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The Term Structure and Time Series Properties of Nominal Interest Rates: Implications from Theory

1. INTRODUCTION

There are few relationships in economics and finance that have been studied as often and with such variety of techniques as the term structure of interest rates. Yet, while a host of empirical methods have been employed in these studies, the majority have shared a common theoretical perspective based on the expectations hypothesis. And, as detailed in the survey by Shiller (forthcoming), these tests have also shared the common conclusion of rejecting the empirical implications of the expectations hypothesis. Consequently, there have been renewed efforts to develop an empirically consistent theory of the term structure which, at the same time, maintains the intellectual appeal of the expectation hypothesis (for example, see Campbell's (1986) analysis of the term structure of real interest rates).

The empirical failure of the expectations hypothesis has been interpreted as implying (or, more correctly, defining) the existence of a term premium; more recently, it has also been recognized that the data implies that the term premium varies over time (again, see Shiller (forthcoming)). These observations suggest that a necessary characteristic of any proposed theory of the term structure offered as a replacement for the expectations hypothesis is the potential for generating a time-varying term premium. Since recently developed theories of nominal interest rates based on the representative agent asset pricing paradigm (see Lucas 1982 and LeRoy 1984) do indeed imply a time-varying term premium and, moreover, these

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models include the expectations hypothesis as a special case, the question is raised whether the restrictions on the nominal term structure implied by these theories are consistent with the data.

In order to address this question, a model of one- and two-period nominal interest rates based on Lucas's (1982) cash-in-advance model is developed in the following section. This analysis generates a set of testable restrictions on the time series behavior of nominal interest rates. Included among these are (1) the volatility of long rates relative to that of short rates, (2) the correlation of the spread between the forward and one-period rates with the subsequent change in the one-period rate, and (3) the sign of the term premium. In section 3, these predictions are compared to the corresponding sample moments using quarterly data on three- and six-month Treasury bills over the period 1959:1–1984:4. This comparison demonstrates that while the model is consistent with restrictions (1) and (2), it predicts the wrong sign of the term premium. Hence, much like the equity premium puzzle studied by Mehra and Prescott (1985) within the barter economy analog of the model employed here, the behavior of the nominal term premium remains unexplained.

2. A MODEL OF ONE- AND TWO-PERIOD INTEREST RATES

In this section, the first-order conditions that describe optimal choices of one- and two-period nominal bonds from Lucas's (1982) cash-in-advance model are studied in order to determine the qualitative equilibrium characteristics of the two interest rates. It is assumed that agents are identical and maximize expected lifetime utility over an infinite horizon. In addition, the utility function is assumed to be time separable with the one-period utility function having the functional form:

$$U(c_t) = \begin{cases} \dfrac{c_t^{1-\gamma} - 1}{1 - \gamma} & \gamma \neq 1 \\ \ln c_t & \gamma = 1 \end{cases} \tag{1}$$

where c_t denotes consumption in period t. Agents' subjective discount rate is denoted by β, $\beta \epsilon (0, 1)$.

Agents face uncertainty due to random growth rates of the endowment (equal to consumption in equilibrium) (g_t), money (μ_t) and the nominal price level (ρ_t). The state of the world is defined to be the realization of the three growth rates; $s_t \equiv (g_t, \mu_t, \rho_t)$. It is assumed that s_t follows a finite state stationary Markov process with transition matrix **II**. An element of **II** is denoted $\pi(s, s')$ and is defined by

$$\pi(s, s') = \text{Prob}\,(g_{t+1} = g' \,,\, \mu_{t+1} = \mu' \,,\, \rho_{t+1} = \rho' | g_t = g \,,\, \mu_t = \mu \,,\, \rho_t = \rho) \,.$$

After the revelation of the state s, agents visit the asset market where returns on previously purchased bonds are received and asset holdings are revised. In particular, new quantities of one- and two-period nominal bonds as well as money are chosen. It is assumed that the purchase price of both bonds is \$1 with one-period bonds returning $\$[n_1(s)]$ in period $t+1$ while two-period bonds return $\$[n_2(s)]^2$ in period $t+2$. Agents then visit the goods market where consumption choices are purchased with money.

