

# **Business cycle accounting for monetary economies**

**(PRELIMINARY DRAFT)**

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## **Abstract**

This paper extends business cycle accounting to investigate the quantitative importance of various classes of frictions for the joint dynamics of real and nominal variables over the business cycle. The extended method is then applied to the 1973 and the 1982 US recessions. The findings show that: (i) frictions affecting total factor productivity (TFP) and the labour market account for virtually all of the fluctuations in real variables in both periods; (ii) during the 1973 recession, TFP was the key determinant of inflation while financial market frictions were key for the behaviour of the nominal interest rate; (iii) during the 1982 recession, a fall in TFP and worsening labour market distortions prevented a faster decline of inflation brought about by a monetary policy change; (iv) in both periods frictions distorting investment decisions were unimportant for both real and nominal variables; and (v) nominal price rigidities did not play an important role in either recession.

**Key words:** Business cycle accounting, inflation, nominal interest rate, 1973 recession, 1982 recession

**JEL classification:** E31, E32, E43, E52



## Summary

[TO BE ADDED]





## 1 Introduction

Chari, Kehoe and McGrattan (2007a) develop a data analysis method to investigate the quantitative importance of various classes of market frictions for aggregate fluctuations. This method, which they label ‘business cycle accounting’, is intended to guide researchers in making decisions about where to introduce frictions in their models so that they generate fluctuations like those in the data. Chari *et al* (2007a), henceforth CKM, focus on fluctuations in four key real variables: output, hours, investment, and consumption. This paper extends the method to fluctuations in two key nominal variables: inflation and the nominal interest rate. The purpose of this extension is to investigate what types of frictions and propagation mechanisms drive the joint dynamics of real and nominal variables over the business cycle.

Business cycle accounting rests on the insight that a large class of detailed models with various market frictions can be mapped into a prototype model with a number of time-varying ‘wedges’ that distort the equilibrium decisions of agents operating in otherwise competitive markets.<sup>(1)</sup> Using the equilibrium conditions of the prototype model and data on the model’s endogenous variables the wedges are backed out from the data and fed back into the model, separately and in various combinations, in order to determine their contributions to the observed movements in the data. By construction, all wedges together account for all of the fluctuations in the data.<sup>(2)</sup>

CKM provide mappings between a number of detailed models with various market frictions and a prototype stochastic growth model with four time-varying wedges, henceforth referred to as the CKM economy. At face value these wedges look like fluctuations in total factor productivity, taxes on labour income, taxes on investment, and government consumption. CKM label these wedges *efficiency*, *labour*, *investment*, and *government consumption wedges*, respectively. They demonstrate that input-financing frictions are equivalent to efficiency wedges, labour market distortions, such as sticky wages, are equivalent to labour wedges, investment-financing frictions are equivalent to investment wedges, and net exports in a model with international borrowing and lending are equivalent to government consumption wedges. Applying the method to the Great Depression and the postwar US business cycle they show that promising models of the business cycle have to include frictions that are equivalent to efficiency and labour wedges,

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<sup>(1)</sup> Other researchers besides CKM, for example Hall (1997), Mulligan (2002a) and Mulligan (2002b), also interpret wedges in equilibrium conditions of a competitive economy as reflecting some underlying market distortions.

<sup>(2)</sup> Other papers besides CKM that discuss the method include Christiano and Davis (2006), who express a criticism of the method, and Chari, Kehoe and McGrattan (2007b), who provide a reply to Christiano and Davis’s critique.

but can safely abstract from frictions that are equivalent to investment and government consumption wedges.

While in many cases the real side of the economy is the only focus of investigation, economists are often also interested in the behaviour of nominal variables, and their interaction with economic activity. In order to make the method applicable to fluctuations in both real and nominal variables, this paper constructs a prototype monetary economy— a straightforward extension of the stochastic growth model in which consumers hold money and nominal bonds, in addition to physical capital, and in which, in line with much of the current literature, the nominal rate of return on bonds is controlled by a monetary authority that follows a Taylor (1993)-type rule, i.e. it sets the nominal interest rate in response to movements in output and inflation. Besides the four wedges in the CKM prototype economy, the prototype monetary economy has two additional wedges: an *asset market wedge* that distorts a no-arbitrage condition between capital and nominal bonds, and a *monetary policy wedge* that resembles a monetary policy shock.

In order to demonstrate that an important class of monetary models of the business cycle can be mapped into the prototype model, this paper provides mappings for four detailed economies considered in the literature. In particular, it shows that an economy with nominal price rigidities is equivalent to the prototype economy with equal investment and labour wedges, and that an economy with limited participation, such as that of Christiano and Eichenbaum (1992), is equivalent to the prototype economy with an asset market wedge. The paper also shows that sticky wages are equivalent to a labour wedge, and that fluctuations in energy prices in a model with capital utilisation, such as that of Finn (1996), are equivalent to fluctuations in an efficiency wedge. Furthermore, the paper shows that detailed monetary policy rules, such as those with random regime changes, are equivalent to a prototype Taylor rule with a monetary policy wedge.

The realised values of the six wedges are then uncovered using data on output, hours, investment, consumption, the GDP deflator, and the yield on 3-month Treasury bills for the postwar period in the United States. The wedges are then fed back into the model, one at a time and in various combinations, in order to determine how much of the observed movements in the six variables can be attributed to each wedge. The decomposition is applied to two postwar downturns, the 1973 and the 1982 recessions, which are used as case studies in order to demonstrate how the method works. The two recessions are interesting because they are the two most severe downturns in the postwar US business cycle. In addition, they are usually thought to have been caused by different shocks: the 1973 recession by high oil prices (a ‘supply shock’), and the 1982 recession by tight

monetary policy intended to reduce inflation (a ‘demand shock’). Furthermore, the two recessions have different inflation dynamics. While inflation sharply increased following the oil-price shock in 1973, the 1982 recession was characterised by a sustained decline in the growth rate of prices.

The main findings obtained for these two episodes can be summarised as follows: (i) frictions affecting efficiency and labour wedges account for virtually all of the fluctuations in real variables in both periods; (ii) during the 1973 recession, fluctuations in the efficiency wedge were the key determinant of inflation dynamics while financial market frictions (fluctuations in the asset market wedge) were key for the behaviour of the nominal interest rate; (iii) during the 1982 recession, a decline of the efficiency wedge and worsening labour market distortions prevented a faster decline of inflation brought about by a monetary policy change; (iv) in both periods frictions distorting investment decisions were unimportant for fluctuations not only in real variables, as CKM find, but also in nominal variables; and (v) movements of the investment and labour wedges in the two recessions, as well as during the entire postwar period, are inconsistent with nominal price rigidities being the key friction driving fluctuations in the data

More specifically, in the case of the 1973 recession, the efficiency wedge is crucial for capturing the sharp decline of economic activity following the oil crisis, while the labour wedge accounts for the subsequent slow recovery. In terms of the two nominal variables, the efficiency wedge alone captures essentially all of the fluctuations in inflation during the recession, suggesting that models in which high oil prices negatively affect the production possibility frontier, such as that of Finn (1996), are promising models of both the decline of economic activity and high inflation during the 1973 downturn. However, in order to account for fluctuations in the nominal interest rate, the asset market wedge must be included in the model. This wedge, which at face value looks like a tax on nominal bond purchases, falls sharply during the recession. Without this wedge the model does not produce a fall in the nominal interest rate observed in the data.

In the case of the 1982 recession, both efficiency and labour wedges play a crucial role for the decline of economic activity as well as for its subsequent recovery. In addition, both wedges produce a rise in inflation and the nominal interest rate at the start of the recession similar to that in the data, and the subsequent decline of these variables during recovery. However, the wedges generate turning points for these two variables that occur later than in the data and predict substantially higher inflation at the end of the recession than in the data. In order to fully account for the decline of inflation, the monetary policy wedge has to be included in the model. In line with much of the literature, this suggests that a change in monetary policy that occurred with the appointment of Paul Volcker as the chairman of

the Federal Reserve was the key factor in the decline of inflation during the 1980s. The decomposition, however, provides an additional insight. It shows that without a fall in the efficiency wedge and worsening labour market distortions during the recession inflation in the 1980s would decline more rapidly.

In both recessions investment wedges play only a minor role for fluctuations in both real and nominal variables. In addition, in both recessions, as well as during the entire post-war period, fluctuations in investment and labour wedges in the data are inconsistent with nominal price rigidities being the key frictions driving the movements in the data. The mapping established for an economy with sticky prices demonstrates that such an economy is equivalent to the prototype economy with equal investment and labour wedges. Therefore, if sticky prices were the key propagation mechanism, we would have to observe in the data the two wedges move in the same direction. However, they move in opposite directions. Although this does not mean that sticky prices in isolation cannot be an important propagation mechanism, it does mean that other distortions that move the two wedges in opposite directions play a more important role.

Besides CKM, the paper is related to at least two strands of the literature. In terms of method it is related to a number of papers that apply business cycle accounting to particular episodes in different countries (Crucini and Kahn (2003), Ahearne, Kydland and Wynne (2005), Chakraborty (2005), Kobayashi and Inaba (2006), and Kersting (2007)). These studies, however, focus only on fluctuations in real variables. The paper is also related to a large literature that studies the joint dynamics of real and nominal variables in estimated dynamic general equilibrium models with a host of frictions and primitive shocks (e.g. Ireland (2003), Ireland (2004), Christiano, Eichenbaum and Evans (2005), Primiceri, Schaumburg and Tambalotti (2006), and Smets and Wouters (2007)). In contrast to this literature, business cycle accounting imposes less structure on the data in the sense that specific frictions are not assumed from the start. Instead, the method itself determines what classes of frictions should be included in a model if the model is to exhibit fluctuations such as those in the data.

The paper proceeds as follows. Section 2 describes the prototype monetary economy. Section 3 provides two examples of mappings between the prototype economy and detailed economies with market frictions. The realised values of the wedges are uncovered from the data in Section 4, while Section 5 carries out the data decompositions. Section 6 investigates the sensitivity of the results to alternative parameterisations of the monetary policy rule while Section 7 concludes. Two appendices contain proofs of the equivalence results of Section 3 and two additional examples of mappings between detailed economies and the prototype.

## 2 The prototype monetary economy

The prototype economy is a straightforward monetary extension of the CKM prototype economy. It is a stochastic growth model with the addition of money, nominal bonds and a Taylor (1993)-type monetary policy rule, such as that constructed by Dittmar, Gavin and Kydland (2005). It has six exogenous stochastic variables, referred to as wedges. These wedges distort first-order conditions and resource constraints in the model and at face value resemble total factor productivity, government consumption, monetary policy shocks, and taxes on labour income, investment in capital and investment in nominal bonds. In this economy money is almost neutral – the only real effects are due to small inflation tax effects, as in Cooley and Hansen (1989). The mappings established in the next section and in Appendix B, however, demonstrate how propagation of shocks due to various market frictions, including nominal rigidities, in specific economic environments is equivalent to fluctuations in particular combinations of the wedges in the prototype economy. Thus, although at a mechanical level money is almost neutral in the prototype economy, the real effects of money due to underlying market frictions are captured by the fluctuations in the wedges.

### 2.1 The economic environment

The prototype economy is inhabited by an infinitely lived representative consumer and a representative producer. Both are price takers in all markets. In addition, there is a government that taxes the consumer and issues money. In each period  $t$  the economy experiences one of finitely many events  $z_t$ . Let  $z^t = (z_0, \dots, z_t)$  denote the history of events up through and including period  $t$ ,  $Z^t$  the set of all possible histories  $z^t$ ,  $\mathcal{Z}^t$  the appropriate  $\sigma$ -algebra, and  $\mu_t(z^t)$  the probability measure associated with this  $\sigma$ -algebra. The initial event  $z_0$  is given. The probability space of this economy is thus defined by the triplet  $(Z^t, \mathcal{Z}^t, \mu_t(z^t))$ . Furthermore, let  $\mu_t(z^{t+1}|z^t)$  denote the conditional probability  $\mu_{t+1}(z^{t+1})/\mu_t(z^t)$ . The economy has six exogenous random variables all of which are functions of the history of events  $z^t$ : the *efficiency wedge*  $A_t(z^t)$ , the *labour wedge*  $\tau_{lt}(z^t)$ , the *investment wedge*  $\tau_{xt}(z^t)$ , the *government consumption wedge*  $g_t(z^t)$ , the *asset market wedge*  $\tau_{bt}(z^t)$ , and the *monetary policy wedge*  $\tilde{R}_t(z^t)$ . The first four wedges are the same as those in the CKM economy and will therefore be sometimes referred to as the *CKM wedges*. They distort the same first-order conditions and resource constraints as in the CKM economy. The asset market wedge and the monetary policy wedge are new.

The consumer maximises expected utility over stochastic paths for per capita consumption

$c_t(z^t)$  and per capita leisure  $h_t(z^t)$  <sup>(3)</sup>

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t \mu_t(z^t) u(c_t(z^t), h_t(z^t)) (1 + \gamma_n)^t \quad (1)$$

where  $\beta$  is a discount factor and  $\gamma_n$  is a population growth rate, subject to three constraints. First, the consumer has to satisfy the time constraint

$$h_t(z^t) + l_t(z^t) + s_t(z^t) = 1 \quad (2)$$

where  $l_t(z^t)$  is time spent working and  $s_t(z^t)$  is time spent shopping, which is determined by the function

$$s_t(z^t) = s\left(\frac{c_t(z^t)}{(1 + \gamma_n)m_t(z^t)/p_t(z^t)}\right) \quad (3)$$

Unless specified otherwise, the function  $s(\cdot)$  is assumed to be smooth, increasing and strictly convex. It is also assumed to satisfy the condition  $s(0) = 0$ , ie, shopping time is zero when the amount of purchases is zero. Second, the consumer has to satisfy the budget constraint

$$\begin{aligned} c_t(z^t) + [1 + \tau_{xt}(z^t)] x_t(z^t) + (1 + \gamma_n) \frac{m_t(z^t)}{p_t(z^t)} \\ + [1 + \tau_{bt}(z^t)] \left[ (1 + \gamma_n) \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} - \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} \right] \\ = [1 - \tau_{lt}(z^t)] w_t(z^t) l_t(z^t) + r_t(z^t) k_t(z^{t-1}) + \frac{m_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{T_t(z^t)}{p_t(z^t)} \end{aligned}$$

Here,  $x_t(z^t)$  is investment in capital,  $m_t(z^t)$  is money balances,  $p_t(z^t)$  is the price of goods in terms of money,  $b_t(z^t)$  is bonds that pay a net nominal rate of return  $R_t(z^t)$  in all states of the world  $z_{t+1}$  and are in net zero supply,  $w_t(z^t)$  is the real wage rate,  $r_t(z^t)$  is the real rental rate for capital,  $k_t(z^{t-1})$  is capital held by the consumer at the start of period  $t$ , and  $T_t(z^t)$  is government transfers. The third constraint is the law of motion for capital

$$(1 + \gamma_n)k_{t+1}(z^t) = (1 - \delta)k_t(z^{t-1}) + x_t(z^t) \quad (4)$$

where  $\delta$  is a depreciation rate.

The producer operates an aggregate constant-returns-to-scale production function

$$y_t(z^t) = A_t(z^t) F(k_t(z^{t-1}), (1 + \gamma_A)^t l_t(z^t)) \quad (5)$$

where  $\gamma_A$  is the growth rate of labour-augmenting technological progress. The producer maximises per period profits  $y_t(z^t) - w_t(z^t)l_t(z^t) - r_t(z^t)k_t(z^{t-1})$  by setting the marginal products of capital and labour equal to  $r_t(z^t)$  and  $w_t(z^t)$ , respectively. The aggregate resource constraint requires that

$$c_t(z^t) + x_t(z^t) + g_t(z^t) = y_t(z^t) \quad (6)$$

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<sup>(3)</sup> All quantities in the model are in per capita terms.

Many existing models used to study the joint dynamics of real and nominal variables are closed by specifying a monetary policy rule, like that of Taylor (1993). In order to preserve the structure of this class of models, the government in the prototype economy also sets the nominal interest rate according to such a rule

$$R_t(z^t) = (1 - \rho_R)R_t^*(z^t) + \rho_R R_{t-1}(z^{t-1}) + \tilde{R}_t(z^t) \quad (7)$$

where

$$R_t^*(z^t) = R + \omega_y (\ln y_t(z^t) - \ln y) + \omega_\pi (\pi_t(z^t) - \pi)$$

Here,  $\rho_R$  is a parameter of persistence,  $\pi_t(z^t) \equiv \ln p_t(z^t) - \ln p_{t-1}(z^{t-1})$  is the inflation rate and a variable's symbol without a time subscript denotes the variable's steady-state value.

Finally, the government's budget constraint is given by

$$g_t(z^t) + \frac{T_t(z^t)}{p_t(z^t)} = \tau_{xt}(z^t)x_t(z^t) + \tau_{bt}(z^t) \left[ (1 + \gamma_n) \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} - \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} \right] \\ + \tau_{lt}(z^t)w_t(z^t)l_t(z^t) + (1 + \gamma_n) \frac{m_t(z^t)}{p_t(z^t)} - \frac{m_{t-1}(z^{t-1})}{p_t(z^t)}$$

## 2.2 Equilibrium

A *competitive equilibrium of the prototype economy* is a set of allocations  $(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t), b_t(z^t))$  and a set of prices  $(p_t(z^t), R_t(z^t), r_t(z^t), w_t(z^t))$  such that the allocations are optimal for the consumer and the producer, the nominal interest rate is set according to the policy rule (7),  $b_t(z^t)$  is equal to zero, and the resource constraint (6) is satisfied.

In equilibrium, the consumer's optimal behaviour can be summarised by the following first-order conditions for labour, capital, bonds, and money holdings, respectively

$$[1 - \tau_{lt}(z^t)] A_t(z^t)(1 + \gamma_A)^t F_{lt}(z^t) \quad (8) \\ = \frac{u_{ht}(z^t)}{u_{ct}(z^t)} \{1 + s_{ct}(z^t) [1 - \tau_{lt}(z^t)] A_t(z^t)(1 + \gamma_A)^t F_{lt}(z^t)\}$$

$$[1 + \tau_{xt}(z^t)] (1 + \gamma_n) \quad (9) \\ = \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \{ [1 + \tau_{x,t+1}(z^{t+1})] (1 - \delta) + A_{t+1}(z^{t+1}) F_{k,t+1}(z^{t+1}) \}$$

$$\sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \frac{[1 + \tau_{x,t+1}(z^{t+1})] (1 - \delta) + A_{t+1}(z^{t+1}) F_{k,t+1}(z^{t+1})}{1 + \tau_{xt}(z^t)} \quad (10) \\ = \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \frac{1 + \tau_{b,t+1}(z^{t+1})}{1 + \tau_{bt}(z^t)} [1 + R_t(z^t)] \frac{p_t(z^t)}{p_{t+1}(z^{t+1})}$$

and

$$(1 + \gamma_A) - \frac{u_{ht}(z^t)s_{mt}(z^t)}{[u_{ct}(z^t) - u_{ht}(z^t)s_{ct}(z^t)]} = \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} \quad (11)$$

where

$$Q_t(z^{t+1}|z^t) = \beta \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}(z^{t+1}) - u_{h,t+1}(z^{t+1})s_{c,t+1}(z^{t+1})}{u_{ct}(z^t) - u_{ht}(z^t)s_{ct}(z^t)} \quad (12)$$

Here, and throughout the paper,  $u_{ct}$ ,  $u_{ht}$ ,  $s_{ct}$ ,  $s_{mt}$ ,  $F_{kt}$ , and  $F_{lt}$  denote the derivatives of the utility, shopping time and production functions with respect to their arguments. Notice that in the absence of shopping time, equations (8)-(10) become the standard optimality conditions in a stochastic growth model.

As in the CKM prototype economy, the labour wedge in our prototype economy distorts the intratemporal optimality condition for labour while the investment wedge distorts the intertemporal optimality condition for investment in capital. In addition to the distortionary effects of these two wedges, for given investment wedges, the asset market wedge distorts the no-arbitrage condition for capital and bonds. The other new wedge, the monetary policy wedge, generates deviations of the nominal interest rate from the level  $R^*$  that is due to systematic responses of the monetary authority to output and inflation. The efficiency and government consumption wedges play the same role here as in the CKM economy. The efficiency wedge determines the amount of output produced for a given amount of inputs, while the government consumption wedge determines the amount of output available for consumption and investment.

Since at a mechanical level the prototype economy is a real business cycle model, money has real effects only through an inflation tax, which affects shopping time and thus the consumer's time available for leisure and work. As in Cooley and Hansen (1989), these effects are small. It is therefore convenient to think of the prototype economy as being block recursive: first, the consumer's optimality conditions (8) and (9), together with the production function (5), the resource constraint (6), and the law of motion for capital (4) determine the equilibrium  $c_t(z^t)$ ,  $x_t(z^t)$ ,  $y_t(z^t)$ ,  $l_t(z^t)$ ,  $k_{t+1}(z^t)$ ; then the no-arbitrage condition (10) and the monetary policy rule (7) determine equilibrium  $p_t(z^t)$  and  $R_t(z^t)$ ; and finally, the optimality condition for money (11) determines equilibrium  $m_t(z^t)$ . As a result of this (approximately) recursive structure, the CKM wedges affect *all* endogenous variables, whereas the asset market wedge and the monetary policy wedge have (significant) effects only on inflation, the nominal interest rate and money.

The usefulness of this setup in which money is almost neutral is its generality: a large class of models with various market frictions and propagation mechanisms, including models with nominal rigidities, can be mapped into the prototype economy. The underlying frictions in specific economic environments will show up in the prototype economy as wedges. Introducing from the start into the prototype economy frictions that lead to significant real effects of money would defeat the purpose of business cycle accounting as



a data analysis technique that precedes the construction of detailed models. The accounting procedure described in Section 4, together with the equivalence results, will determine which classes of frictions are important and which can be safely abstracted from.

