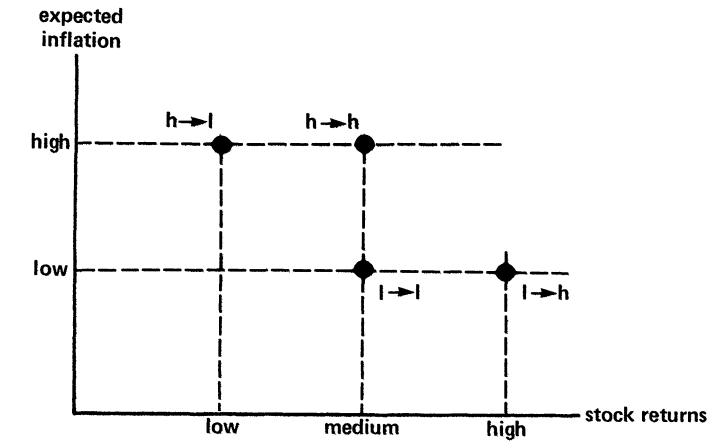

FIG. 3.—Stock returns and expected inflation

VII. On Monetarism

Comparison of the results of this paper with those of the preceding paper, in which the effects of monetary shocks were analyzed, has implications for the dispute about monetarism that used to engage the attention of macroeconomists. Monetarists assert that (1) the major cause of changes in nominal income is the money stock. Also, econo-

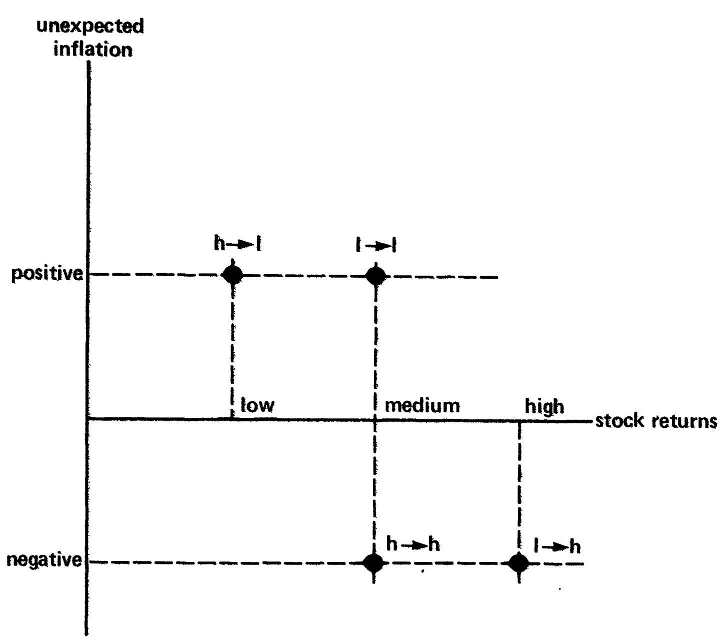

FIG. 4.—Stock returns and unexpected inflation

mists sympathetic to monetarism are likely to accept the propositions (2) that real interest rates are constant and (3) that changes in nominal interest rates therefore primarily reflect changing inflationary expectations, as implied by the Fisher relation, and not changes in real interest rates or risk premia. In finance, monetarists tend to use (4) efficient-markets models, which in practice means Martingale models. Finally, (5) monetarists rely on the expectations hypothesis in thinking about the term structure of interest rates.[8] Nonmonetarists, on the other hand, stress the importance of variables other than the money stock in causing changes in nominal GNP. Further, they characteristically reject the proposition that real interest rates are constant, do not rely on Martingale-efficient markets models of securities prices, and are skeptical of the expectations hypothesis.

The results of this paper suggest that these seemingly unrelated assertions in fact have an internal logic. If the monetarists are correct that monetary changes are the major source of shocks to nominal GNP, then the other propositions asserted by monetarists follow, at least in the model analyzed in the preceding paper: the real interest rate is approximately constant, the Fisher analysis is approximately correct, the Martingale property approximately obtains (on the latter point, see LeRoy [1982*a*] and the papers surveyed there), and the expectations hypothesis is correct. But if the monetarists' critics are correct that variables other than the money stock are important in causing changes in nominal GNP, then from the present paper they are also correct in denying that real interest rates are constant, that the Fisher relation is satisfied, that securities prices conform to the Martingale property, and that (in the absence of near risk neutrality), the expectations hypothesis is correct.

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8. Economists associated with the University of Chicago have prominently advocated all five of these assertions: Friedman (1) and (3), Fama (2) and (4), and Meiselman (5).