At the agent's optimum, the utility foregone from the purchase of an additional bond must, at the margin, be equal to the expected utility the bond's return implies. This intertemporal marginal relation implies the following necessary conditions:

$$\frac{U_t'(s)}{P_t(s)} = [n_1(s)]\ \beta\ E_s\left[\frac{U_{t+1}'}{P_{t+1}}\right] \tag{2}$$

$$\frac{U_t'(s)}{P_t'(s)} = [n_2(s)]^2\ \beta^2\ E_s\left[\frac{U_{t+2}'}{P_{t+2}}\right] \tag{3}$$

where $E_s\,[\cdot]$ denotes expectations conditional on state s and $P_t(s)$ is the nominal price level. I now use the assumptions on preferences and the growth rate processes to simplify equations (2) and (3).

Specifically, these assumptions permit equation (2) to be written as

$$[n_1(s)]^{-1} = \beta E_s\left[\frac{c_{t+1}^{-\gamma}(s')}{c_t^{-\gamma}(s)} \cdot \frac{P_t(s)}{P_{t+1}(s')}\right]. \tag{4}$$

Or, in terms of growth rates

$$[n_1(s)]^{-1} = \beta E_s\,\{[1 + \xi(s')]\} \tag{5}$$

where

$$[1 + \xi(s)] \equiv [1 + g(s)]^{-\gamma}\,[1 + \rho(s)]^{-1}\,. \tag{6}$$

An immediate implication of equation (5) is that the stochastic properties of the function $\xi(s)$ determine the equilibrium behavior of $n_1(s)$. That is, $\xi(s)$ is the relevant state variable for one-period (and, as shown below, two-period) nominal interest rates. Hence, for all that follows, discussion will be in terms of the derived state variable, $\xi(s)$, and its time-series characteristics.

Note that if the cash-in-advance constraint is always binding (which will be the case if $n_1(s) > 0$), then $\rho(s) = \mu(s) - g(s)$. Hence, it would be possible to express $\xi(s)$ directly in terms of the exogenous growth rates of money and the endowment. However, defining $\xi(s)$ in terms of the inflation rate permits a *more* general test of the intertemporal marginal conditions [equations (2) and (3)] associated with one- and two-period nominal interest rates. That is, these expressions and the restrictions they place on the time series properties of nominal interest rates, inflation, and consumption are also consistent with a money-in-the utility model like that studied in LeRoy (1984) as well as a transaction cost model as used in Marshall (1989). By not imposing a theory of price level determination implied by a particular monetary model, the restrictions I derive represent a broader evaluation of the theory of nominal interest rates as captured in equations (2) and (3).

To simplify equation (3), note that the law of iterated expectations implies the following:

$$[n_2(s)]^{-2} = \beta^2 E_s \left\{ \frac{U'_{t+1}}{U'_t} \cdot \frac{P_t}{P_{t+1}} E_s{}' \left[\frac{U'_{t+2}}{U'_{t+1}} \cdot \frac{P_{t+1}}{P_{t+2}} \right] \right\} . \tag{7}$$

Using the assumption on preferences and the definition given in equation (6) results in

$$[n_2(s)]^{-2} = \beta^2 E_s \{[1 + \xi(s')] E_{s'} [1 + \xi(s'')]\} \tag{8}$$

where s'' is used to denote the state in period $t + 2$.

These expressions for equilibrium one- and two-period nominal interest rates imply the following characterization of equilibrium behavior:

PROPOSITION 1 (P1): *If $\xi(s)$ is independently distributed, then $n_1(s) = n_2(s) = n$.*

PROOF: Follows immediately from equations (5) and (8).

To provide some intuition behind (P1), note that using the approximation $\ln(1 + x) = x$ implies

$$\xi(s) = -[\gamma g(s) + \rho(s)] . \tag{9}$$

Suppose γ and/or $g(s) = 0$ for all s implying that either agents are risk-neutral or there is no real risk in the economy. The only uncertainty, therefore, is due to inflation, and with expected inflation equal across all states, all nominal interest rates are constant. At the other extreme, suppose $\rho(s) = 0$ for all s so that there is no inflation. Now nominal interest rates are constant because the expected change in the marginal utility of consumption is constant across all states.