### 2.3 Dynamics of inflation and the nominal interest rate

In order to understand the dynamics of inflation and the nominal interest rate in this model, it is useful to log-linearise the equilibrium conditions (10) and (7) in the neighborhood of the model's steady state

$$a_1 E_t \hat{\tau}_{x,t+1} - a_2 \hat{\tau}_{xt} + a_3 E_t \hat{A}_{t+1} + a_4 E_t \hat{l}_{t+1} - a_5 E_t \hat{k}_{t+1} \quad (13)$$

$$= a_6 E_t \hat{\tau}_{b,t+1} - a_7 \hat{\tau}_{bt} + a_8 \hat{R}_t - a_9 E_t \hat{\pi}_{t+1}$$

$$\hat{R}_t = (1 - \rho_R) \omega_y \hat{y}_t + (1 - \rho_R) \omega_\pi \hat{\pi}_t + \rho_R \hat{R}_{t-1} + \hat{R}_t \quad (14)$$

Here,  $a_1 = (1 - \delta)/(1 + \tau_x)$ ,  $a_2 = [(1 - \delta)(1 + \tau_x) + AF_k]/(1 + \tau_x)^2$ ,  $a_3 = F_k A/(1 + \tau_x)$ ,  $a_4 = AF_{kl}/(1 + \tau_x)$ ,  $a_5 = -AF_{kk}k/(1 + \tau_x)$ ,  $a_6 = (1 + R)/[(1 + \pi)(1 + \tau_b)]$ ,  $a_7 = (1 + R)/[(1 + \pi)(1 + \tau_b)]$ ,  $a_8 = 1$ ,  $a_9 = (1 + R)/(1 + \pi)^2$ , and variables with a 'hat' denote percentage deviations from steady state, in the case of the efficiency wedge, labour, capital, and output, and percentage point deviations from steady state, in the case of the investment, asset market, and monetary policy wedges, the inflation rate, and the nominal interest rate. Notice that all of the coefficients in equation (13) are positive. Assuming, for illustration, that each wedge follows an AR(1) process, and combining equations (13) and (14), inflation in period  $t$  can be expressed as

$$\hat{\pi}_t = \frac{1}{(1 - \rho_R) \omega_\pi} [-(a_2 - a_1 \rho_x) \hat{\tau}_{xt} + a_3 \rho_A \hat{A}_t + a_4 E_t \hat{l}_{t+1} - a_5 E_t \hat{k}_{t+1} \quad (15)$$

$$+ (a_7 - a_6 \rho_b) \hat{\tau}_{bt} - (1 - \rho_R) \omega_y \hat{y}_t - \rho_R \hat{R}_{t-1} - \hat{R}_t + a_9 E_t \hat{\pi}_{t+1}]$$

Here,  $(a_2 - a_1 \rho_x) > 0$ ,  $(a_7 - a_6 \rho_b) > 0$ , and  $\rho_x$ ,  $\rho_A$  and  $\rho_b$  are the autocorrelation coefficients of the AR(1) processes for the investment, efficiency and asset market wedges, respectively. By appearing in equation (15), investment, efficiency, asset market, and monetary policy wedges have a direct effect on inflation. The first two wedges, however, together with labour and government consumption wedges, also have an indirect effect on inflation by affecting output, labour and capital in equation (15).

Equation (15) characterises inflation dynamics in all models that can be mapped into the prototype economy. Consider, for example, a real business cycle model, such as that of Dittmar *et al* (2005), in which the only source of fluctuations are shocks to total factor productivity (our efficiency wedge), and in which the central bank follows a Taylor rule. A persistent fall in total factor productivity has a direct negative effect on inflation by reducing the expected real return on capital,  $a_3 \rho_A \hat{A}_t + a_4 E_t \hat{l}_{t+1} - a_5 E_t \hat{k}_{t+1}$ . But, as long as

$\omega_y > 0$ , it also has an indirect positive effect by reducing output. When  $\omega_y$  is sufficiently large, which is the case for  $\omega_y = 0.125$  used by Taylor (1993) (and also used in our baseline calibration), the latter effect dominates and output and inflation move in opposite directions following a technology shock.<sup>(4)</sup>

As another example, consider a sticky-price model, such as the one constructed by Ireland (2004). As the next section shows, an economy with sticky prices is equivalent to the prototype economy with equal investment and labour wedges. A negative ‘demand’ shock, such as a positive shock to the nominal interest rate, usually leads in these models to a fall in both output and the inflation (see Ireland (2004), Figure 1). Viewed through the lens of the prototype economy, the propagation of this shock through sticky prices is equivalent to an increase in labour and investment wedges. By distorting labour and investment decisions, such an increase leads to a fall in output and thus an increase in inflation (here we are ignoring, for simplicity, the effect of the wedges on  $E_t \hat{l}_{t+1}$  and  $E_t \hat{k}_{t+1}$ ). The direct effect of an increase in  $\hat{\tau}_{xt}$  on inflation, however, works in the opposite direction. When this effect is sufficiently strong (or equivalently when  $\omega_y$  is sufficiently small), inflation falls following a monetary policy tightening.

### 3 Equivalence results

This section provides mappings between two detailed monetary economies and the prototype monetary economy described above. First, it shows that an economy with sticky prices is equivalent to the prototype economy with equal investment and labour wedges. Then it shows that an economy with limited participation in the money market, like that of Christiano and Eichenbaum (1992), is equivalent to the prototype economy with asset market wedges. This section also demonstrates how detailed monetary policy rules considered in the literature, including rules with regime changes, can be mapped into the prototype policy rule (7). Appendix B then provides additional equivalence results. It shows that an economy with sticky wages considered by CKM is equivalent to the prototype economy with labour wedges, and that an economy with capital utilisation and fluctuations in energy prices in world markets, like that of Finn (1996), is equivalent to the prototype economy with efficiency wedges. The mappings established in this section and in the Appendix complement those established by CKM for a non-monetary prototype economy.

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<sup>(4)</sup> Indeed, equation (15) is just a stochastic difference equation in inflation, not a particular solution for inflation. However, since the terms containing  $E_t \hat{\pi}_{t+1}$  drop out of a particular solution that excludes explosive paths for inflation (which is the case when  $\omega_\pi$  is sufficiently above one), we can discuss the effects of changes in the exogenous variables on inflation using equation (15).

Other detailed models than those considered here can be potentially mapped into the prototype economy. It is not the purpose of this paper to provide an exhaustive list of such mappings. Rather it is to illustrate how the key frictions and propagation mechanisms considered in the literature that studies the co-movement between real and nominal variables map into the wedges. Therefore, when a wedge associated with a particular friction considered here turns out to be important for fluctuations in the data, it does not mean that the friction is the only possible mechanism that can generate the data. As emphasised by CKM, business cycle accounting does not uniquely identify a model. It only determines a class of frictions that are promising by identifying which equilibrium conditions in the prototype economy need to be distorted so as to capture the nature of the fluctuations.

Throughout this section we retain the notation of Section 2. For new variables, notation will be introduced as we go. For brevity, this section abstracts from population and technology growth.

### 3.1 *An economy with sticky prices*

#### 3.1.1 *The underlying economy*

Consider an economy with monopolistic competition in product markets and nominal price rigidities. The underlying probability space of this economy is the same as that of the prototype economy described in the previous section; i.e. it is given by  $(Z^t, \mathcal{Z}^t, \mu_t(z^t))$ . There are two types of producers: identical final good producers and intermediate good producers indexed by  $j \in [0, 1]$ . Final good producers take all prices as given and solve

$$\max_{y_t(z^t), \{y_t(j, z^t)\}, j \in [0, 1]} p_t(z^t)y_t(z^t) - \int p_t(j, z^t)y_t(j, z^t)dj$$

subject to a production function

$$y_t(z^t) = \left[ \int y_t(j, z^t)^{\varepsilon_t(z^t)} dj \right]^{1/\varepsilon_t(z^t)}$$

Here,  $y_t(z^t)$  is aggregate output,  $y_t(j, z^t)$  is input of an intermediate good  $j$ ,  $p_t(j, z^t)$  is its price, and  $\varepsilon_t(z^t)$  is a shock that determines the degree of monopoly power of intermediate good producers.<sup>(5)</sup> The solution to this problem is characterised by a demand function for an intermediate good  $j$

$$y_t(j, z^t) = \left( \frac{p_t(z^t)}{p_t(j, z^t)} \right)^{\frac{1}{1-\varepsilon_t(z^t)}} y_t(z^t) \quad j \in [0, 1] \quad (16)$$

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<sup>(5)</sup> In the context of sticky-price economies, a number of different types of shocks have been considered in the literature, including preference, investment-specific, government consumption, mark-up, and monetary policy shocks. The main conclusion regarding the distortionary effects of sticky prices, however, does not depend on the choice of a particular shock.

and a price aggregator

$$p_t(z^t) = \left[ \int p_t(j, z^t)^{\frac{\varepsilon_t(z^t)}{\varepsilon_t(z^t)-1}} dj \right]^{\frac{\varepsilon_t(z^t)-1}{\varepsilon_t(z^t)}}$$

The problem of an intermediate good producer  $j$  can be split into two sub-problems. First, for a given level of output  $y_t(j, z^t)$  the producer solves

$$\min_{l_t(j, z^t), k_t(j, z^t)} w_t(z^t)l_t(j, z^t) + r_t(z^t)k_t(j, z^t)$$

subject to

$$F(k_t(j, z^t), l_t(j, z^t)) = y_t(j, z^t)$$

where  $l_t(j, z^t)$  and  $k_t(j, z^t)$  are labour and capital, respectively, employed by producer  $j$ .

Denoting the value function for this cost minimisation problem by

$\vartheta(y_t(j, z^t), w_t(z^t), r_t(z^t))$ , in the second step of the optimisation problem the producer chooses the price  $p_t(j, z^t)$  to maximise the present value of profits

$$\sum_{t=0}^{\infty} \sum_{z^t} Q_t(z^t) \left[ \frac{p_t(j, z^t)y_t(j, z^t)}{p_t(z^t)} - \vartheta(y_t(j, z^t), w_t(z^t), r_t(z^t)) - \frac{\phi}{2} \left( \frac{p_t(j, z^t)}{\pi p_{t-1}(j, z^{t-1})} - 1 \right)^2 \right]$$

subject to the demand function **(16)**. Here,  $Q_t(z^t)$  is an appropriate discount factor and the last term in the square brackets is a price adjustment cost as in Rotemberg (1982).<sup>(6)</sup> Given the symmetry among the producers, all of them choose the same price, capital and labour.

The consumer maximises the utility function **(1)**, subject to the time constraint **(2)**, the law of motion for capital **(4)** and the budget constraint

$$\begin{aligned} c_t(z^t) + x_t(z^t) + \frac{m_t(z^t)}{p_t(z^t)} + \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} \\ = w_t(z^t)l_t(z^t) + r_t(z^t)k_t(z^{t-1}) + \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{m_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{T_t(z^t)}{p_t(z^t)} + \psi_t(z^t) \end{aligned}$$

where  $\psi_t(z^t)$  is profits from intermediate good producers, and where in the utility function, the shopping time function, and in the capital accumulation law  $\gamma_n = 0$ .

The government follows the monetary policy rule

$$R_t(z^t) = (1 - \rho_R) [R + \omega_y (\ln y_t(z^t) - \ln y) + \omega_\pi (\pi_t(z^t) - \pi)] + \rho_R R_{t-1}(z^{t-1}) \quad \mathbf{(17)}$$

and its budget constraint is

$$T_t(z^t) = m_t(z^t) - m_{t-1}(z^{t-1}) + p_t(z^t) \frac{\phi}{2} \left( \frac{p_t(z^t)}{\pi p_{t-1}(z^{t-1})} - 1 \right)^2$$

Here, we assume that the price adjustment cost acts like a tax that is rebated back to the consumer.

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<sup>(6)</sup> The equivalence result also holds for Calvo and Taylor-style price setting behaviour.

An equilibrium of this sticky-price economy is a set of allocations  $(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t), b_t(z^t))$  and a set of prices  $(p_t(z^t), R_t(z^t), r_t(z^t), w_t(z^t))$  that satisfy: (i) a set of the consumer's first-order conditions for labour, capital, bonds, and money, respectively

$$u_{ct}(z^t)w_t(z^t) = u_{ht}(z^t) [1 + s_{ct}(z^t)w_t(z^t)] \quad (18)$$

$$\sum_{z_{t+1}} Q_t(z^{t+1}|z^t) [1 + r_{t+1}(z^{t+1}) - \delta] = 1 \quad (19)$$

$$\sum_{z_{t+1}} Q_t(z^{t+1}|z^t) [1 + R_t(z^t)] \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} = 1 \quad (20)$$

$$-\frac{u_{ht}(z^t)s_{mt}(z^t)}{u_{ct}(z^t) - u_{ht}(z^t)s_{ct}(z^t)} + \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} = 1 \quad (21)$$

where

$$Q_t(z^{t+1}|z^t) = \beta \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}(z^{t+1}) - u_{h,t+1}(z^{t+1})s_{c,t+1}(z^{t+1})}{u_{ct}(z^t) - u_{ht}(z^t)s_{ct}(z^t)}$$

(ii) a set of optimality conditions for the cost minimisation problem of intermediate good producers

$$\frac{F_{kt}(z^t)}{F_{lt}(z^t)} = \frac{r_t(z^t)}{w_t(z^t)} \quad (22)$$

$$y_t(z^t) = F(k_t(z^{t-1}), l_t(z^t)) \quad (23)$$

(iii) a first-order condition for the profit maximisation problem of intermediate good producers (the so-called 'New-Keynesian Phillips Curve')

$$\begin{aligned} \Phi(p_t(z^t), p_{t-1}(z^{t-1}), \eta_t(z^t), y_t(z^t), \varepsilon_t(z^t)) \\ + \sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \Psi(p_t(z^t), p_{t+1}(z^{t+1}), y_{t+1}(z^{t+1}), \varepsilon_{t+1}(z^{t+1})) = 0 \end{aligned} \quad (24)$$

where  $\eta_t(z^t) \equiv \partial \vartheta_t(z^t) / \partial y_t(z^t)$  is a marginal cost and  $\Phi(\cdot, \cdot, \cdot, \cdot, \cdot)$  and  $\Psi(\cdot, \cdot, \cdot, \cdot)$  are smooth functions; (iv) the resource constraint  $c_t(z^t) + x_t(z^t) = y_t(z^t)$ ; (v) the capital accumulation law (4); (vi) the monetary policy rule (17); and (vii) the bond market clearing condition  $b_t(z^t) = 0$ .

Notice that in this model  $r_t(z^t)$  and  $w_t(z^t)$  are not set equal to the marginal products of capital and labour. Instead, imperfect competition and sticky nominal prices lead to a time-varying mark-up of prices over marginal costs, given implicitly by the equilibrium condition (24).

### 3.1.2 The associated prototype economy

Consider now a version of the prototype economy of Section 2. The prototype economy is the same as that of Section 2, except that it has an investment wedge that resembles a tax

on capital income rather than a tax on investment. The consumer's budget constraint therefore is

$$\begin{aligned} c_t(z^t) + x_t(z^t) + \frac{m_t(z^t)}{p_t(z^t)} + [1 + \tau_{bt}(z^t)] \left[ \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} - \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} \right] \\ = [1 - \tau_{lt}(z^t)] w_t(z^t) l_t(z^t) + [1 - \tau_{kt}(z^t)] r_t(z^t) k_t(z^{t-1}) + \frac{m_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{T_t(z^t)}{p_t(z^t)} \end{aligned}$$

where  $\tau_{kt}(z^t)$  is the capital income tax. In equilibrium, the consumer's first-order condition for capital accumulation (4) becomes

$$\sum_{z^{t+1}} Q_t(z^{t+1}|z^t) \{ [1 - \tau_{k,t+1}(z^{t+1})] A_{t+1}(z^{t+1}) F_{k,t+1}(z^{t+1}) + (1 - \delta) \} = 1 \quad (25)$$

where  $Q_t(z^{t+1}|z^t)$  is given as before by equation (12).

**PROPOSITION 1:** *Consider equilibrium allocations of the economy with sticky prices  $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t))$  and prices  $(p_t^*(z^t), R_t^*(z^t), r_t^*(z^t), w_t^*(z^t))$  that support these allocations. Let the wedges in the prototype economy satisfy:  $A_t(z^t) = 1$ ,  $\tau_{bt}(z^t) = g_t(z^t) = \tilde{R}_t(z^t) = 0$ , and*

$$\tau_{kt}(z^t) = \tau_{lt}(z^t) = 1 - \frac{r_t^*(z^t)}{F_{kt}^*(z^t)} \quad (26)$$

for all  $z^t$ , where  $F_{kt}^*(z^t)$  is evaluated at the equilibrium of the sticky-price economy. Then  $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t))$  and  $(p_t^*(z^t), R_t^*(z^t))$  are also equilibrium allocations and prices of the prototype economy.

For the proof, see Appendix A.

The key point here is that sticky prices have the same distortionary effects as capital and labour income taxes.<sup>(7)</sup> Fluctuations in the data due to sticky prices thus show up in the prototype economy as equal movements in investment and labour wedges.

## 3.2 An economy with limited participation in the money market

### 3.2.1 The underlying economy

Consider now an economy in which consumers do not participate in the money market, such as that of Christiano and Eichenbaum (1992). The probability space underlying this economy is again the same as that of the prototype economy described in Section 2. The consumer chooses plans for consumption  $c_t(z^t)$ , investment  $x_t(z^t)$ , capital  $k_{t+1}(z^t)$ , leisure  $h_t(z^t)$ , labour  $l_t(z^t)$ , money balances  $m_t(z^t)$ , and deposits with financial intermediaries

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<sup>(7)</sup> A similar point has been made by Goodfriend and King (1998).

$q_t(z^{t-1})$  to maximise the utility function **(1)** subject to three constraints. First, the consumer has to satisfy the budget constraint

$$c_t(z^t) + x_t(z^t) + \frac{m_t(z^t)}{p_t(z^t)} = [1 + R_t(z^t)] \frac{q_t(z^{t-1})}{p_t(z^t)} + w_t(z^t)l_t(z^t) + r_t(z^t)k_t(z^{t-1}) + \frac{m_{t-1}(z^{t-1}) - q_t(z^{t-1})}{p_t(z^t)} + \frac{\psi_t(z^t)}{p_t(z^t)}$$

where  $\psi_t(z^t)$  is profits from the financial intermediaries. Second, the consumer has to satisfy a cash-in-advance constraint

$$c_t(z^t) = \frac{m_{t-1}(z^{t-1}) - q_t(z^{t-1})}{p_t(z^t)} \quad (27)$$

The third constraint is the capital accumulation law **(4)**. Again, the population growth rate is equal to zero. Notice that in the consumer's problem deposits in period  $t$  are a function of a history only up through and including period  $t - 1$ .

The producer has access to an aggregate production function

$$y_t(z^t) = F(k_t(z^{t-1}), l_t(z^t)) \quad (28)$$

and it finances a fraction  $\phi_t$  of the wage bill  $w_t(z^t)l_t(z^t)$  through loans from the financial intermediaries.<sup>(8)</sup> The intermediaries operate in a perfectly competitive market so that the interest rate on loans is equal to the interest rate on deposits. The producer maximises profits  $F(k_t(z^{t-1}), l_t(z^t)) - [1 + \phi_t(z^t)R_t(z^t)]w_t(z^t)l_t(z^t) - r_t(z^t)k_t(z^{t-1})$  by setting marginal products of capital and labour equal to their effective prices, which in the case of labour is  $[1 + \phi_t(z^t)R_t(z^t)]w_t(z^t)$ .

The government sets the nominal interest rate according to the rule

$$R_t(z^t) = (1 - \rho_R) [R + \omega_y (\ln y_t(z^t) - \ln y) + \omega_\pi (\pi_t(z^t) - \pi)] + \rho_R R_{t-1}(z^{t-1}) + \xi_t(z^t) \quad (29)$$

where  $\xi_t(z^t)$  is a monetary policy shock. In terms of the prototype economy of Section 2, this shock can be considered as a part of the monetary policy wedge, though, as the next subsection shows, the wedge is a much broader object.<sup>(9)</sup> The government implements the nominal interest rate dictated by this rule through money transfers  $\eta_t(z^t)$  to the financial intermediaries. This mechanism is similar to open market operations carried out by the Fed. Total loanable funds at the disposal of the financial intermediaries are therefore  $q_t(z^{t-1}) + \eta_t(z^t)$  and the money stock evolves as  $m_t(z^t) = m_{t-1}(z^{t-1}) + \eta_t(z^t)$ . Since the consumer is excluded from the money market, the producer has to hold the extra cash.

<sup>(8)</sup> In the original model by Christiano and Eichenbaum (1992)  $\phi_t(z^t) = 1$ .

<sup>(9)</sup> The choice of this particular shock is not crucial for the main result of this subsection. A more elaborate model of financial intermediation would allow us to consider other shocks, such as shocks to bank reserves, that would have similar effect on the money market equilibrium as monetary policy shocks.

Clearing the money market therefore requires that the supply of loanable funds is equal to their demand

$$q_t(z^{t-1}) + \eta_t(z^t) = \phi_t(z^t)p_t(z^t)w_t(z^t)l_t(z^t) \quad (30)$$

Since there are no deposits held by the consumer against the transfer  $\eta_t(z^t)$ , the gross interest that the intermediaries earn from lending these transfers to the producer is the intermediaries' profit  $\psi_t(z^t) = (1 + R_t(z^t))\eta_t(z^t)$ , which is paid to the consumer.