To characterize interest rate behavior when states are serially correlated, assume that $\xi(s)$ follows a two-state Markov process with possible realizations $\xi(h) > \xi(\ell)$. In addition, it is assumed that the transition probability matrix is symmetric with the probability of remaining in the same state denoted

$$\pi = Prob\ (\xi_{t+1} = \xi(i) | \xi_t = \xi(i)] \ ; \ i = (h, \ \ell) \ .$$

This characterization implies states are positively or negatively autocorrelated as π is greater than or less than ½. These assumptions permit the following:

PROPOSITION 2 (P2): *$n_1(h) > (<) n_1(\ell)$ and $n_2(h) > (<) n_2(\ell)$ as $\xi(s)$ is negatively (positively) autocorrelated.*

PROOF: The restrictions on the $\xi(s)$ process implies

$$E_h [1 + \xi(s')] > (<) E_\ell [1 + \xi(s')] \text{ as } \pi > (<) ½ .$$

This establishes the behavior of $n_1 (s)$. Furthermore, it is straightforward to establish that the assumptions imply

$$E_h \{[1 + \xi(s')] E_{s'} [1 + \xi(s'')]\} > (<) E_\ell \{[1 + \xi(s')] E_{s'} [1 + \xi(s'')]\}$$
$$\text{as } \pi > (<) ½$$

which establishes the behavior of $n_2(s)$.

To understand these results, assume $\pi > ½$ and consider extreme values for γ, $g(s)$, and $\rho(s)$. That is, suppose $\gamma = 0$ and/or $g(s) = 0$ for all s, implying [from equation (6)] the inflation rate is *lower* in the high state and, because of positive autocorrelation, the expected inflation rate is also lower which results in a lower nominal interest rate. If $\rho(s) = 0$ for all s, then the realized and, more importantly, expected consumption growth rate is lower in the high state. The increased demand for savings causes equilibrium nominal interest rates to fall.

An important corollary of (P2) that will be used later in determining the sign of the term premia is that the contemporaneous covariance, $Cov[n_i, \xi]$; $i = 1, 2$ is determined by the pattern of serial correlation for $\xi(s)$. Specifically, $Cov[n_i, \xi] > (<) 0$ as $\pi < (>) ½$, that is, $\xi(s)$ is negatively (positively) autocorrelated.

3. THE TIME SERIES PROPERTIES OF NOMINAL INTEREST RATES

The expressions for $n_1(s)$ and $n_2(s)$ are now used to develop a set of restrictions on the time series behavior of the two nominal interest rates. To facilitate this goal, the implied forward rate is defined as

$$f_1(s) \equiv \frac{[n_2(s)]^2}{n_1(s)} \tag{10}$$

so that equation (8) implies

$$[f_1(s)]^{-1} = \beta E_s \left\{ [1 + \xi(s')] \frac{n_1 (s)}{n_1 (s')} \right\} . \tag{11}$$

This expression, in conjunction with (P1) and (P2), yields the following proposition (again, assuming the two-state process for $\xi(s)$ described earlier):

PROPOSITION 3 (P3): *If ξ(s) is serially uncorrelated, then* $f_1(h) = f_2(\ell) = n$. *If states are positively autocorrelated then*

$$n_1(h) > f_1(h)\ ; f_1(\ell) > n_1(\ell)$$

If states are negatively autocorrelated, then

$$f_1(h) > n_1(h)\ ;\ n_1(\ell) > f_1(\ell).$$

PROOF: The properties of $f_1(s)$ under serial independence are immediately given (P1). For the case of autocorrelation, note that equation (11) implies

$$\begin{aligned} [f_1(h)]^{-1} &= \beta \left\{ \pi[1 + \xi(h)] + (1 - \pi)[1 + \xi(l)]\ \frac{n_1(h)}{n_1(l)} \right\}; \\ [f_1(l)]^{-1} &= \beta \left\{ \pi[1 + \xi(l)] + (1 - \pi)[1 + \xi(h)]\ \frac{n_1(l)}{n_1(h)} \right\}. \end{aligned} \qquad (12)$$