An *equilibrium of this economy with limited participation* is a set of allocations  $(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t), q_t(z^{t-1}))$  and a set of prices  $(p_t(z^t), R_t(z^t), r_t(z^t), w_t(z^t))$  that satisfy: (i) a set of the consumer's first-order conditions for deposits, labour and capital, respectively

$$\sum_{z_t} \mu_{t-1}(z^t|z^{t-1}) \frac{u_{ct}(z^t)}{p_t(z^t)} = \beta \sum_{z_t} \mu_{t-1}(z^t|z^{t-1}) \frac{u_{c,t+1}(z^{t+1})}{p_{t+1}(z^{t+1})} (1 + R_t(z^t)) \quad (31)$$

$$u_{ht}(z^t) \frac{1 + \phi_t(z^t)R_t(z^t)}{F_{lt}(z^t)} = \beta \sum_{z_{t+1}} \mu_t(z^{t+1}|z^t) u_{c,t+1}(z^{t+1}) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} \quad (32)$$

$$u_{ht}(z^t) \frac{1 + \phi_t(z^t)R_t(z^t)}{F_{lt}(z^t)} = \beta \sum_{z_{t+1}} \mu_t(z^{t+1}|z^t) u_{h,t+1}(z^{t+1}) \times \frac{1 + \phi_{t+1}(z^{t+1})R_{t+1}(z^{t+1})}{F_{l,t+1}(z^{t+1})} [1 + F_{k,t+1}(z^{t+1}) - \delta] \quad (33)$$

(ii) the producer's first-order conditions  $w_t(z^t) = F_{lt}(z^t)/[1 + \phi_t(z^t)R_t(z^t)]$  and  $r_t(z^t) = F_{kt}(z^t)$ ; (iii) the cash-in-advance constraint (27); (iv) the money market clearing condition (30); (v) the aggregate resource constraint  $c_t(z^t) + x_t(z^t) = y_t(z^t)$ , where  $y_t(z^t)$  is given by the production function (28); (vi) the capital accumulation law (4); and (vii) the interest rate rule (29). Notice, that the expectations in the first-order condition for deposits are conditional on a history  $z^{t-1}$ , rather than  $z^t$ , as in the case of the other first-order conditions.

### 3.2.2 The associated prototype economy

Consider now a version of the prototype economy of Section 2. In particular, suppose that the shopping time function (3) has the following form

$$s_t(z^t) = \begin{cases} 0 & \text{if } p_t(z^t)c_t(z^t) = m_t(z^t) \\ 1 & \text{otherwise} \end{cases} \quad (34)$$

Effectively, the consumer faces zero costs of the first shopping trip, and infinite costs of any subsequent trip. Since in equilibrium the consumer always chooses  $s_t(z^t) = 0$ , this shopping time function implies that in equilibrium the consumer has to satisfy the cash-in-advance constraint  $p_t(z^t)c_t(z^t) = m_t(z^t)$ .



The consumer's first-order conditions with respect to bonds, labour and capital, respectively, now become

$$\frac{[1 + \tau_{bt}(z^t)] u_{ht}(z^t)}{[1 - \tau_{lt}(z^t)] A_t(z^t) F_{lt}(z^t)} \quad (35)$$

$$= \beta \sum_{z^{t+1}} \mu_t(z^{t+1}|z^t) \frac{[1 + \tau_{b,t+1}(z^{t+1})] u_{h,t+1}(z^{t+1})}{[1 - \tau_{l,t+1}(z^{t+1})] A_{t+1}(z^{t+1}) F_{l,t+1}(z^{t+1})} (1 + R_t(z^t)) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})}$$

$$\frac{u_{ht}(z^t)}{[1 - \tau_{lt}(z^t)] A_t(z^t) F_{lt}(z^t)} = \beta \sum_{z^{t+1}} \mu_t(z^{t+1}|z^t) u_{c,t+1}(z^{t+1}) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} \quad (36)$$

$$[1 + \tau_{xt}(z^t)] \frac{u_{ht}(z^t)}{[1 - \tau_{lt}(z^t)] A_t(z^t) F_{lt}(z^t)} \quad (37)$$

$$= \beta \sum_{z^{t+1}} \mu_t(z^{t+1}|z^t) \frac{u_{h,t+1}(z^{t+1})}{[1 - \tau_{l,t+1}(z^{t+1})] A_{t+1}(z^{t+1}) F_{l,t+1}(z^{t+1})}$$

$$\times \{ (1 - \delta) [1 + \tau_{x,t+1}(z^{t+1})] + A_{t+1}(z^{t+1}) F_{k,t+1}(z^{t+1}) \}$$

and the first-order condition for money **(11)** is replaced by the cash-in-advance constraint.

**PROPOSITION 2:** *Consider equilibrium allocations of the economy with limited participation  $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t), q_t^*(z^{t-1}))$  and prices  $(p_t^*(z^t), R_t^*(z^t), r_t^*(z^t), w_t^*(z^t))$  that support these allocations. Let the wedges in the prototype economy satisfy:  $A_t(z^t) = 1$ ,  $\tau_{xt}(z^t) = g_t(z^t) = 0$ ,  $\tilde{R}_t(z^t) = \xi_t(z^t)$*

$$[1 - \tau_{lt}(z^t)] = \frac{1}{1 + \phi_t(z^t) R_t^*(z^t)} \quad (38)$$

and

$$\frac{u_{ct}^*(z^t)}{p_t^*(z^t)} \Omega_t^*(z^t) [1 + \tau_{bt}(z^t)] \quad (39)$$

$$= (1 + R_t^*(z^t)) \beta \sum_{z^{t+1}} \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}^*(z^{t+1})}{p_{t+1}^*(z^{t+1})} \Omega_{t+1}^*(z^{t+1}) [1 + \tau_{b,t+1}(z^{t+1})]$$

where

$$\Omega_t^*(z^t) \equiv \sum_{z^{t+1}} \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}^*(z^{t+1})}{u_{ct}^*(z^t)} \frac{p_t^*(z^t)}{p_{t+1}^*(z^{t+1})}$$

for all  $z^t$ , where  $u_{ct}^*$  is evaluated at the equilibrium of the detailed economy. Then  $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t))$  and  $(p_t^*(z^t), R_t^*(z^t))$  are also equilibrium allocations and prices of the prototype economy.

For the proof, see Appendix A.

Consider now a special case of Proposition 2. Suppose that the fraction of the wage bill financed through loans from financial intermediaries  $\phi_t(z^t)$  fluctuates so as to offset the effects of changes in the interest rate on the effective wage rate. In this case, open market

operations by the central bank lead to fluctuations in  $\tau_{bt}(z^t)$  but not in  $\tau_{lt}(z^t)$ . The main idea here is that limited participation in the money market distorts the no-arbitrage condition between capital and nominal bonds. This distortion, which Fuerst (1992) calls a ‘liquidity effect’, is equivalent to a tax on investment in bonds in the prototype economy. Fluctuations in the data due to limited participation in the money market will therefore show up in the prototype economy as fluctuations in the asset market wedge.

### 3.3 *The monetary policy wedge*

As we saw above, a monetary policy shock – an innovation to the nominal interest rate in a monetary policy rule – can be a part of the monetary policy wedge. However, the wedge is a much broader object. It captures all aspects of monetary policy beyond the responses of the monetary authority to output and inflation as specified by the prototype policy rule (7). As an example, consider a monetary policy rule with fluctuations in the inflation target, as in Gavin, Kydland and Pakko (2007). In particular, suppose that the underlying monetary policy rule is

$$R_t(z^t) = R + \omega_y (\ln y_t(z^t) - \ln y) + \omega_\pi (\pi_t(z^t) - \bar{\pi}_t(z^t)) \quad (40)$$

where  $\bar{\pi}_t(z^t)$  is a stochastic inflation target that fluctuates around a steady-state inflation rate  $\pi$ . This policy rule is equivalent to the prototype policy rule (7) where the inflation target is constant and the monetary policy wedge is given by  $\tilde{R}_t(z^t) = -\omega_\pi(\bar{\pi}_t(z^t) - \pi)$ . In a similar fashion, responses of the monetary authority to variables other than inflation and output show up as fluctuations in the monetary policy wedge.<sup>(10)</sup>

## 4 **Measuring the realised wedges**

Before taking the prototype economy of Section 2 to the data we need to make assumptions about the stochastic process for the events  $z_t$ . Following CKM we assume that the events are governed by a stationary Markov process of the form  $\mu(z^t|z^{t-1})$  and that there is a one-to-one and onto mapping between the events and the wedges. The latter assumption implies that the wedges uniquely identify the underlying events. We can therefore replace in the prototype economy the probability measures for the events with probability measures for the wedges. Since the stochastic process for the events is Markov, the stochastic process for the wedges is also Markov. In particular, following CKM we specify a vector autoregressive AR(1) process for the wedges

$$\omega_{t+1} = P_0 + P\omega_t + \varepsilon_{t+1} \quad (41)$$

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<sup>(10)</sup> For example, Sims and Zha (2006) argue that the Fed was responding to money growth in the period before 1979.

where  $\omega_t = (\log A_t, \tau_{lt}, \tau_{xt}, \log g_t, \tau_{bt}, \tilde{R}_t)$  and the shock  $\varepsilon_{t+1}$  is iid over time and is distributed normally with mean zero and a covariance matrix  $V = BB'$ . There are no restrictions imposed on this stochastic process except stationarity. In particular, the off-diagonal elements of  $P$  and  $V$  are allowed to be non-zero.

Measurement of the realised wedges involves three steps. The first step is to choose functional forms of the utility, production and shopping-time functions and their parameter values, as well as the parameter values of the monetary policy rule. The second step is to estimate the parameters of the stochastic process for the wedges  $P_0$ ,  $P$  and  $B$ . In the third step the equilibrium decision rules and pricing functions of the prototype economy are used to uncover the wedges from the data.

As a part of steps two and three we need to compute the equilibrium decision rules and pricing functions of the prototype economy. Since the state space is large (there are nine state variables in the model), the equilibrium is computed using a linear-quadratic approximation method described by Hansen and Prescott (1995). The outcome of this method is a set of linear functions that express equilibrium allocations and prices in terms of a state vector  $(\omega_t, p_{t-1}, R_{t-1}, k_t)$ . A linear-quadratic approximation method is also used for the experiments in Section 5.<sup>(11)</sup> The rest of this section describes the three steps in more detail.

Calibration of the model is summarised in Table A. We set one period in the model equal to one quarter. As in CKM, the utility function is assumed to have the functional form  $u(.,.) = \lambda \log c_t + (1 - \lambda) \log h_t$  and the production function to have the form  $F(.,.) = k_t^\alpha ((1 + \gamma_A) l_t)^{1-\alpha}$ . These functional forms are standard in the real business cycle literature. Following Dittmar *et al* (2005), the shopping-time function takes the form

$$s(.,.) = \nu_1 \left( \frac{c_t}{m_t/p_t} \right)^{\nu_2}$$

where  $\nu_1 \in (0, \infty)$  and  $\nu_2 \in [1, \infty)$ . Wherever possible, parameter values are the same as those used by CKM. In particular, the population growth rate  $\gamma_n$  is set equal to 0.0037, technology growth  $\gamma_A$  is set equal to 0.004, the depreciation rate  $\delta$  is set equal to 0.0118, and the capital share of output  $\alpha$  is set equal to 0.35. As in Dittmar *et al* (2005), the curvature parameter in the shopping time function  $\nu_2$  is set equal to one, which implies a long-run money demand function with interest elasticity of -0.5, found by many studies for the US data (eg Lucas (2000)).

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<sup>(11)</sup> Linear approximations to underlying decision rules are fairly accurate for the postwar US business cycle. The reason is that linear approximations work well in the neighborhood of a steady state and, unlike in the Great Depression period, deviations of key variables from trend after the end of the WWII were relatively small.

The parameters of the monetary policy rule are set equal to standard values used in the literature: the weight on output is set equal to 0.125, the weight on inflation is set equal to 1.5 and the smoothing parameter  $\rho_R$  is set equal to 0.75 (see Woodford (2003), Chapter 1). Steady-state inflation  $\pi$  is set equal to the average quarterly inflation rate in the postwar period equal to 0.91%. As discussed above, any changes in the parameters of the monetary policy rule due to policy regime changes are captured by fluctuations in the monetary policy wedge. We therefore keep the parameter values of the monetary policy rule fixed for the entire postwar period and think of the prototype policy rule as an average policy rule for the postwar period.

Values of the remaining parameters  $\lambda$ ,  $\beta$  and  $\nu_1$  are chosen so that, for the estimated steady-state values of the wedges, the model matches three calibration targets:  $l$  equal to 0.26,  $k/y$  equal to 11.2 and  $py/m$  equal to 0.58 – the average quarterly velocity of the MZM aggregate in the postwar period. These calibration targets imply  $\lambda$  equal to 0.266,  $\beta$  equal to 0.995 and  $\nu_1$  equal to 0.0319.

The parameters  $P_0$ ,  $P$  and  $B$  of the stochastic process for the wedges are estimated using a maximum likelihood procedure (eg McGrattan (1994)). The resulting estimates are contained in Table B. The likelihood function is based on a state-space representation consisting of the stochastic process for the wedges (41) and equilibrium decision rules for  $y_t$ ,  $x_t$ ,  $g_t$ , and  $l_t$ , and equilibrium pricing functions for  $p_t$  and  $R_t$ . The decision rules and pricing functions are linear functions of the state vector  $(\omega_t, p_{t-1}, R_{t-1}, k_t)$ , where linearity comes from the linear-quadratic approximation of the model. Estimation is carried out using data on output (the sum of GDP and imputed services from consumer durables), investment (which includes consumer durables), hours, the sum of government consumption and net exports, the GDP deflator, and a yield on 3-month Treasury bills for the period 1959.Q1-2004.Q4. Data on output, investment, hours, and the sum of government consumption and net exports are in per capita terms. In addition, a common trend of 1.6% at an annual rate is removed from the data on output, investment, and the sum of government consumption and net exports, and a trend of 3.7% is removed from the price level. Capital is computed recursively using the law of motion (4), data on investment, and an initial capital stock.

Once we have the stochastic process for the wedges, we can compute the equilibrium of the model associated with this stochastic process and uncover from the data the realised values of the wedges, denoted by  $\omega_t^d = (\log A_t^d, \tau_{lt}^d, \tau_{xt}^d, \log g_t^d, \tau_{bt}^d, \tilde{R}_t^d)$ . The realised values of  $g_t$  are observed directly from the data as the sum of government consumption and net exports. The realised values of the remaining wedges are then obtained from the equilibrium

decision rules and pricing functions  $y_t = y(\omega_t, p_{t-1}, R_{t-1}, k_t)$ ,  $x_t = x(\omega_t, p_{t-1}, R_{t-1}, k_t)$ ,  $l_t = l(\omega_t, p_{t-1}, R_{t-1}, k_t)$ ,  $p_t = p(\omega_t, p_{t-1}, R_{t-1}, k_t)$ , and  $R_t = R(\omega_t, p_{t-1}, R_{t-1}, k_t)$ . Again, as at the estimation stage, we use linear approximations of these functions in the actual implementation of the procedure. The linear functions constitute a system of five equations that in each period can be solved for the five unknown values of  $\log A_t$ ,  $\tau_{lt}$ ,  $\tau_{xt}$ ,  $\tau_{bt}$ , and  $\tilde{R}_t$  using data on current output, investment, hours, and the sum of government consumption and net exports, and data on the current and lagged price level and the nominal interest rate. We do not use capital stock data. Instead, capital stock is computed recursively from the data on investment using the law of motion (4). As at the estimation stage, the data on output, investment, and government consumption and net exports are first detrended with a common linear trend of 1.6%, and the price level is detrended with a linear trend of 3.7%.

Notice, that in this procedure  $\log A_t^d$ ,  $\tau_{lt}^d$ ,  $\tau_{xt}^d$ ,  $\tau_{bt}^d$ , and  $\tilde{R}_t^d$  are effectively obtained from five equilibrium conditions for the prototype economy: the production function (5), the monetary policy rule (7) and the consumer's first-order conditions (8)-(10), once real money balances have been eliminated by substitution from the first-order condition (11). Notice also that in measuring the realised wedges, the estimated stochastic process (41) plays a role only in measuring labour, investment and asset market wedges. Efficiency wedges and monetary policy wedges are obtained, respectively, from the production function (5), together with the law of motion for capital (4), and the monetary policy rule (7). These two equations do not contain expectations and therefore the stochastic process is not required to back out the two wedges. In contrast, in order to uncover labour, investment and asset market wedges we do need to know the stochastic process because the optimality conditions (8)-(10) have expectations on the right-hand side (the optimality condition for labour contains expectations once real money balances are substituted into equation (8) from the optimality condition (11)).<sup>(12)</sup>

Tables C and D provide some summary statistics for the realised wedges. Table C shows the standard deviations of the wedges, relative to output, and the correlations of the

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<sup>(12)</sup> In the CKM prototype economy the optimal labour decision is purely intratemporal and therefore the labour wedge does not depend on the stochastic process for the wedges. Notice, that if we used data on money, we would not need to eliminate real money balances from the optimality condition for labour and the labour wedge would not depend on the stochastic process. However, to do so in a way consistent with the principles of business cycle accounting would require us to include the first-order condition for money balances, with a new wedge, in the system of equilibrium conditions used to back out the wedges. We do not proceed this way for the following reasons. First, introducing an additional wedge would increase the number of parameters in the stochastic process for the wedges that need to be estimated. Second, most of the recent literature studying the joint dynamics between real and nominal variables in quantitative dynamic general equilibrium models only focuses on the dynamics of inflation and the nominal interest rate (eg Ireland (2004), Primiceri *et al* (2006), and Smets and Wouters (2007)). And third, since most of the outstanding money stock in the US economy is inside money (deposits), a prototype model addressing the dynamics of money should also contain a banking sector. We leave such extensions for future research.

wedges with output at various leads and lags (both the wedges and output have been detrended with HP-filter before computing the statistics). Focusing on the CKM wedges first, we see that the efficiency and investment wedges are much less volatile than output (only about 63% and 50% as volatile as output, respectively), while the government consumption wedge is more volatile than output (1.5 times) and the labour wedge is about as volatile as output. In addition, both the efficiency and investment wedges are procyclical, with no apparent phase shift, while the labour and government consumption wedges are countercyclical. The labour wedge also lags output by one quarter with a negative sign while the government consumption wedge, whose cyclical behaviour is primarily driven by net exports, leads output by two quarters with a negative sign.

The cyclical behaviour of the efficiency, labour and government consumption wedges found here is essentially the same as that reported by CKM. Of course this is what we would expect for efficiency and government consumption wedges, since they do not depend on expectations, and to some extent for labour wedges, since they depend on expectations only due to the presence of real money balances in the first-order condition (8). The realised values of investment wedges are, however, different from those obtained by CKM. We obtain  $\tau_{xt}$  that is procyclical, while their  $\tau_{xt}$  is countercyclical. The reason behind this difference is that in order to uncover the investment wedges, we need to know the stochastic process (41). Because we have two more wedges in our prototype economy, this stochastic process differs from that in the CKM prototype economy. As a result, expectations about the future evolution of the wedges in the first-order condition for capital in our prototype economy are different from those in the CKM economy. Nevertheless, as we show below, the differences in the measured values of the investment wedge have little effect on the substantive result of CKM that investment wedges play only a minor role in aggregate fluctuations.

Looking at the cyclical behaviour of the two new wedges, we see that the asset market wedge is highly volatile (more than 2.5 times as volatile as output) and strongly procyclical. High volatility of the asset market wedge reflects the well-known failure of Euler equations with power utility functions to price financial assets. Since the real return on Treasury bills is more volatile than the marginal rate of substitution, the asset market wedge has to be volatile enough for the first-order condition for bonds to hold. Although it is possible to interpret the asset market wedge as a measure of goodness of fit of the Euler equation, we have shown that it can also be interpreted as summarising some underlying frictions in financial markets. The strong positive comovement of the asset market wedge with output suggests that these frictions worsen in expansions. In contrast to the cyclical behaviour of the asset market wedge, the monetary policy wedge is very smooth and only

weekly correlated with output at all leads and lags.

Table D displays contemporaneous correlations of HP-filtered wedges with each other. We see that in general the wedges are correlated with each other, and that for some of them the correlations are strong. In particular, the asset market wedge is strongly positively correlated with the efficiency and investment wedges, and strongly negatively correlated with the labour wedge. Furthermore, the efficiency wedge is strongly positively correlated with the investment wedge. In contrast, the monetary policy wedge is only weakly correlated with the other wedges, perhaps with the exception of the asset market wedge. Notice also that the labour wedge is negatively correlated with the investment wedge. This finding is in a sharp contrast with the predictions of sticky price models. According to Proposition 1, nominal rigidities in the form of sticky prices are equivalent to investment and labour wedges that move together. We conclude from this finding that nominal rigidities in the form of sticky prices played at most a modest role in driving aggregate fluctuations in the postwar US economy. Other frictions, which drive investment and labour wedges in opposite directions, were more important.

## **5 Assessing the contributions of the wedges to aggregate fluctuations**

In this section we decompose fluctuations in output, hours, investment, consumption, inflation, and the nominal interest rate into movements driven by each of the six wedges, and by their various combinations.<sup>(13)</sup> The decomposition is applied to two US postwar recessions: the 1973 and the 1982 recessions. These two recessions are interesting because they were the two most severe ones in the postwar US history. In addition, they were presumably caused by different shocks. It is commonly thought that the 1973 recession was caused by high oil prices (a ‘supply shock’), while the 1982 recession was caused by tight monetary policy intended to reduce inflation (a ‘demand shock’). The two recessions have also different dynamics. The 1973 recession is characterised by a sharp fall in economic activity, followed by a slow recovery, whereas the 1982 recession is characterised by a prolonged decline in activity but a relatively fast recovery. It is therefore interesting to investigate whether also different wedges, or their different combinations, are needed to generate the fluctuations in the data during the 1973 recession than during the 1982 recession. Of course, it is unlikely that any single wedge, or any combination of the wedges (except the one that contains all six of them), would account for all of the fluctuations in all six variables. What we are interested in is to see which wedges can broadly capture the nature of each recession and the subsequent recoveries.

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<sup>(13)</sup> See Chari *et al* (2007b) for a discussion of how business cycle accounting decomposition is related to VAR decompositions.

We use the same decomposition methodology as CKM.<sup>(14)</sup> In this methodology, the decomposition is carried out as follows. Suppose that we are interested in the movements in the data due to the efficiency wedge only. In this case, we need to compare the data to the predictions of a version of the prototype model of Section 2, in which, as before, the efficiency wedge is a function of the underlying events, but in which all the other wedges are constant in all states of the world; ie the wedges vector is  $(A_t(z^t), \bar{\tau}_l, \bar{\tau}_x, \bar{g}, \bar{\tau}_b, \bar{R})$ . As emphasised by Chari *et al* (2007b), this experiment isolates the distortionary effects of the efficiency wedge on equilibrium quantities and prices, without altering the consumer's expectations about the future evolution of the underlying events.

As in the previous section, in the actual implementation of this experiment, expectations for the events are replaced with expectations for the wedges; ie we solve a version of the prototype model in which the consumer is faced with the stochastic process (41), with the parameter values in Table B, but in which, in the budget and resource constraints, and in the monetary policy rule, all wedges except the efficiency wedge are kept constant at their steady-state values. In this economy, all wedges play a forecasting role for the evolution of the underlying events, but only the efficiency wedge distorts the equilibrium. Let  $y^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$ ,  $x^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$ ,  $c^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$ ,  $l^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$ ,  $p^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$ , and  $R^A(\omega_t, p_{t-1}, R_{t-1}, k_t)$  denote the equilibrium decision rules and pricing functions for this modified economy. Starting from  $p_{-1}$ ,  $R_{-1}$  and  $k_0$  for some base period, these decision rules and pricing functions are used in each period together with  $\omega_t^d$  – the vector of the realised wedges – to compute the efficiency wedge component of output, investment, consumption, labour, inflation, and the nominal interest rate. The capital accumulation law (4) is then used to obtain the capital stock for the next period. The components of the movements in the endogenous variables due to the other wedges, or their various combinations, are computed similarly. Indeed, the model with all six wedges exactly reproduces the data.

### 5.1 The 1973 recession

The findings for the 1973 recession are displayed in Figures 1-9. We employ a working definition of the 1973 recession as the period from the start of the oil crisis in 1973.Q4 to full recovery in output in 1978.Q4. Figure 1 shows the actual data and the realised values of the wedges for this period. Panel A of the figure displays percentage deviations of output, investment, consumption, and government consumption from a linear trend of 1.6%, and percentage deviations of hours from their postwar average. The data are

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<sup>(14)</sup> See Christiano and Davis (2006) for an alternative methodology, and Chari *et al* (2007b) for a comparison of the two methodologies.



normalised so that in 1973.Q3, one quarter before the oil crisis, the deviations are zero. We see that output declines by about 7% below trend by the first half of 1975 and does not fully recover until the end of 1978. A similar pattern is also observed for hours, investment and consumption, although the fall in investment is much sharper (27% below trend by the end of 1975) while the fall in consumption is milder (5.4% below trend by the first quarter of 1975).

Panel B plots the deviations of quarterly inflation and the nominal interest rate (expressed at annual rates) from their 1973.Q3 levels. The surge in inflation following the oil crisis clearly stands out in the chart. By the end of 1974 inflation is 4 percentage points higher than before the start of the crisis. However, after this initial increase inflation falls below its 1973.Q3 level and starts to pick up only towards the end of the recession. Except for the initial peak, the nominal interest rate follows a similar pattern as inflation, although it is less volatile. Notice also that during the entire period, the nominal interest rate is relatively lower than the inflation rate, implying that during the recession the real interest rate is below its 1973.Q3 level.

Panels C and D of Figure 1 display the deviations of the wedges. For all six wedges, their relative volatilities and their comovement with output during the recession are consistent with their behaviour throughout the entire postwar period, as summarised by Table C. In panel C of the figure we see that the efficiency wedge  $A$  falls by 3.5% below trend by 1975 and does not fully recover until the end of the recession. The labour wedge  $\tau_l$  increases by almost 6 percentage points by the end of the first half of 1975 and falls below its pre-crisis level by the end of the period. In contrast, throughout the whole period the investment wedge  $\tau_x$  fluctuates below its pre-crisis level. As mentioned above, such behaviour of investment and labour wedges is inconsistent with sticky prices being the primary friction driving the fluctuations in the data. Panel D plots the asset market and monetary policy wedges. As can be seen, the monetary policy wedge  $\tilde{R}$  does not fluctuate much relative to the asset market wedge  $\tau_b$  and stays below its pre-crisis level for the entire period. In contrast, the asset market wedge falls sharply (by 17 percentage points by 1975.Q1) and stays below its pre-crisis level until the middle of 1978.

Plotting the data and the wedges is useful for getting an idea about the nature of the recession. However, what matters for assessing the quantitative importance of the different wedges for aggregate fluctuations are the responses of the model when we feed the wedges back into the model. Recall that putting the wedges back into the model involves re-computing the equilibrium of the model under the assumption that only the wedges under investigation distort the budget and resource constraints in the prototype economy.

We start by putting the wedges back into the model one at a time. As mentioned in Section 2, the CKM wedges affect all endogenous variables in the prototype economy, whereas the asset market and monetary policy wedges have significant effects only on inflation and the nominal interest rate. Since we are interested in the *joint* behaviour of real and nominal variables, we feed back individually only the CKM wedges. The marginal contributions to fluctuations in inflation and the nominal interest rate of the asset market and monetary policy wedges will be studied only in combinations with the CKM wedges.

Consider the efficiency wedge first. In Figure 2 we see that the efficiency wedge alone accounts for nearly all of the decline in output (86%), but it generates a more rapid recovery than in the data. It also accounts for a large fraction of the decline in labour input (68%) and for essentially all of the decline in investment (93%). However, as in the case of output, for both variables the efficiency wedge generates a more rapid recovery than actually occurred. In terms of consumption, the model accounts for more than half of its decline and predicts a more subdued recovery than in the data. As can also be seen in Figure 2, throughout the entire period the efficiency wedge alone generates fluctuations in the inflation rate that closely mimic those in the data. The efficiency wedge alone cannot, however, account for the behaviour of the nominal interest rate. The model predicts an increase in the nominal interest rate, while the interest rate falls in the data.

The findings for the model with the efficiency wedge alone are interesting in light of our equivalence results. In particular, Appendix B shows that fluctuations in energy prices in a model with capital utilisation are equivalent to fluctuations in the efficiency wedge in the prototype economy. Our findings thus suggest that models in which oil prices affect the production possibility frontier are promising models of the decline in economic activity and the behaviour of inflation in the 1970s. Such models can abstract from nominal price rigidities, and other frictions that do not manifest themselves as efficiency wedges, without their ability to account either for the decline in activity or inflation dynamics being significantly affected.

Figure 3 shows the responses of the model to the labour wedge. We see that the model generates a fall in output that is not as sharp as in the data, and that is also smaller than in the case of the efficiency wedge (75% vs 86%). This is despite the fact that the labour wedge accounts for more than the observed sharp fall in hours. However, the labour wedge accounts for the slow recovery in output, as well as the recovery in hours, investment and consumption not captured by the efficiency wedge, suggesting that worsening labour market frictions prevented the economy from a quick recovery following the oil-price shock. However, unlike the efficiency wedge, the labour wedge does not generate the

observed movements in inflation. Its ability to account for the fall in the nominal interest rate is as poor as in the case of the efficiency wedge.

Figure 4 shows the responses of the model to the investment wedge. In contrast to the efficiency and labour wedges, the investment wedge generates a mild expansion of output, hours and investment. This result is in line with the findings of CKM that the investment wedge played only a minor role in the postwar US business cycle. In addition, as we can see in the Figure, once the nominal side of the economy is taken into account, it turns out that the investment wedge is also unimportant for the fluctuations in the nominal interest rate and inflation. As for the government consumption wedge, Figure 5 shows that its effect on real variables is relatively small and that the wedge drives the nominal interest rate and inflation in opposite directions that in the data.

Now we put the wedges back into the model in various combinations. In these experiments, we always put back all wedges except the wedge whose contribution we want to assess. Recall that when we feed back all six wedges, we exactly reproduce the data. Leaving a wedge out thus measures its marginal contribution to the fluctuations in the data. We start by leaving out the efficiency wedge. Figure 6 shows the results of this experiment. We see that without the efficiency wedge the model predicts a recession that is much milder and that occurs a year later than in the data. Furthermore, the model without the efficiency wedge does not capture the inflation dynamics. Leaving out the efficiency wedge, however, has only little effect on the ability of the model to account for the behaviour of hours and the nominal interest rate.

Figure 7 shows the responses of the model when we leave out the labour wedge. We see that without the labour wedge output falls only half as much as in the data, even though the timing of the fall coincides with that in the data. The model also predicts much faster recovery than actually occurred. As can also be seen in the Figure, without the labour wedge the model completely misses the behaviour of hours, predicting relatively flat hours at the start of the recession and an increase after that. Leaving out the labour wedge, however, has little effect on the predictions of the model for inflation and the nominal interest rate. Although the levels of the two variables do not exactly coincide with the levels in the data, especially in the middle of the period, the model captures well the general pattern of the two variables.

In contrast to the two previous experiments, leaving out the investment wedge has essentially no effect on the ability of the model to account for the data, as Figure 8 shows. Without the investment wedge, the model predicts only somewhat deeper recession than in

the data and somewhat faster recovery in hours.

Figures 9 and 10 show the marginal contributions of the monetary policy and asset market wedges. In Figure 9 we see that leaving out the monetary policy wedge has an effect on both, inflation and the nominal interest rate, but the effect on inflation is bigger than on the nominal interest rate. In particular, without the monetary policy wedge the model still predicts a decline in the nominal interest rate after the first half of 1974 and some of its pick up after 1977, but the model completely misses the behaviour of inflation.

This result seems to contradict our previous result that the efficiency wedge alone can account for most of the observed movements in inflation. It is, however, important to realise that in the present experiment we measure the *marginal* contribution of the monetary policy wedge once the labour and asset market wedges are added to the efficiency wedge. The labour and asset market wedges generate movements in the inflation rate that need to be offset by the monetary policy wedge for the predictions of the model to be close to the data. This suggests that during the recession monetary policy actions captured by the monetary policy wedge interacted with frictions that manifest themselves as the labour and asset market wedges. Once such frictions are included in a detailed model, perhaps in order to capture the slow recovery, monetary policy that manifests itself as fluctuations in the monetary policy wedge needs to be also included in the model, if the model is to generate the inflation dynamics observed in the data.

Figure 10 shows the effects of leaving out the asset market wedge. As we can see in the Figure, the asset market wedge is crucial for the behaviour of the nominal interest rate. Without the asset market wedge, the model predicts an increase in the nominal interest rate, while in the data the nominal interest falls and stays below its pre-crisis level throughout the episode. In terms of inflation, the model generates a path that closely co-moves with the path in the data, even though the inflation rate in the model is much higher and more volatile than in the data.

To summarise our results for the 1973 recession, we find that the efficiency wedge is crucial for capturing the sharp decline in economic activity following the oil-crisis while the labour wedge accounts for the subsequent slow recovery. In addition, the efficiency wedge alone captures most of the fluctuations in inflation during the recession. The monetary policy wedge becomes important for inflation dynamics only once the labour and asset market wedges are included in the model. In contrast, the asset market wedge is crucial for the nominal interest rate, regardless of which other wedges are included in the model. The investment and government consumption wedges play only a minor role for

fluctuations in both real and nominal variables. Thus models in which the underlying frictions and propagation mechanisms manifest themselves as efficiency wedges are promising models for the decline in economic activity and inflation dynamics in the 1970s. In order to capture the slow recovery, and at the same time to capture the behaviour of inflation, the models should also include frictions that show up as labour, asset market and monetary policy wedges.

## 5.2 *The 1982 recession*

The results for the 1982 recession are displayed in Figures 10-18. We define the 1982 recession as the period from 1979.Q3, the point when Paul Volcker became the chairman of the Federal Reserve, which many regard as a shift in the US monetary policy towards a tougher stance on inflation, to the point of full recovery in output in the last quarter of 1985.<sup>(15)</sup> Notice that our definition of the start of the 1982 recession roughly coincides with our definition of the end of the 1973 recession. Panel A of Figure 11 shows the deviations of output and its components from a common trend of 1.6%, as well as the deviations of hours from their postwar average. As before, the data are normalised so that the deviations are zero at the start of the recession. Notice that this recession was more severe than the 1973 recession – we see that output falls below trend by nearly 10% by the end of 1982, compared with the maximum deviation of 7% below trend during the 1973 recession.

Panel B plots the deviations of quarterly inflation and the nominal interest rate (expressed at annual rates) from their 1979.Q3 levels. We see that inflation increases until the end of 1980, when it is 2.4 percentage points higher than in 1979.Q3. After that it starts to decline and by the end of the recession it is six percentage points below its 1979 level. In contrast, the nominal interest rate increases until the middle of 1981 (two quarters longer than inflation), when it is five percentage points above its 1979.Q3 level. After that, it declines to 2.4 percentage points below its 1979.Q3 level at the end of the recession. Notice that unlike in the 1973 recession, the nominal interest rate is relatively higher than the inflation rate throughout much of the period, implying that during the recession the real interest rate is above its 1979 level.

Panels C and D plot the realised values of the wedges. Overall, as in the case of the 1973 recession, the cyclical behaviour of the wedges during the 1982 recession is in line with their cyclical behaviour throughout the entire postwar period. As can be seen from Panel

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<sup>(15)</sup> CKM define the start of the recession as the first quarter of 1979. As a result of this difference in the base year, the deviations of the data and the wedges reported below are slightly different from those in their paper.

C, the efficiency wedge  $A$  and the investment wedge  $\tau_x$  decline by about 4% by the middle of 1982. But while the efficiency wedge fully recovers by the end of the recession, the investment wedge is at the end of the recession still 1.3 percentage points below its 1979 level. In contrast, the labour wedge  $\tau_l$  increases by more than 6 percentage points by 1983, but falls sharply after that and by the end of the recession is almost 3 percentage points below its 1979 level. In Panel D of Figure 1 we see that the monetary policy wedge  $\tilde{R}$  fluctuates above its 1979 level throughout the entire period, in contrast to its behaviour during the 1973 recession when it fluctuated below its pre-oil crisis level. The asset market wedge  $\tau_b$  falls sharply, reaching its trough in the last quarter of 1982, but recovers rapidly after that.

The realised values of the efficiency and labour wedges are the same as in CKM. The investment wedge, however, looks different than in their paper. The reason is that, as discussed in Section 4, the stochastic process for the wedges in our prototype economy is different from that in CKM. As a result of that, expectations about the future realisations of the wedges in the first-order condition for capital are different. However, as we show below, this does not change the substantive result of CKM that the investment wedge plays only a minor role for fluctuations in the data during the 1982 recession.

Again, we start by feeding back into the model the efficiency wedge alone. In Figure 12 we see that the predicted output mimics the data extremely well until 1982. After that the model predicts flat output and faster recovery than in the data. Predicted investment tracks the actual investment well throughout the entire period, but the model does not capture a substantial fraction of the decline in hours. In terms of inflation and the nominal interest rate, the model captures the general pattern of these variables, but misses their turning points. In particular, the model correctly predicts an increase in the nominal interest rate and inflation at the start of the recession, and a decline in these variables during the recovery. However, while in the model inflation and the nominal interest rate increase until 1982.Q2, in the data they start to decline in the first half of 1981. The model also misses their levels at the end of the recession, by about 6.5 percentage points in the case of inflation and 1.5 percentage points in the case of the nominal interest rate.

Figure 13 shows the responses to the labour wedge. As we can see, like the efficiency wedge, the labour wedge produces a somewhat milder recession than in the data. By itself, however, it accounts for essentially all of the fluctuations in hours. In addition, like the efficiency wedge, it captures the increase in the nominal interest rate and inflation at the start of the recession and their decline during the recovery, but misses the turning points, predicting the start of the decline almost two years later than in the data. As in the 1973

recession, the investment and government consumption wedges play only a minor role for fluctuations in both real and nominal variables, as Figures 14 and 15 show.

Now we start putting the wedges back into the model in various combinations. Figure 16 shows the responses when we leave out the efficiency wedge. We see that without the efficiency wedge, the model produces a recession of a much smaller magnitude than in the data. In terms of the nominal interest rate and inflation, the model generates paths that mimic the actual paths well, but both the nominal interest rate and inflation in the model are in general lower than in the data. Notice also that the decline in inflation is much sharper than in the data. This suggests that in the absence of frictions that manifest themselves as efficiency wedges, the ‘conquest’ of US inflation would be faster. We obtain similar results when we leave out the labour wedge, as Figure 17 shows. In contrast, Figure 18 shows that leaving out the investment wedge has only small effects on the ability of the model to account for both real and nominal data.

Figures 19 and 20 show the responses of the model when we leave out either the asset market or the monetary policy wedge. In Figure 19 we see that leaving out the monetary policy wedge leads to fluctuations in the nominal interest rate and inflation that positively co-move with the data, but that have much higher levels and are more volatile. In addition, without the monetary policy wedge the model predicts inflation at the end of the recession about 4 percentage points above actual inflation. Figure 20 shows the predictions of the model when we leave out the asset market wedge. Interestingly, unlike in the 1973 recession, leaving out the asset market wedge has a bigger effect on inflation than on the nominal interest rate.

To summarise, we find that both efficiency and labour wedges are crucial for capturing the behaviour of economic activity during the 1982 recession. Not surprisingly these results are the same as those of CKM. However, including nominal variables into the analysis provides some additional insights. In particular, we find that the efficiency and labour wedges can account for the rise in inflation and the nominal interest rate at the start of the recession, and for their declines during the recovery. However, neither wedge predicts the turning points correctly. In particular, the model predicts that the turning points occur about one to two years later than in the data. In addition neither wedge predicts correctly the level of the inflation rate at the end of the recession.

These results suggest that explanations of the 1982 recession based purely on frictions that manifest themselves as either efficiency or labour wedges will reproduce the general pattern of inflation and the nominal interest rate, but will not generate the exact timing and

the extent of the disinflation process. Models intended to account for both real and nominal variables during this period therefore need to also include frictions that manifest themselves as asset market wedges, and monetary policy actions that manifest themselves as fluctuations in the monetary policy wedge above its 1979 level. On this latter point, notice that a fall in the Fed's implicit inflation target in the monetary policy rule (40) would produce such fluctuations in the monetary policy wedge.

## 6 Alternative parameterisations of the monetary policy rule

A number of researchers have argued that the coefficients of the Fed's reaction function have changed following the appointment of Volcker as a Chairman of the Fed in 1979 (see, for example, Woodford (2003), Chapter 1, for a brief review of the literature and Sims and Zha (2006) for an alternative view that the coefficients remained broadly unchanged). There is, however, less agreement on the exact values of the parameters of the reaction function before and after 1979. In this section we therefore investigate the sensitivity of our key results to alternative weights on output and inflation in the policy rule (7). For space constraints, we only focus on the importance of the efficiency wedge for inflation dynamics during the 1973 recession. In particular, first we split the sample into two subsamples: 1959.Q1-1979.Q3 (the pre-Volcker period) and 1979.Q4-2004.Q4 (the post-Volcker period). Then, for each subsample we back out the wedges and feed them back into the model for under six alternative parameterisations of the policy rule:  $\omega_\pi = \{1.3, 1.5, 1.7\}$  and  $\omega_y = \{0.08, 0.0125, 0.175\}$ . Notice that the values  $\omega_\pi = 1.5$  and  $\omega_y = 0.125$  correspond to our baseline parameterisation of the policy rule.<sup>(16)</sup>

Figure 21 plots the realised wedges during the 1973 recession for alternative parameterisations of the policy rule (in this case the pre-Volcker sub-sample is used to estimate the stochastic process for the wedges). We only plot labour, investment, asset market, and monetary policy wedges since efficiency and government consumption wedges are not affected by the parameters of the monetary policy rule. For comparison, we also plot the original wedges, which have been obtained for the baseline parameterisation and the stochastic process estimated for the whole sample. As can be seen, although the exact values of the wedges differ across the different parameterisations of the policy rule, their general behaviour during the period is unaffected. This is also true for the responses of the model when we feed the wedges back, as Figures 22-25 show.

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<sup>(16)</sup> Some researchers, for example Lubik and Schorfheide (2004), have argued that the pre-Volcker period is characterised by  $\omega_\pi < 1$  and thus indeterminacy of equilibria. We abstract from this possibility here in order to avoid all the complications associated with multiple equilibria. In fact, in our case, values for  $\omega_\pi$  less than 1.28 result in indeterminacy.