Except for the ratio for one-period interest rates term, these expressions are identical to those for $[n_1\ (s)]^{-1}$; $s = (h, \ell)$ as given in equation (5). Hence, whether $f_1\ (s) \gtrless n_1\ (s)$ is determined by whether this ratio is greater or less than unity. Since Proposition 2 implies

$$\frac{n_1(h)}{n_1(l)} > (<)\ 1 \text{ as } \pi < (>)\ 1/2\ ,$$

the stated qualitative relationships between $f_1\ (s)$ and $n_1\ (s)$ are obtained.

Two corollaries are generated from (P3). First, the definition of $f_1(s)$ implies that long rates ($n_2(s)$) will be less volatile than $n_1(s)$ regardless of the serial correlation properties of $\xi(s)$. To see this, note that for positive serial correlation the relationship between $f_1(s)$ and $n_1(s)$ can be rewritten (using the definition of $f_1(s)$) as

$$[n_1(h)]^2 < [n_2(h)]^2 < [n_2(\ell)]^2 < [n_1(\ell)]^2$$

where the middle inequality is due to (P2). In the case of negative autocorrelation, (P3) implies

$$[n_1(\ell)]^2 < [n_2(\ell)]^2 < [n_2(h)]^2 < [n_1(h)]^2\ .$$

Again, (P2) establishes the middle inequality. Hence for both patterns of serial correlation, long rates vary over a smaller interval than short rates. The implication of this result is that the volatility of the conditional term premia generated from this model is not of great enough magnitude to overturn the volatility implications of the expectation hypothesis. That is, in the absence of a time-varying term premium, long rates are a geometric average of current and expected short rates so that the volatility relationship implied by (P3) is predicted. However, for the current model, it is possible for long rates to be more volatile than short rates if the variation in the term premia is great enough; (P3) implies this is not the case.

An additional implication of (P3) is that the unconditional covariance between the forward premium (that is, the spread between $f_1(s)$ and $n_1(s)$) and the subsequent change in one-period rates is always positive. That is, defining

$$fp(s) \equiv f_1(s) - n_1(s) , \tag{13}$$

$$\Delta n_1 (s, s') \equiv n_1(s') - n_1(s) , \tag{14}$$

Proposition 3 implies Cov $[fp(s), \Delta n_1(s, s')] > 0$ for all values of $\pi \neq ½$. Of course, the relationship between the forward premium and the change in the short rates has been extensively studied in empirical tests of the expectations hypothesis of the term structure. According to this hypothesis, the certain return from holding a two-period bond to maturity is equal to the expected return from purchasing a sequence of one-period bonds.[1] That is,

$$[n_2(s)]^2 = [n_1(s)]\, E_s\, [n_1 (s')] \tag{15}$$

or

$$f_1(s) = E_s\, [n_1 (s')] . \tag{16}$$

This relationship can be tested by the regression

$$\Delta n_{1,t+1} = \alpha + \beta fp_t + u_t \tag{17}$$

where $\Delta n_{1,t+1} \equiv \Delta n_1 (s, s')$, $fp_t \equiv fp(s)$ and u_t is an error term. Under the expectations hypothesis, the null hypothesis is ($\alpha = 0$, $\beta = 1$). As discussed in Shiller (forthcoming), the overwhelming majority of tests based on this regression reject the null values. On the other hand, the model developed here implies that, while the forward premium should on average predict the direction of movements of one-period rates ($\beta > 0$), it does not imply $\beta = 1$. That is, the forward rate is a biased predictor of the future spot rate with the bias being interpreted as a term premium. Specifically, defining the conditional term premium in terms of the rolling premium (the difference between the return on a two-period nominal bond and the expected return from the purchase of a sequence of one-period nominal bonds)[2] results in

$$tp(s) = [n_2(s)]^2 - [n_1(s)]\, E_s\, [n_1(s')] \tag{18}$$

[1]As discussed in Cox, Ingersoll, and Ross (1981), this is the "return to maturity" version of the expectations hypothesis.