## 7 Conclusions

The purpose of business cycle accounting is to guide researchers in their decisions about where to introduce frictions in their models so that they exhibit fluctuations like those in the data. The method, as developed by CKM, has focused only on real variables. This paper extends business cycle accounting to fluctuations in two key nominal variables: inflation and the nominal interest rate. The purpose of this extension is to investigate what classes of frictions and propagation mechanisms drive the joint dynamics of real and nominal variables over the business cycle. To this end the paper constructs a prototype monetary economy – a monetary extension of the stochastic growth model in which, in line with much of the current literature, the monetary authority follows a Taylor (1993)-type rule. This prototype economy has six time-varying wedges that summarise the equilibrium effects of a number of frictions widely considered in the literature.

In order to demonstrate how the extended method works, the method is applied here to two postwar US downturns: the recessions of 1973 and 1982. Besides being the two most severe downturns in postwar US history, these two periods are interesting because of their different inflation dynamics: a sharp increase of inflation following the oil crisis in 1973, and a steady decline (after a small initial increase) during the 1982 recession. Application of business cycle accounting to these two periods shows that while in the case of the 1973 recession the efficiency wedge accounts for essentially all of the fluctuations in inflation, in the case of the 1982 recession the monetary policy wedge plays a crucial role for inflation dynamics. Nevertheless, the efficiency, as well as the labour wedge, still plays an important role for inflation behaviour during the 1982 downturn: a fall in the efficiency wedge and a rise in the labour wedge prevented a much more rapid decline of inflation. In both recessions, efficiency and labour wedges also account for nearly all of the fluctuations in economic activity. These findings suggest that models in which high energy prices negatively affect the production possibility frontier are promising models of the fall of economic activity and the increase of inflation during the 1973 recession. And models intended to account for the steady decline of inflation during the 1982 recession should include changes in a monetary policy rule, as well as frictions that reduce the efficiency with which factors of production are employed, or frictions that distort labour decisions.

Application of the method to the two recessions also shows that financial market frictions – frictions distorting a no-arbitrage condition between capital and bonds – are crucial for the behaviour of the nominal interest rate during the 1973 recession, and that frictions distorting investment decisions play a minimal role not only for the dynamics of real variables, as CKM find, but also for the behaviour of nominal variables. In addition, the

comovement between investment and labour wedges found in the data is inconsistent with nominal price rigidities being the key frictions driving aggregate fluctuations, both during the 1973 and 1982 recessions, as well as during the entire postwar business cycle.

There are various ways in which the method could be applied. One way is to apply the method to particular episodes as is done in this paper. Alternatively, the method could be applied to the entire postwar business cycle in an attempt to address some outstanding anomalies. For example, one outstanding anomaly in the literature, which is related to nominal data, is the phase shift of inflation and the nominal interest rate. These two variables are strongly negatively correlated with future output and strongly positively correlated with past output. Applying the method to the entire postwar business cycle could shed light on what types of frictions can account for this lead-lag pattern.

## Appendix A: Proofs of Propositions 1 and 2

### PROOF OF PROPOSITION 1

The proof proceeds by comparing the equilibrium conditions of the detailed economy with those of the prototype economy. Notice that when in the prototype economy  $A_t(z^t) = 1$ ,  $\tau_{bt}(z^t) = 0$ ,  $g_t(z^t) = 0$ , and  $\tilde{R}_t(z^t) = 0$ , the equilibrium conditions in the two economies are the same except: (i) in the prototype economy the capital rental rate is set equal to the marginal product of capital, whereas in the detailed economy this equilibrium condition is replaced by a condition for optimal price setting **(24)**; and (ii) in the prototype economy the wage and the rental rate are subject to taxes, whereas they are not taxed in the detailed economy. Since in the detailed economy  $r_t^*(z^t) \neq F_{kt}^*(z^t)$ , it follows from the equilibrium condition **(22)** that also  $w_t^*(z^t) \neq F_{lt}^*(z^t)$ . The two economies thus only differ in terms of the prices of capital and labour services that the consumers face. We can, however, eliminate these differences by appropriately choosing  $\tau_{kt}(z^t)$  and  $\tau_{lt}(z^t)$  in the prototype economy. In particular, let  $\tau_{kt}(z^t)$  and  $\tau_{lt}(z^t)$  satisfy  $r_t^*(z^t) = (1 - \tau_{kt}(z^t))F_{kt}^*(z^t)$  and  $w_t^*(z^t) = (1 - \tau_{lt}(z^t))F_{lt}^*(z^t)$  for every history  $z^t$ . Then the first-order conditions for capital and labour in the two economies are the same and the equilibrium allocations  $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t))$  and the equilibrium prices  $(p_t^*(z^t), R_t^*(z^t))$  of the detailed economy are also equilibrium allocations and prices of the prototype economy. In addition, since in the detailed economy  $w_t^*(z^t) = [F_{lt}^*(z^t)/F_{kt}^*(z^t)]r_t^*(z^t)$ , the labour income tax satisfies  $r_t^*(z^t) = (1 - \tau_{lt}(z^t))F_{kt}^*(z^t)$  and therefore  $\tau_{lt}(z^t) = \tau_{kt}(z^t)$ . *Q.E.D*

### PROOF OF PROPOSITION 2

The proof proceeds again by comparing the equilibrium conditions of the detailed economy with those of the prototype economy. Notice that when  $A_t(z^t) = 1$ ,  $\tau_{xt}(z^t) = 0$ ,  $g_t(z^t) = 0$ , and  $\tilde{R}_t(z^t) = \xi_t(z^t)$  in the prototype economy, the two economies differ only in terms of the first-order conditions for bonds (deposits), labour and capital. We will choose  $\tau_{lt}(z^t)$  and  $\tau_{bt}(z^t)$  so that the equilibrium allocations and prices of the detailed economy satisfy the three first-order conditions in the prototype economy. First, compare the first-order conditions for labour and capital. It follows immediately that when  $\tau_{lt}(z^t)$  in the prototype economy is given by the condition **(38)** of the Proposition for every history  $z^t$ , equilibrium allocations and prices of the detailed economy also satisfy the first-order conditions **(36)** and **(37)** of the prototype economy. Second, compare the equilibrium conditions for bonds in the two economies. To make them more easily comparable, substitute in the prototype economy the left-hand side of the first-order condition for labour **(36)** into the first-order condition for bonds **(35)**. The resulting equation can be expressed as

$$\begin{aligned} & \frac{u_{ct}(z^t)}{p_t(z^t)} \Omega_t(z^t) [1 + \tau_{bt}(z^t)] - \beta (1 + R_t(z^t)) \\ & \times \sum_{z^{t+1}} \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}(z^{t+1})}{p_{t+1}(z^{t+1})} \Omega_{t+1}(z^{t+1}) [1 + \tau_{b,t+1}(z^{t+1})] = 0 \quad (\mathbf{A-1}) \end{aligned}$$

where

$$\Omega_t(z^t) \equiv \sum_{z^{t+1}} \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}(z^{t+1})}{u_{ct}(z^t)} \frac{p_t(z^t)}{p_{t+1}(z^{t+1})}$$

Then, using the law of iterated expectations, rewrite the first-order condition for deposits **(31)** in the detailed economy as

$$\sum_{z^t} \mu_{t-1}(z^t|z^{t-1})\Lambda_t(z^t) = 0 \quad (\mathbf{A-2})$$

where

$$\Lambda_t(z^t) = \frac{u_{ct}(z^t)}{p_t(z^t)} - \beta(1 + R_t(z^t)) \sum_{z^{t+1}} \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}(z^{t+1})}{p_{t+1}(z^{t+1})} \quad (\mathbf{A-3})$$

Notice that if  $\Omega_t(z^t)$ ,  $\Omega_{t+1}(z^{t+1})$ ,  $\tau_{bt}(z^t)$ , and  $\tau_{b,t+1}(z^{t+1})$  were absent from equation **(A-1)**, and if the left-hand side of equation **(A-3)** was zero, the equilibrium conditions for bonds in the two economies would be the same. Fuerst (1992) calls the term  $\Lambda(z^t)$  a ‘liquidity effect’. We will choose  $\tau_{bt}(z^t)$  so that it has the same effect on the equilibrium of the prototype economy as the liquidity effect. To do so, consider equilibrium allocations  $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t))$  and prices  $(p_t^*(z^t), R_t^*(z^t))$  of the detailed economy. Evaluating the left-hand side of equation **(A-1)** at these equilibrium allocations and prices and choosing sequences for  $\tau_{bt}(z^t)$  such that the right-hand side is equal to zero for every history  $z^t$  implicitly defines  $\tau_{bt}(z^t)$  that has the same effect on the equilibrium as the liquidity effect. *Q.E.D*

## Appendix B: Additional equivalence results

This appendix provides two additional equivalence results. First, it shows that an economy with sticky wages considered by CKM is equivalent to the prototype monetary economy with labour wedges. This result complements the mapping established by CKM between the sticky-wage economy and a non-monetary prototype economy. Second, it shows that an economy with exogenous fluctuations in energy prices and capital utilisation, like that of Finn (1996), is equivalent to the prototype monetary economy with efficiency wedges. Unless specified otherwise, the notation in this Appendix is the same as in Section 2 and we abstract from population and technology growth.

### B.1 An economy with sticky wages

#### B.1.1 The underlying economy

Consider an economy populated by a continuum of infinitely lived consumers differentiated by a labour type  $j \in [0, 1]$ , a representative perfectly competitive producer, and a government. The consumers can be thought of as being organised in a continuum of unions indexed by  $j$ . The producer has access to an aggregate production function

$$y_t(z^t) = F(k_t(z^{t-1}), l_t(z^t)) \quad (\mathbf{B-1})$$

where

$$l_t(z^t) = \left[ \int l_t(j, z^t)^{\varepsilon_t(z^t)} dj \right]^{1/\varepsilon_t(z^t)} \quad (\mathbf{B-2})$$

is a labour aggregate and  $\varepsilon_t(z^t)$  is a shock to the degree of monopoly power of the unions. The producer's problem can be described in two steps. First, for a given  $l_t(z^t)$ , the producer solves

$$\min_{\{l_t(j, z^t)\}_{j \in [0, 1]}} \int W_t(j, z^{t-1}) l_t(j, z^t) dj$$

subject to **(B-2)**, where  $W_t(j, z^{t-1})$  is the nominal wage rate for labour of type  $j$ . The solution to this cost minimisation problem gives the producer's demand function for each labour type

$$l_t(j, z^t) = \left[ \frac{W_t(j, z^{t-1})}{W_t(z^{t-1})} \right]^{\frac{1}{\varepsilon_t(z^t)-1}} l_t(z^t) \quad (\mathbf{B-3})$$

where

$$W_t(z^{t-1}) = \left[ \int W_t(j, z^{t-1})^{\frac{\varepsilon_t(z^t)}{\varepsilon_t(z^t)-1}} dj \right]^{\frac{\varepsilon_t(z^t)-1}{\varepsilon_t(z^t)}}$$

is the aggregate nominal wage rate. In the second step, the producer chooses  $k_t(z^{t-1})$  and  $l_t(z^t)$  to maximise profits

$$F(k_t(z^{t-1}), l_t(z^t)) - r_t(z^t)k_t(z^{t-1}) - \frac{W_t(z^{t-1})}{p_t(z^t)} l_t(z^t)$$

The first-order conditions for this problem equalise the marginal products of capital and labour with their prices.

Union  $j$  is a monopolist in the market for labour of type  $j$  and it sets the nominal wage rate  $W_t(j, z^{t-1})$  before the realisation of  $z_t$ . In addition, it agrees to supply in period  $t$  whatever

labour is demanded at that wage rate. The preferences of a consumer  $j$  are characterised by a utility function

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t \mu_t(z^t) u(c_t(j, z^t), 1 - l_t(j, z^t) - s_t(j, z^t)) \quad (\mathbf{B-4})$$

The consumer/union's problem is to choose plans for  $c_t(j, z^t)$ ,  $x_t(j, z^t)$ ,  $k_{t+1}(j, z^t)$ ,  $l_t(j, z^t)$ ,  $s_t(j, z^t)$ ,  $m_t(j, z^t)$ ,  $b_t(j, z^t)$ , and  $W_{t+1}(j, z^t)$  to maximise **(B-4)** subject to the labour demand function **(B-3)**, a shopping time technology

$$s_t(j, z^t) = s\left(\frac{c_t(j, z^t)}{m_t(j, z^t)/p_t(z^t)}\right)$$

a budget constraint

$$\begin{aligned} c_t(j, z^t) + x_t(j, z^t) + \frac{m_t(j, z^t)}{p_t(z^t)} + \frac{b_t(j, z^t)}{(1 + R_t(z^t))p_t(z^t)} \\ = \frac{W_t(j, z^{t-1})}{p_t(z^t)} l_t(j, z^t) + r_t(z^t) k_t(j, z^{t-1}) + \frac{m_{t-1}(j, z^{t-1})}{p_t(z^t)} + \frac{b_{t-1}(j, z^{t-1})}{p_t(z^t)} + \frac{T_t(z^t)}{p_t(z^t)} \end{aligned}$$

and a capital accumulation law

$$k_{t+1}(j, z^t) = (1 - \delta)k_t(j, z^{t-1}) + x_t(j, z^t)$$

Assuming that  $k_0$ ,  $m_{-1}$  and  $b_{-1}$  are the same for all types, the solution to this problem is symmetric across all consumers.

The government sets the nominal interest rate according to a policy rule

$$R_t(z^t) = (1 - \rho_R) [R + \omega_y (\ln y_t(z^t) - \ln y) + \omega_\pi (\pi_t(z^t) - \pi)] + \rho_R R_{t-1}(z^{t-1}) \quad (\mathbf{B-5})$$

and its budget constraint is given by  $T_t(z^t) = m_t(z^t) - m_{t-1}(z^{t-1})$ .

An *equilibrium of this economy with sticky nominal wages* is a set of allocations  $(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t), b_t(z^t))$  and a set of prices  $(p_t(z^t), R_t(z^t), r_t(z^t), W_t(z^t))$  that satisfy: (i) a set of the consumer's first-order conditions for wages, capital, bonds, and money, respectively

$$\begin{aligned} W_{t+1}(z^t) &= \frac{\sum_{z^{t+1}} \mu_t(z^{t+1}|z^t) u_{h,t+1}(z^{t+1}) l_{t+1}(z^{t+1})}{\sum_{z^{t+1}} \mu_t(z^{t+1}|z^t) \varepsilon_{t+1}(z^{t+1}) \left\{ \frac{l_{t+1}(z^{t+1})}{p_{t+1}(z^{t+1})} [u_{c,t+1}(z^{t+1}) - u_{h,t+1}(z^{t+1}) s_{c,t+1}(z^{t+1})] \right\}} \\ &\sum_{z^{t+1}} Q_t(z^{t+1}|z^t) [1 + r_{t+1}(z^{t+1}) - \delta] = 1 \\ &\sum_{z^{t+1}} Q_t(z^{t+1}|z^t) (1 + R_t(z^t)) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} = 1 \\ &-\frac{s_{mt}(z^t) u_{ht}(z^t)}{u_{ct}(z^t) - u_{ht}(z^t) s_{ct}(z^t)} + \sum_{z^{t+1}} Q_t(z^{t+1}|z^t) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} = 1 \end{aligned}$$

where

$$Q_t(z^{t+1}|z^t) = \beta \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}(z^{t+1}) - u_{h,t+1}(z^{t+1}) s_{c,t+1}(z^{t+1})}{u_{ct}(z^t) - u_{ht}(z^t) s_{ct}(z^t)}$$

(ii) a set of the producer's first-order conditions

$$r_t(z^t) = F_{kt}(z^t)$$

$$\frac{W_t(z^{t-1})}{p_t(z^t)} = F_{lt}(z^t)$$

(iii) the resource constraint  $c_t(z^t) + x_t(z^t) = y_t(z^t)$ , where  $y_t(z^t)$  is given by the production function **(B-1)**; (iv) the capital accumulation law  $k_{t+1}(z^t) = (1 - \delta)k_t(z^{t-1}) + x_t(z^t)$ ; (v) the monetary policy rule **(B-5)**; and (vi) the bond market clearing condition  $b_t(z^t) = 0$ .

### B.1.2 The associated prototype economy

Consider now a version of the prototype economy of Section 2, in which all wedges except the labour wedge are constant in all states of the world. Comparing the equilibrium conditions of the detailed economy with those of the prototype economy we obtain the following proposition.

**PROPOSITION 3:** *Consider equilibrium allocations of the sticky-wage economy  $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t))$  and prices  $(p_t^*(z^t), R_t^*(z^t), r_t^*(z^t), W_{t+1}^*(z^t))$  that support these allocations. Let the wedges in the prototype economy satisfy:  $A_t(z^t) = 1$ ,  $\tau_{xt}(z^t) = \tau_{bt}(z^t) = g_t(z^t) = \tilde{R}_t(z^t) = 0$ , and*

$$\tau_{lt}(z^t) = 1 - \frac{u_{ht}^*(z^t)}{[u_{ct}^*(z^t) - u_{ht}^*(z^t)s_{ct}^*(z^t)] F_{lt}^*(z^t)}$$

for all  $z^t$ , where  $u_{ht}^*$ ,  $u_{ct}^*$ ,  $s_{ct}^*$ , and  $F_{lt}^*$  are evaluated at the equilibrium of the sticky-wage economy. Then  $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t))$  and  $(p_t^*(z^t), R_t^*(z^t))$  are also equilibrium allocations and prices of the prototype economy.

The key point here is that sticky wages are equivalent to labour income taxes in the prototype monetary economy. This mapping is the same as between the sticky-wage economy and the non-monetary prototype economy considered by Chari *et al* (2007a). Sticky wages thus affect inflation and the nominal interest rate only through their distortionary effect on labour decisions.

## B.2 An economy with capital utilisation and energy price shocks

### B.2.1 The underlying economy

Consider now an economy that purchases an intermediate input, called energy, at the world market at a price  $p_t^e(z^t)$ , which it takes as given. In this economy, an infinitely lived representative consumer operates an aggregate production function

$$y_t(z^t) = (v_t(z^t)k_t(z^{t-1}))^\alpha l_t(z^t)^{1-\alpha} \quad \mathbf{(B-6)}$$

where  $\alpha \in (0, 1)$ ,  $v_t(z^t)$  is a rate of capital utilisation and  $v_t(z^t)k_t(z^{t-1})$  is a flow of capital services. Energy  $e_t(z^t)$  is related to capital services according to

$$e_t(z^t) = a(v_t(z^t))k_t(z^{t-1}) \quad \mathbf{(B-7)}$$

where  $a'(\cdot) > 0$  and  $a''(\cdot) > 0$ . Convexity of the function  $a(\cdot)$  captures the idea that less efficient machines have to be operated as capital utilisation increases.

The consumer chooses plans for  $c_t(z^t)$ ,  $x_t(z^t)$ ,  $h_t(z^t)$ ,  $l_t(z^t)$ ,  $s_t(z^t)$ ,  $y_t(z^t)$ ,  $k_{t+1}(z^t)$ ,  $m_t(z^t)$ ,  $b_t(z^t)$ ,  $v_t(z^t)$ , and  $e_t(z^t)$  to maximise the utility function **(1)** subject to the time constraint

(2), the capital accumulation law (4) and the budget constraint

$$\begin{aligned} c_t(z^t) + x_t(z^t) + p_t^e(z^t)e_t(z^t) + \frac{m_t(z^t)}{p_t(z^t)} + \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} \\ = y_t(z^t) + \frac{m_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{T_t(z^t)}{p_t(z^t)} \end{aligned}$$

where  $y_t(z^t)$  is given by the production function (B-6) and  $e_t(z^t)$  is given by the expression (B-7). The government sets the nominal interest rate according to

$$R_t(z^t) = [R + \omega_y (\ln y_t(z^t) - \ln y) + \omega_\pi (\pi_t(z^t) - \pi)] + \rho_R R_{t-1}(z^{t-1}) \quad (\mathbf{B-8})$$

and its budget constraint is given by  $T_t(z^t) = m_t(z^t) - m_{t-1}(z^{t-1})$ .

An *equilibrium of this economy with capital utilisation and energy price shocks* is a set of allocations  $(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), m_t(z^t), k_{t+1}(z^t), b_t(z^t), v_t(z^t))$  and a set of prices  $(p_t(z^t), R_t(z^t))$  that satisfy: (i) the consumer's first-order conditions for capital utilisation, labour, capital, bonds, and money, respectively

$$\begin{aligned} \alpha v_t(z^t)^{\alpha-1} k_t(z^{t-1})^{\alpha-1} l_t^{1-\alpha} &= p_t^e(z^t) a'(v_t(z^t)) k_t(z^{t-1}) \\ u_{ct}(z^t)(1 - \alpha) (k_t(z^{t-1})v_t(z^t))^\alpha l_t(z^t)^{-\alpha} \\ &= u_{ht}(z^t) \left[ 1 + s_{ct}(z^t)(1 - \alpha) (k_t(z^{t-1})v_t(z^t))^\alpha l_t(z^t)^{-\alpha} \right] \\ \sum_{z^{t+1}} Q_t(z^{t+1}|z^t) [\alpha v_{t+1}(z^{t+1})^\alpha k_{t+1}(z^t)^{\alpha-1} l_{t+1}(z^{t+1})^{1-\alpha} \\ &\quad + 1 - \delta - p_{t+1}^e(z^{t+1}) a(v_{t+1}(z^{t+1}))] = 1 \\ \sum_{z^{t+1}} Q_t(z^{t+1}|z^t) (1 + R_t(z^t)) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} &= 1 \\ - \frac{u_{ht}(z^t) s_{mt}(z^t)}{u_{ct}(z^t) - u_{ht}(z^t) s_{ct}(z^t)} + \sum_{z^{t+1}} Q_t(z^{t+1}|z^t) \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} &= 1 \end{aligned}$$

where

$$Q_t(z^{t+1}|z^t) = \beta \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}(z^{t+1}) - u_{h,t+1}(z^{t+1}) s_{c,t+1}(z^{t+1})}{u_{ct}(z^t) - u_{ht}(z^t) s_{ct}(z^t)}$$

(ii) the resource constraint

$$c_t(z^t) + x_t(z^t) + p^e(z^t) a(v(z^t)) k(z^{t-1}) = y(z^t)$$

where  $y_t$  is given by the production function (B-6); (iii) the capital accumulation law (4); (iv) the monetary policy rule (B-8); and (v) the bond market clearing condition  $b_t(z^t) = 0$ .

### B.2.2 The associated prototype economy

Consider now a version of the prototype economy of Section 2 in which the production function has the Cobb-Douglas functional form as in the underlying economy

$$y_t(z^t) = A_t(z^t) k_t(z^{t-1})^\alpha l_t(z^t)^{1-\alpha}$$

and in which the investment wedge resembles a tax on capital income rather than a tax on investment. The consumer's budget constraint is now



$$\begin{aligned}
& c_t(z^t) + x_t(z^t) + \frac{m_t(z^t)}{p_t(z^t)} + \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} \\
&= [1 - \tau_{kt}(z^t)] r_t(z^t) k_t(z^{t-1}) + w_t(z^t) l_t(z^t) + \frac{m_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} + \frac{T_t(z^t)}{p_t(z^t)}
\end{aligned}$$

where  $\tau_{kt}$  is a tax on capital income, and the first-order condition for capital is

$$\sum_{z^{t+1}} Q_t(z^{t+1}|z^t) [(1 - \tau_{k,t+1}(z^{t+1})) \alpha A_{t+1}(z^{t+1}) k_{t+1}(z^t)^{\alpha-1} l_{t+1}(z^{t+1})^{1-\alpha} + (1 - \delta)] = 1$$

Comparing the equilibrium conditions of the detailed economy with those of the prototype economy we obtain the following proposition.

**PROPOSITION 4:** *Consider equilibrium allocations of the detailed economy with capital utilisation and energy price shocks  $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t), v_t^*(z^t), e_t^*(z^t))$  and prices  $(p_t^*(z^t), R_t^*(z^t))$  that support these allocations. Let the wedges in the prototype economy satisfy:  $\tau_{bt}(z^t) = g_t(z^t) = \tilde{R}_t(z^t) = 0$ , and*

$$\begin{aligned}
A_t(z^t) &= v_t^*(z^t)^\alpha \\
\tau_{kt}(z^t) &= \frac{p_t^e(z^t) a(v_t^*(z^t))}{\alpha A_t(z^t) (k_t^*(z^{t-1}))^{\alpha-1} (l_t^*(z^t))^{1-\alpha}} \\
g_t(z^t) &= p_t^e(z^t) a(v_t^*(z^t)) k_t(z^{t-1})
\end{aligned}$$

*Then  $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t))$  and  $(p_t^*(z^t), R_t^*(z^t))$  are also equilibrium allocations and prices of the prototype economy.*

Consider now a special case of the proposition. Suppose that fluctuations in  $v_t^*(z^t)$  are such that they offset fluctuations in  $p_t^e(z^t)$  in a way that leaves  $p_t^e(z^t) a(v_t^*(z^t))$  constant.<sup>(17)</sup> Then fluctuations in energy prices in the detailed economy show up in the prototype economy as fluctuations in efficiency wedges (and small fluctuations in government consumption wedges due to small movements in  $k_t$  over time), but not as fluctuations in investment wedges. The main idea here is that fluctuations in energy prices (or prices of commodities used to produce energy, such as oil) are equivalent to fluctuations in efficiency wedges.

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<sup>(17)</sup> It is trivial to show that  $\partial v_t^* / \partial p_t^e < 0$ .

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**Table A. Baseline parameter values**

Symbol	Value	Definition
<b>Preferences</b>		
$\lambda$	0.266	Consumption share in utility
$\beta$	0.995	Discount factor
$\gamma_n$	0.0037	Population growth rate
<b>Production</b>		
$\gamma_A$	0.004	Technology growth rate
$\delta$	0.0118	Depreciation rate
$\alpha$	0.35	Capital share in production
<b>Shopping time</b>		
$\nu_1$	0.0319	Level parameter
$\nu_2$	1.0	Curvature parameter
<b>Monetary policy</b>		
$\pi$	0.0091	Steady-state inflation rate
$\omega_y$	0.125	Weight on output
$\omega_\pi$	1.5	Weight on inflation
$\rho_R$	0.75	Smoothing parameter

**Table B. Stochastic process for the wedges<sup>a</sup>**

$$P_0 = \begin{bmatrix} -0.0798 & 0.0072 & -0.0338 & 0.0474 & -0.0119 & -0.0019 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.854 & -0.0963 & 0.173 & -0.0061 & -0.0425 & 0.520 \\ -0.0673 & 1.058 & -0.0014 & 0.0097 & 0.0465 & -0.722 \\ -0.0857 & -0.0335 & 1.088 & 0.0026 & -0.0116 & 0.402 \\ 0.0821 & 0.0587 & -0.0974 & 1.0053 & 0.0241 & 0.341 \\ 0.0973 & -0.298 & 0.085 & -0.0076 & 0.826 & 0.12 \\ -0.0217 & 0.0146 & 0.0005 & 0.0004 & 0.0063 & 0.441 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0072 & 0 & 0 & 0 & 0 & 0 \\ 0.0037 & 0.0092 & 0 & 0 & 0 & 0 \\ 0.0058 & -0.0008 & 0.0029 & 0 & 0 & 0 \\ 0.0009 & 0.005 & 0.0117 & 0.0087 & 0 & 0 \\ 0.0005 & -0.0175 & -0.0013 & 0.0014 & 0.0219 & 0 \\ 0.0003 & 8.3e-6 & 0.0001 & -0.0002 & 0.004 & 0.001 \end{bmatrix}$$

<sup>a</sup> The equilibrium conditions of the prototype economy imply that in a steady state the values of  $\tau_b$  and  $\tilde{R}$  are zero. This restriction is imposed in the estimation of  $P_0$ ,  $P$  and  $B$ .

**Table C. Business cycle properties of the wedges, 1959.Q1-2004.Q4<sup>a</sup>**

Wedge	Relative std. dev. <sup>b</sup>	Correlations of output in period $t$ with wedges:									
		$j =$	-4	-3	-2	-1	0	1	2	3	4
$\log A_{t+j}$	0.63		0.33	0.49	0.67	0.77	0.85	0.62	0.38	0.13	-0.05
$\tau_{l,t+j}$	0.92		-0.17	-0.33	-0.50	-0.67	-0.74	-0.78	-0.74	-0.63	-0.43
$\tau_{x,t+j}$	0.50		0.16	0.35	0.54	0.68	0.79	0.62	0.44	0.26	0.13
$\log g_t$	1.51		-0.40	-0.42	-0.45	-0.44	-0.35	-0.24	-0.10	0.04	0.20
$\tau_{b,t+j}$	2.59		0.06	0.27	0.48	0.70	0.82	0.81	0.72	0.58	0.41
$\tilde{R}_{t+j}$	0.12		0.11	0.15	0.13	0.15	0.11	0.01	-0.09	-0.16	-0.17

<sup>a</sup> The statistics are computed after the wedges and output have been detrended with HP-filter.

<sup>b</sup> The standard deviations are measured relative to output.

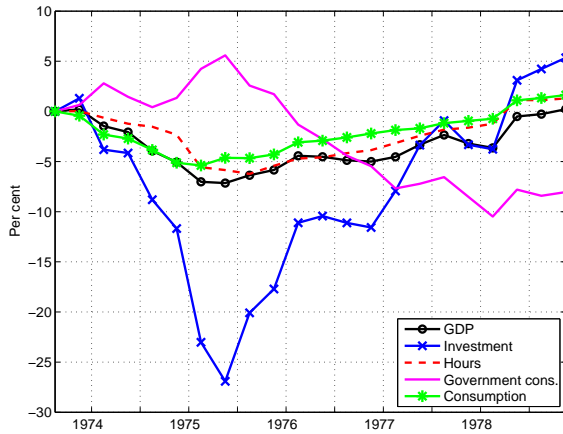
**Table D. Contemporaneous correlations of the wedges with each other: 1959.Q1-2004.Q4<sup>a</sup>**

	$\log A$	$\tau_l$	$\tau_x$	$\log g$	$\tau_b$	$\tilde{R}$
$\log A$	1.00					
$\tau_l$	-0.31	1.00				
$\tau_x$	0.90	-0.28	1.00			
$\log g$	-0.34	0.45	0.01	1.00		
$\tau_b$	0.53	-0.88	0.54	-0.40	1.00	
$\tilde{R}$	0.19	-0.02	0.17	-0.19	0.35	1.00

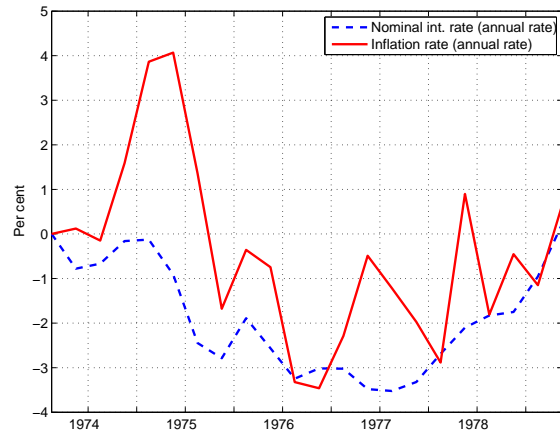
<sup>a</sup> The statistics are computed after the wedges have been detrended with HP-filter.

**Figure 1. The 1973 recession: Data and wedges**

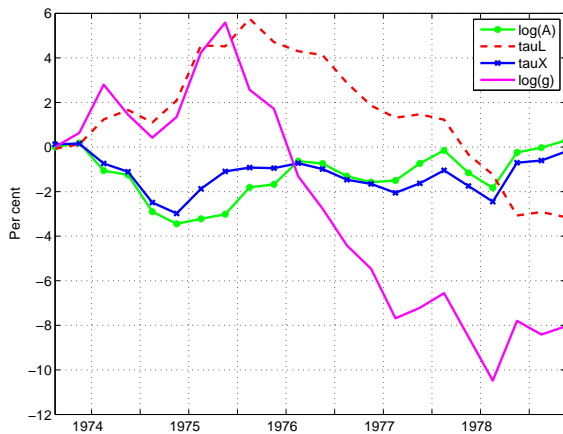
A. Deviations of logged data from trend



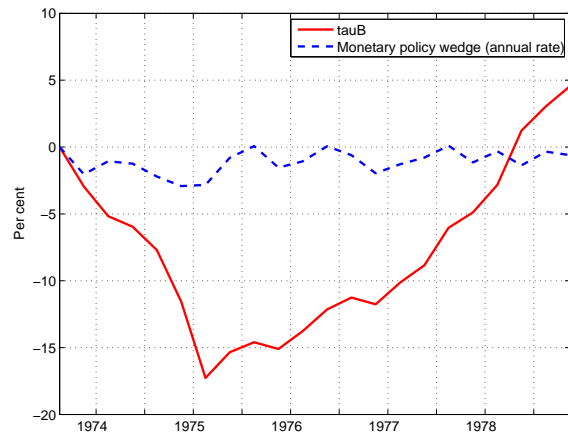
B. Deviations of data from postwar averages



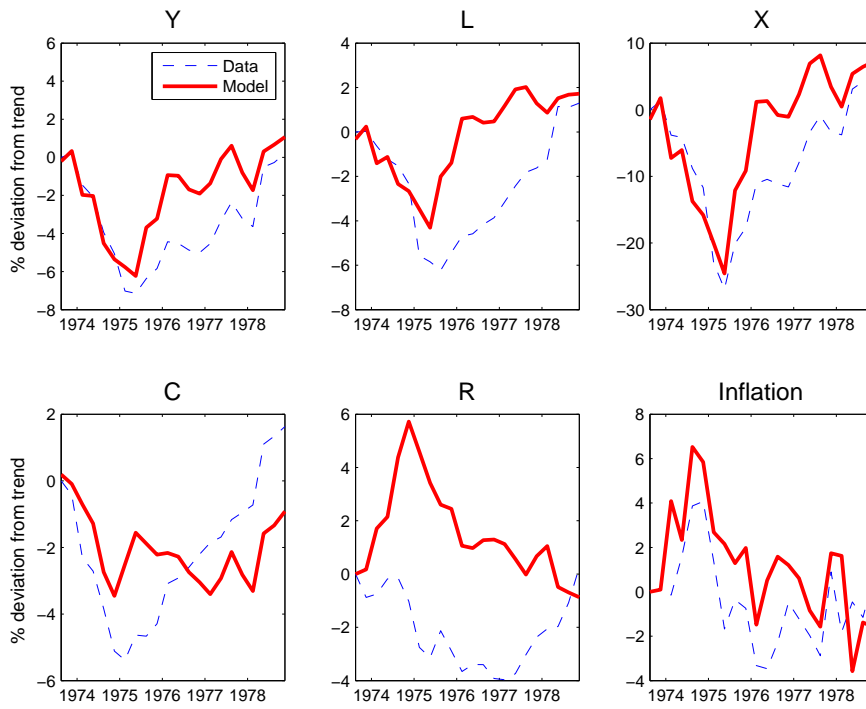
C. Deviations of wedges from trend



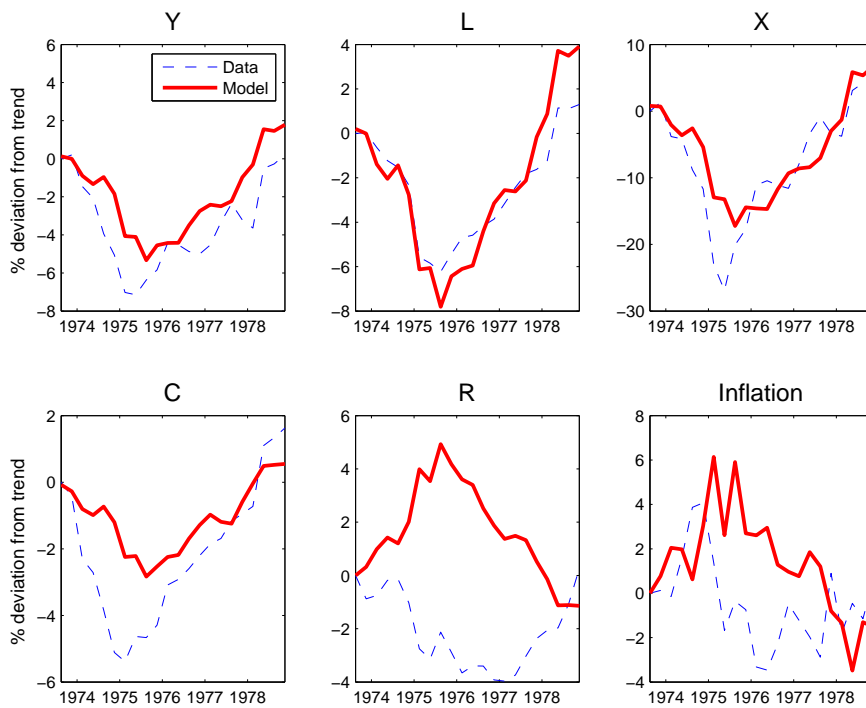
D. Deviations of wedges from trend



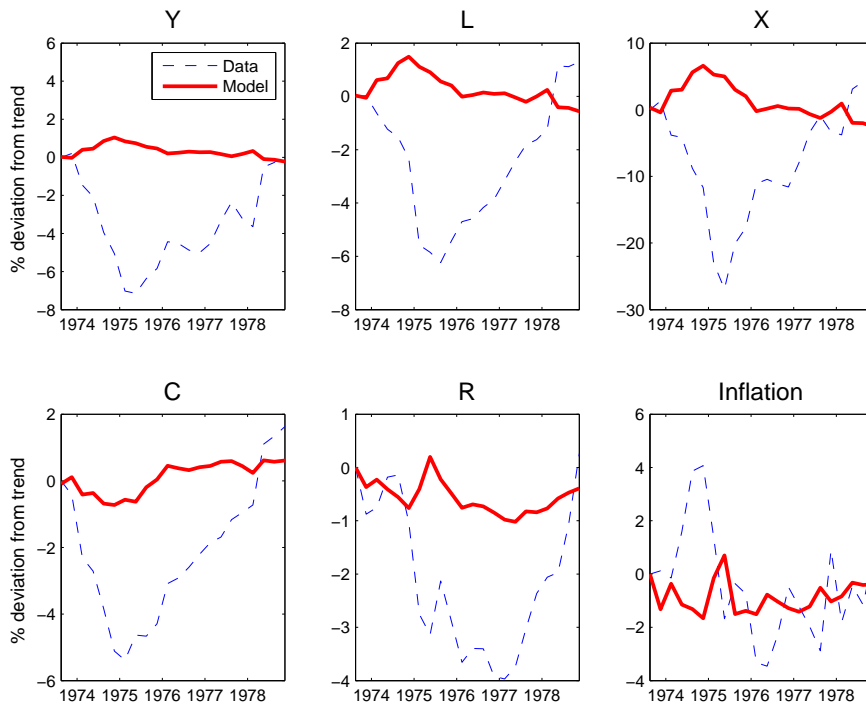
**Figure 2. The 1973 recession: Efficiency wedge only**



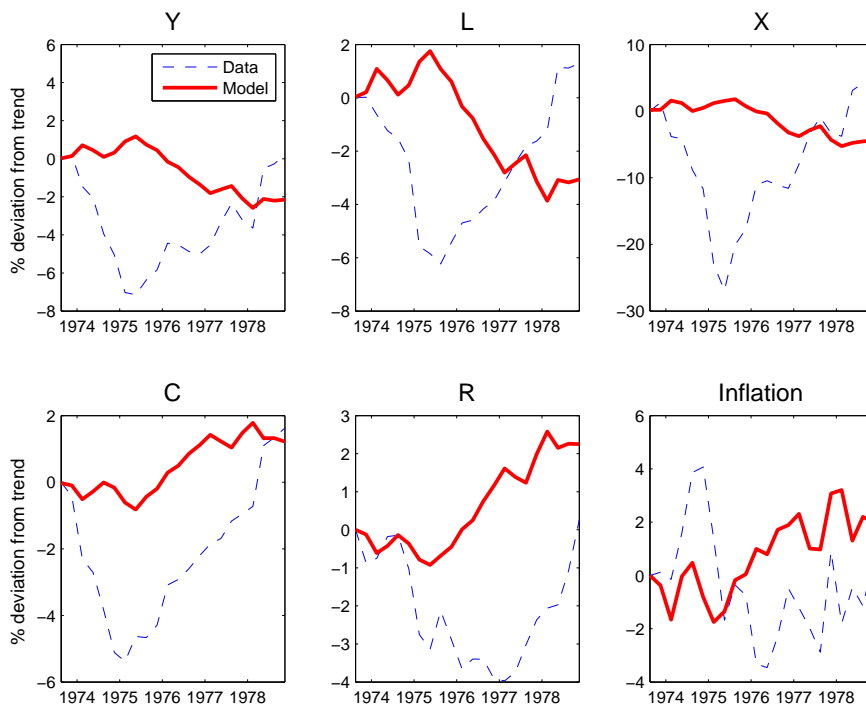
**Figure 3. The 1973 recession: Labour wedge only**



**Figure 4. The 1973 recession: Investment wedge only**

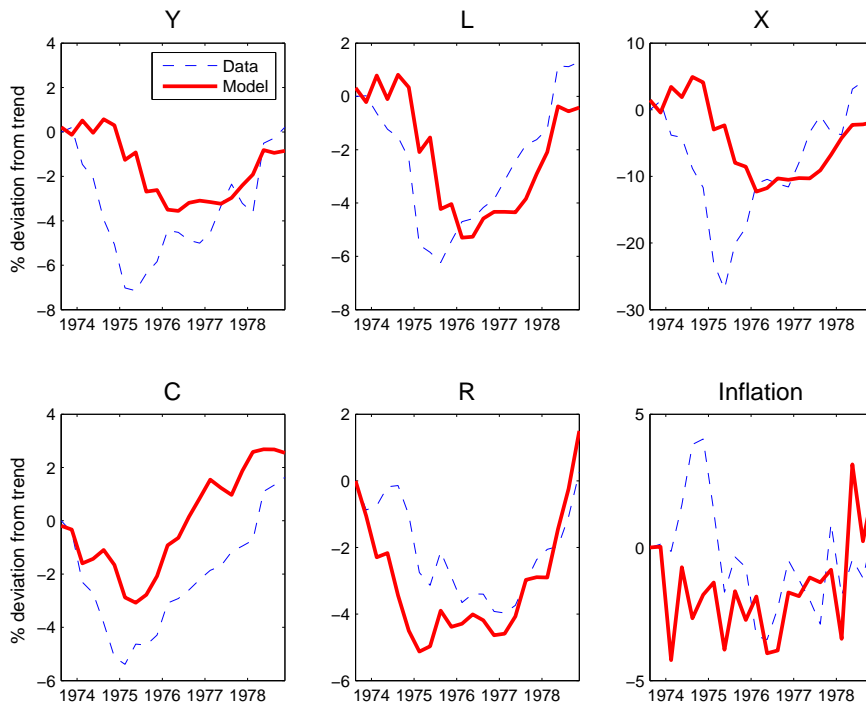


**Figure 5. The 1973 recession: Government consumption wedge only**

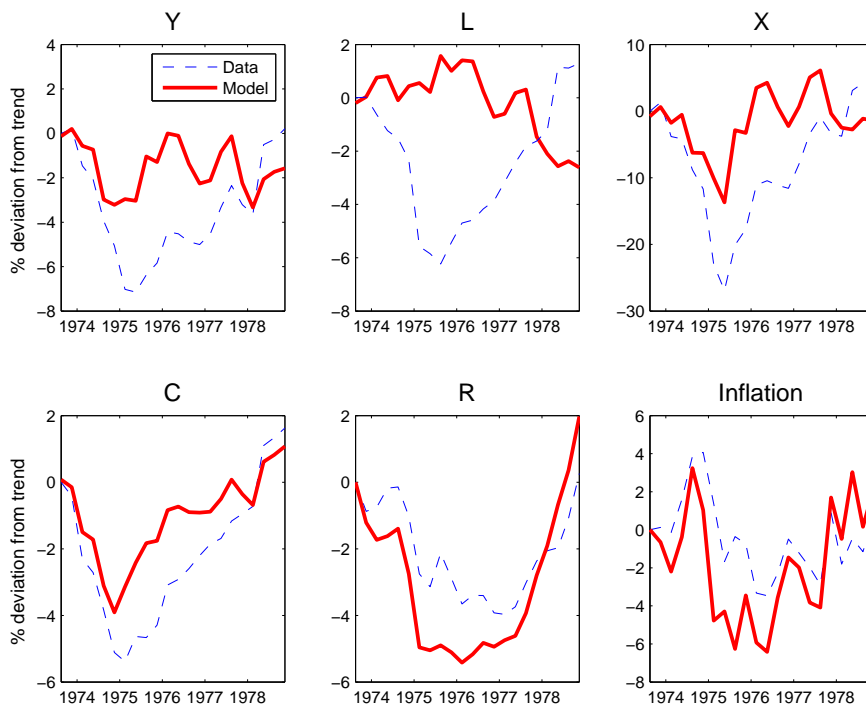




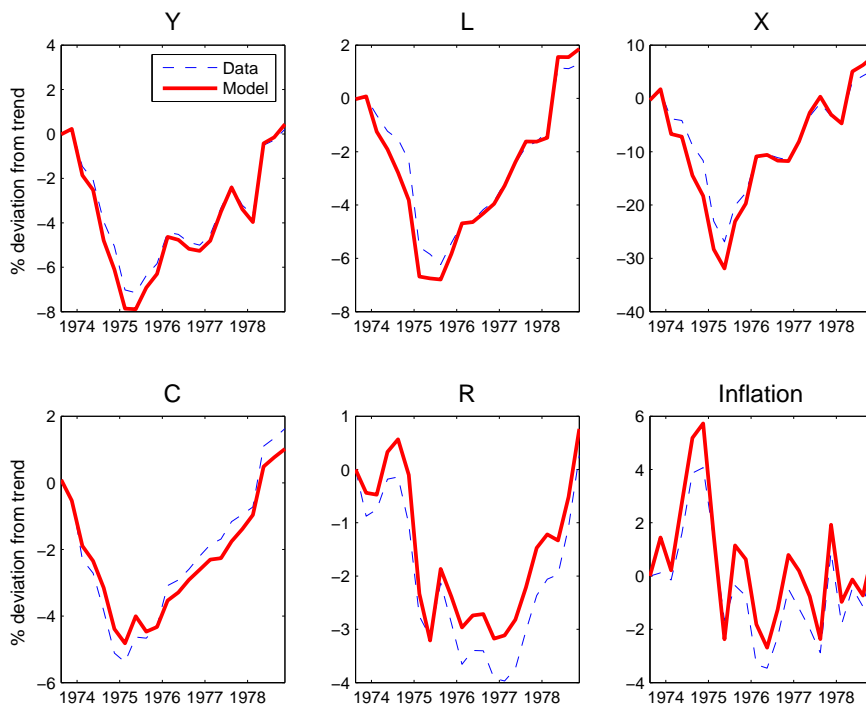
**Figure 6. The 1973 recession: No efficiency wedge**



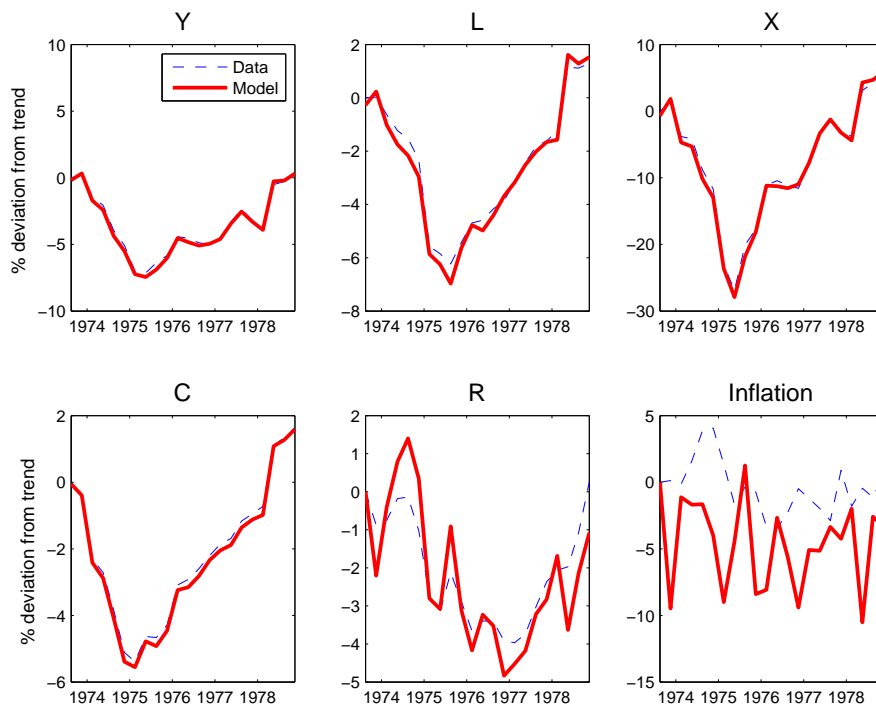
**Figure 7. The 1973 recession: No labour wedge**



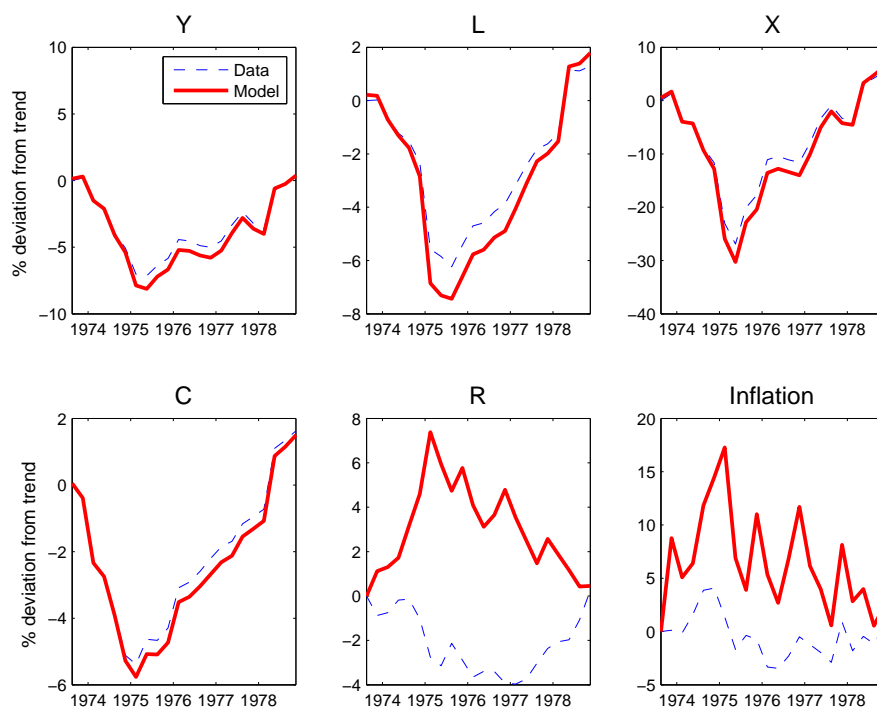
**Figure 8. The 1973 recession: No investment wedge**



**Figure 9. The 1973 recession: No monetary policy wedge**

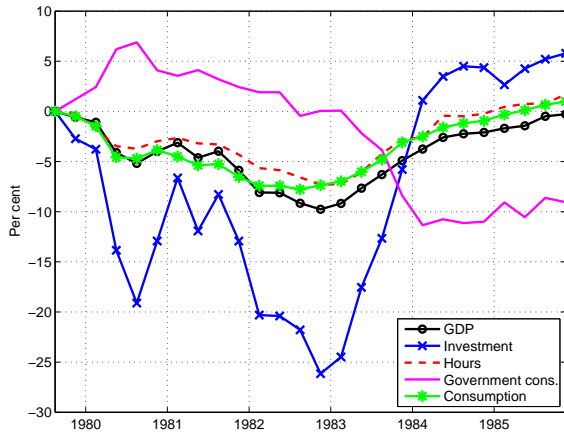


**Figure 10. The 1973 recession: No asset market wedge**

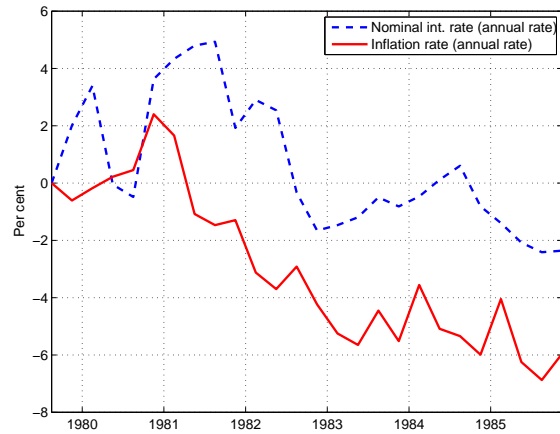


**Figure 11. The 1982 recession: Data and wedges**

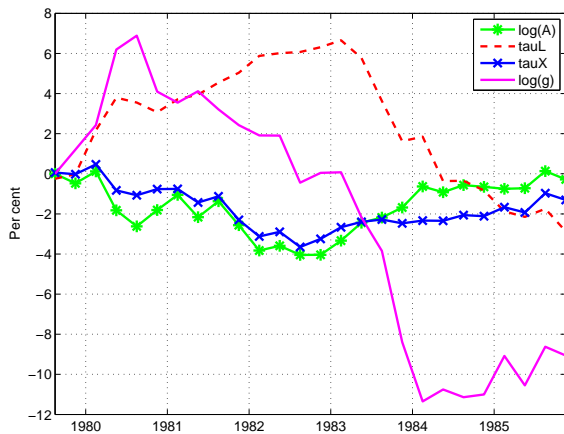
**A. Deviations of logged data from trend**



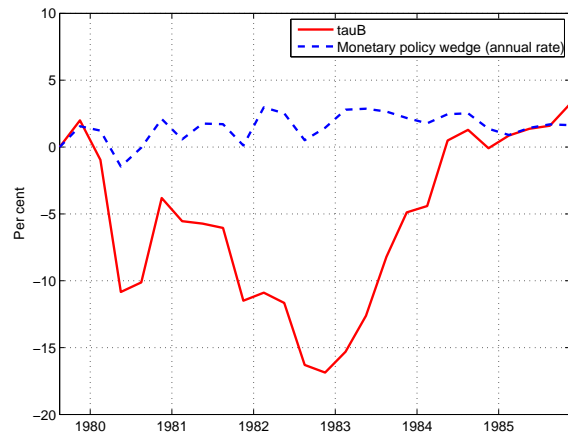
**B. Deviations of data from postwar averages**



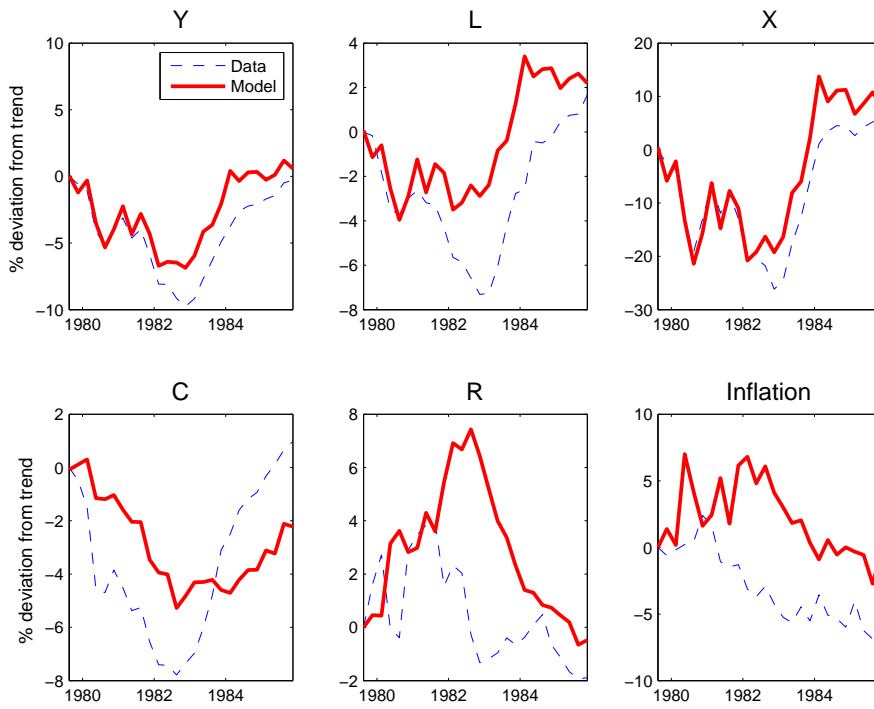
**C. Deviations of wedges from trend**



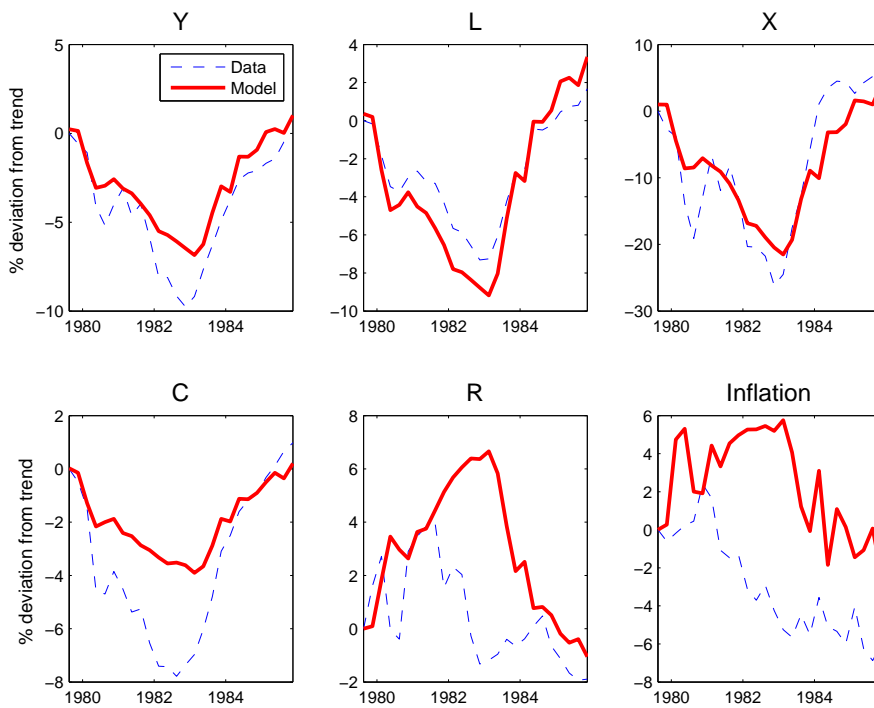
**D. Deviations of wedges from trend**



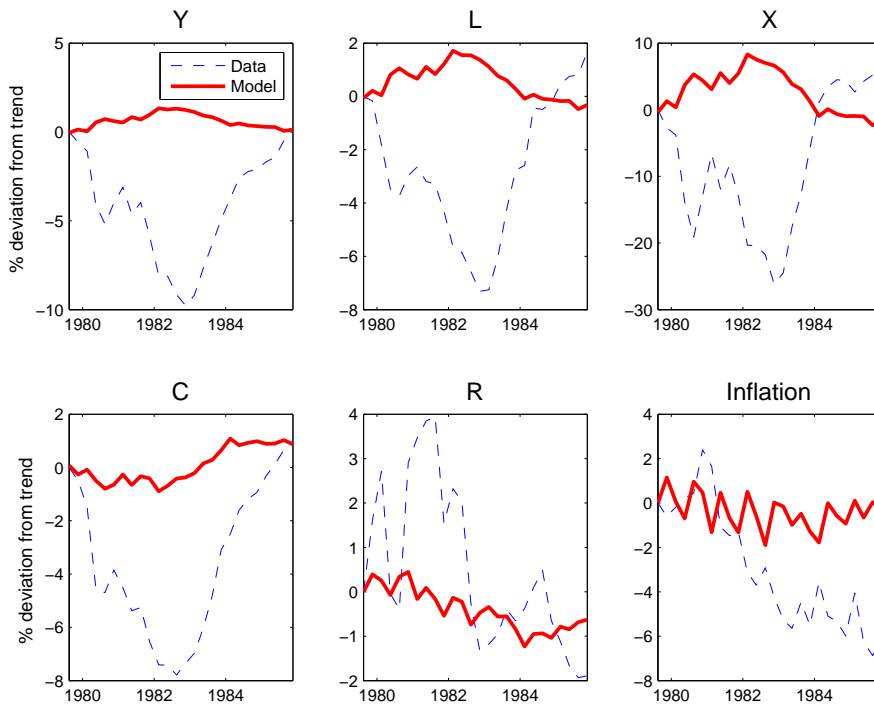
**Figure 12. The 1982 recession: Efficiency wedge only**



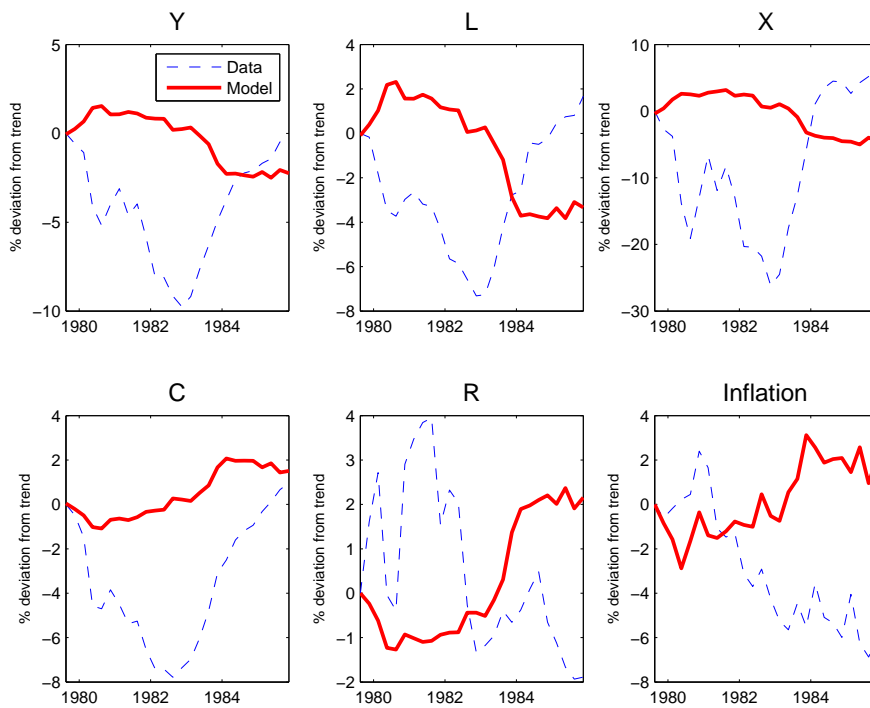
**Figure 13. The 1982 recession: Labour wedge only**



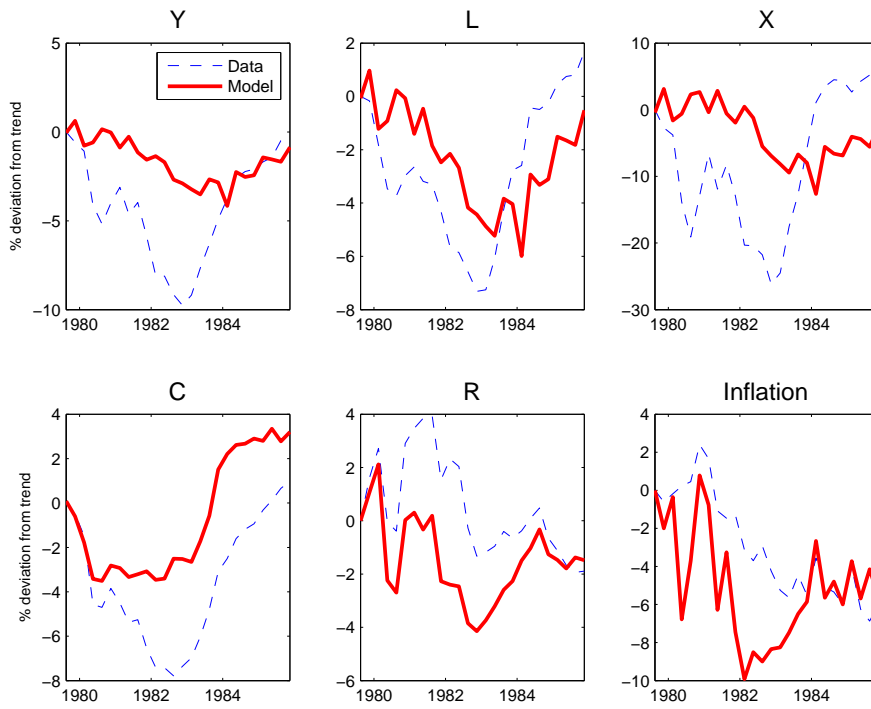
**Figure 14. The 1982 recession: Investment wedge only**



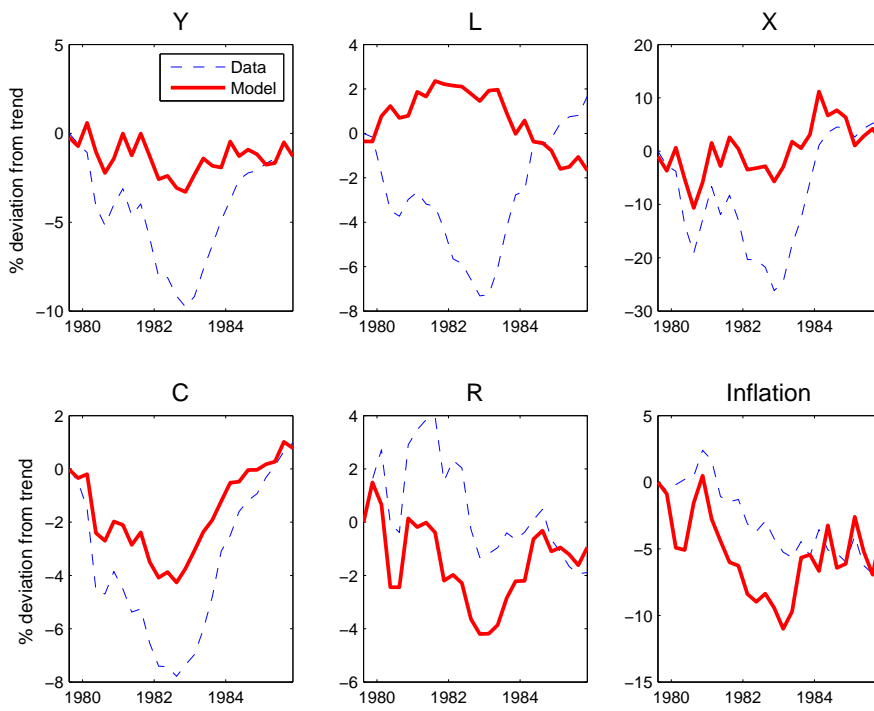
**Figure 15. The 1982 recession: Government consumption wedge only**



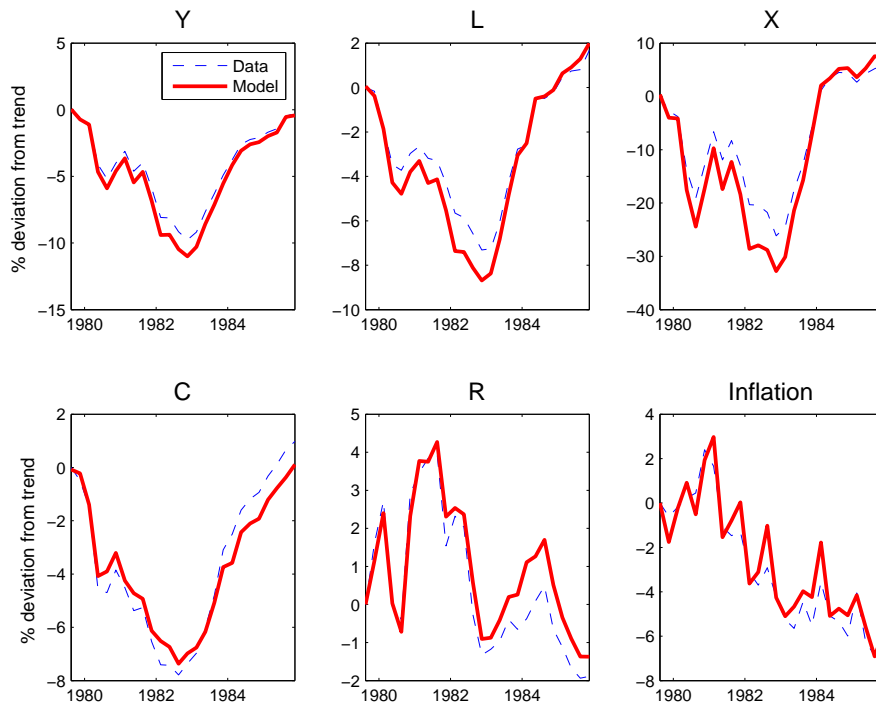
**Figure 16. The 1982 recession: No efficiency wedge**



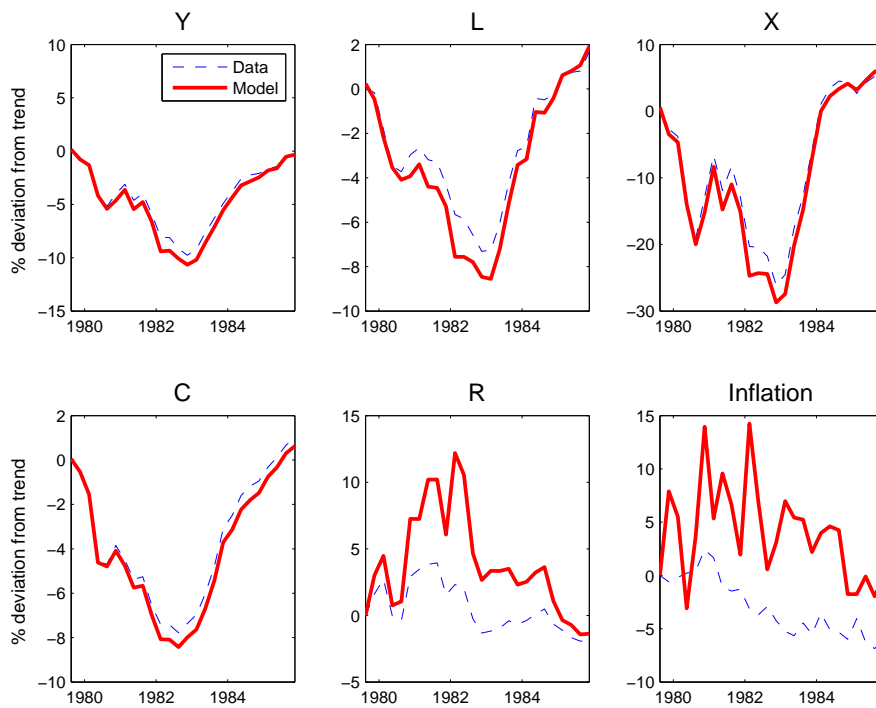
**Figure 17. The 1982 recession: No labour wedge**



**Figure 18. The 1982 recession: No investment wedge**

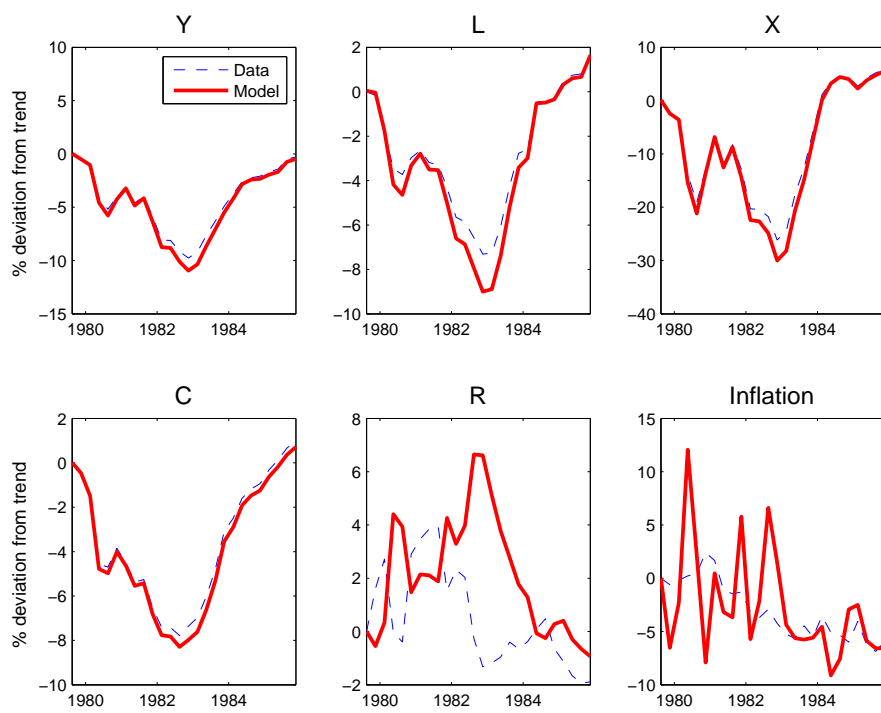


**Figure 19. The 1982 recession: No monetary policy wedge**



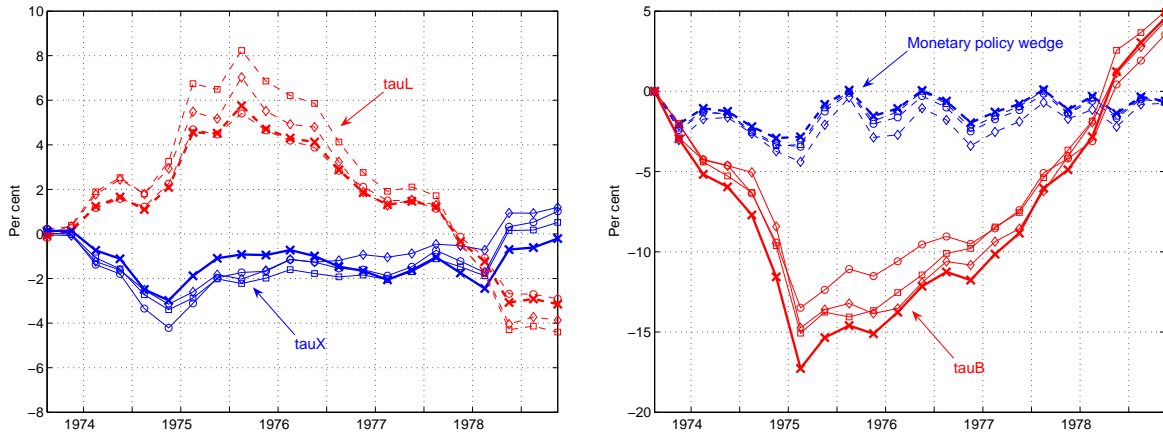


**Figure 20. The 1982 recession: No asset market wedge**



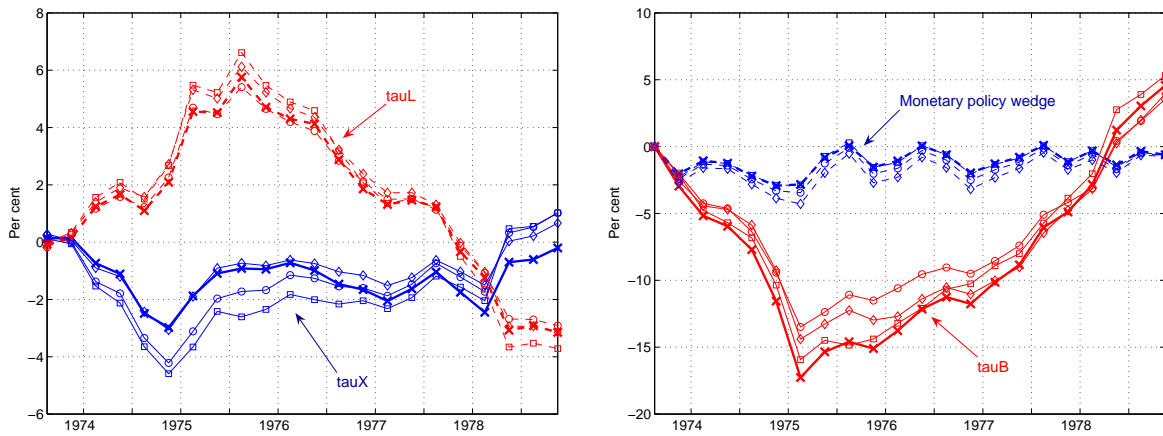
**Figure 21. The 1973 recession: Wedges for alternative parameterisations of the Taylor rule**

**A. Alternative weights on inflation**



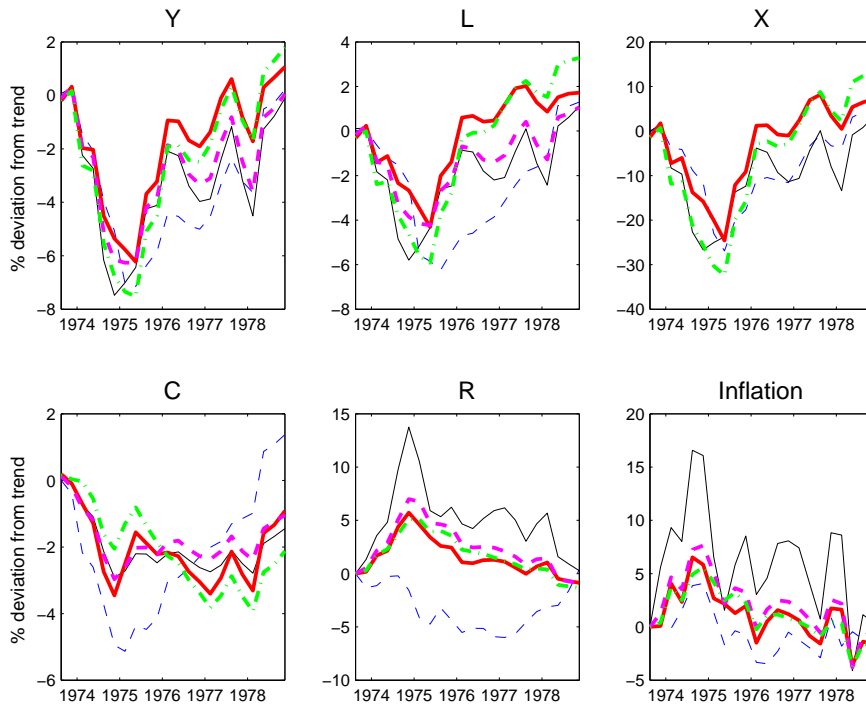
Legend: Thick line – baseline, whole sample; circle – baseline, subsample 1; diamond –  $\omega_{\pi} = 1.3$ ; square –  $\omega_{\pi} = 1.7$

**B. Alternative weights on output**



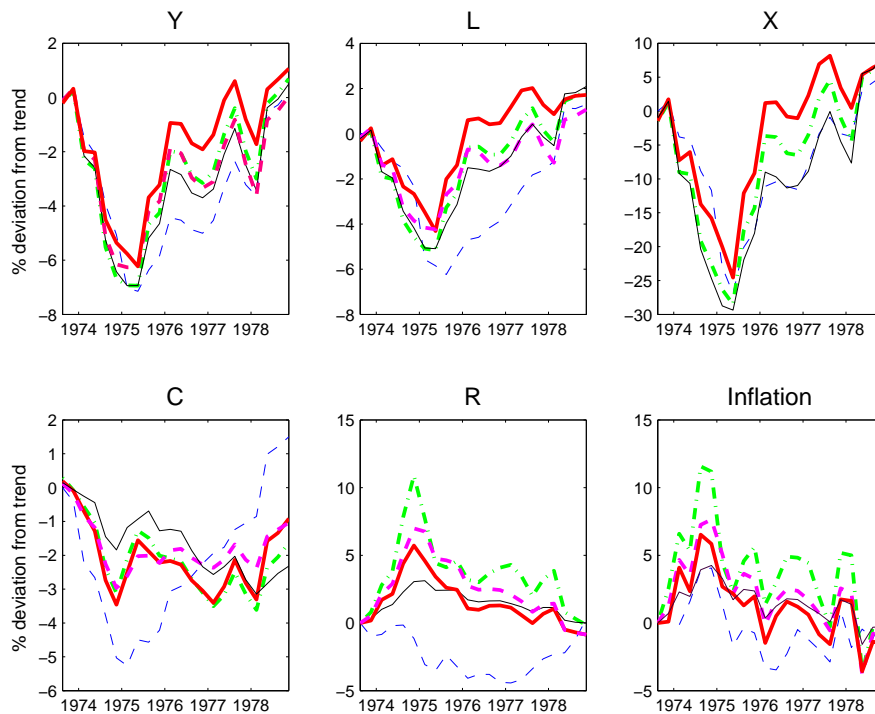
Legend: Thick line – baseline, whole sample; circle – baseline, subsample 1; diamond –  $\omega_y = 0.08$ ; square –  $\omega_y = 0.175$

**Figure 22. The 1973 recession: Efficiency wedge only – alternative weights on inflation**



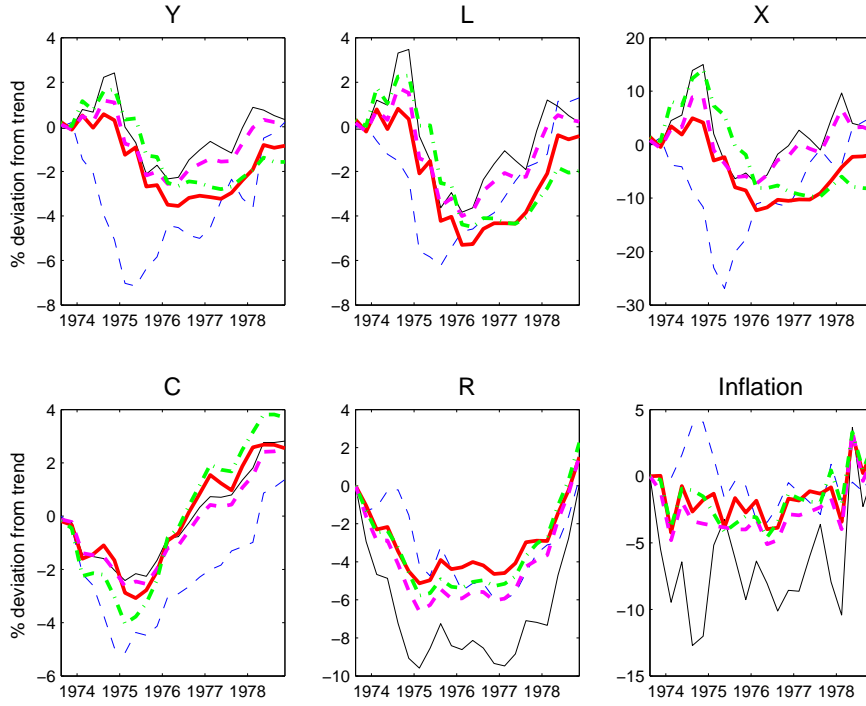
Legend: Thin dashed – data; solid thick – baseline, whole sample; dashed thick – baseline, subsample 1; thin solid –  $\omega_\pi = 1.3$ ; thick dash-dotted –  $\omega_\pi = 1.7$

**Figure 23. The 1973 recession: Efficiency wedge only – alternative weights on output**



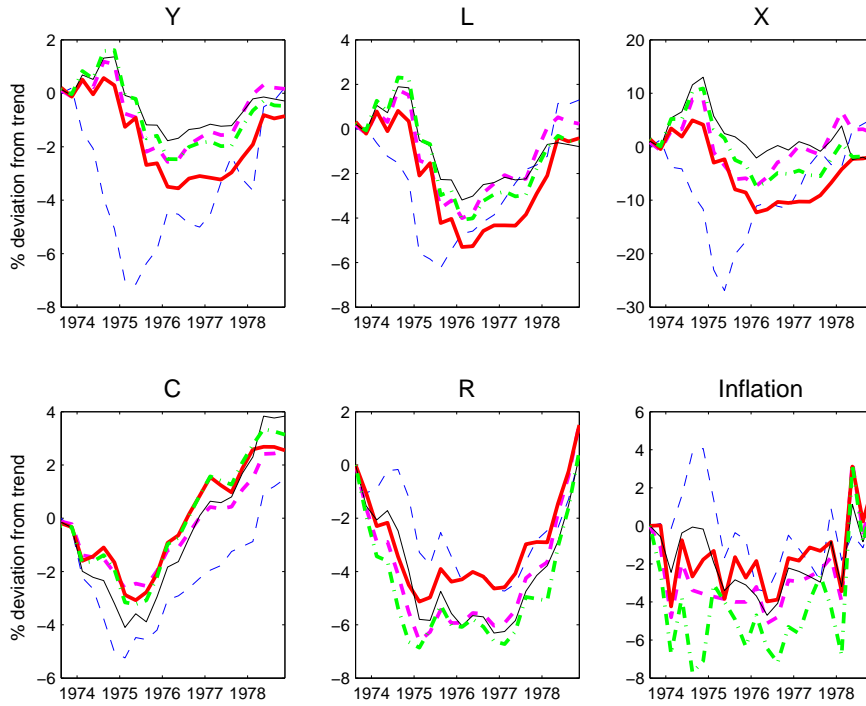
Legend: Thin dashed – data; solid thick – baseline, whole sample; dashed thick – baseline, subsample 1; thin solid –  $\omega_y = 0.08$ ; thick dash-dotted –  $\omega_y = 0.175$

**Figure 24. The 1973 recession: No efficiency wedge – alternative weights on inflation**



Legend: Thin dashed – data; solid thick – baseline, whole sample; dashed thick – baseline, subsample 1; thin solid –  $\omega_\pi = 1.3$ ; thick dash-dotted –  $\omega_\pi = 1.7$

**Figure 25. The 1973 recession: No efficiency wedge – alternative weights on output**



Legend: Thin dashed – data; solid thick – baseline, whole sample; dashed thick – baseline, subsample 1; thin solid –  $\omega_y = 0.08$ ; thick dash-dotted –  $\omega_y = 0.175$