[2]Alternatively, the conditional term premium could be defined as the difference between the expected nominal return from liquidating a two-period bond after one period and the certain nominal return from a one-period bond. The use of this definition, rather than that given in equation (18), does not alter any of the results or conclusions presented in the paper (details of the proof of this statement are available from the author).

or

$$tp(s) = [n_1(s)] \{[f_1(s)] - E_s [n_1(s')]\} . \tag{19}$$

To determine the sign of the term in brackets [that is, the direction of bias in the regression equation (17)], note that using equation (5) and the definition of the covariance allows equation (11) to be written as

$$[f_1(s)]^{-1} = E_s \{[n_1(s')]^{-1}\} + [n_1(s)] \beta \, Cov_s \{[1 + \xi(s')], [n_1(s')]^{-1}\} \tag{20}$$

where $Cov_s(\cdot)$ denotes the covariance conditional on state s. Since Jensen's inequality implies $E(x^{-1}) = [E(x)]^{-1} + \delta; \delta > 0$ for any positively valued random variable x, equation (20) can be expressed as

$$[f_1(s)]^{-1} = \{E_s[1 + n_1(s')]\}^{-1} + [n_1(s)] \beta \, Cov_s \{[1 + \xi(s')], [n_1(s')]^{-1}\} + \delta(s) . \tag{21}$$

Assuming $\delta(s) \simeq 0$ for all s

$$[f_1(s)]^{-1} - \{E_s[n_1(s')]\}^{-1} = [n_1(s)] \beta \, Cov_s \{[1 + \xi(s')], [n_1(s')]^{-1}\} . \tag{22}$$

Since (P2) implies $Cov_s \{[1 + \xi(s')], [n_1 (s')]^{-1}\} > (<) 0$ for all s as $\pi > (<) \frac{1}{2}$ we have

$$[f_1(s)] < (>) E_s [n_1 (s')] \text{ for all } s \text{ as } \pi > (<) \tfrac{1}{2} . \tag{23}$$

Or, from (19)

$$tp(s) < (>) 0 \text{ for all } s \text{ as } \pi > (<) \tfrac{1}{2} .[3] \tag{24}$$

In other words, if states are positively (negatively) autocorrelated, the expected nominal return from a sequence of one-period bonds will be greater (less) than the certain nominal return from a two-period bond.

Some intuition behind the prediction of the sign of the conditional nominal term premium is developed by considering the determinants of the conditional term premium on real bonds (that is, bonds whose price and return are denominated in units of consumption). As discussed in LeRoy (1982), the sign of the conditional real term premium can be explained by appealing directly to the consumption-based

[3]If $\delta(s) > 0$, the prediction of the sign of the term premia when $\pi > \frac{1}{2}$ is true, a fortiori.

CAPM (Breeden 1979). That is, since the conditional real term premium is similar to a risk premium in that it expressed the difference between a certain and expected real return, its sign must reflect the sign of the conditional covariance between the marginal utility of consumption and the return on the risky investment strategy (for example, a rollover of short-term bonds or the liquidation of a long-term bond before maturity). I now demonstrate that, because of the symmetry in the underlying asset pricing equations, an analogous covariance property is reflected in the sign of conditional nominal term premium.

In general, the assumption of time-separable utility implies that the first-order conditions determining optimal choices of assets with a certain return (R) and a risky return (Z) over a two-period horizon are

$$X_t = R_t^2\, \beta^2\, E_t(X_{t+2}) \;, \tag{25}$$

$$X_t = \beta^2\, E_t(Z_{t+2}\, X_{t+2}) \tag{26}$$

where R is expressed as a one-period yield and $E_t(\cdot)$ denotes expectations conditional on the state at date t. For example, if the risky return is due to the purchase of a sequence of one-period bonds then $Z_{t+2} \equiv [1 + i_t]\,[1 + i_{t+1}]$ where i_t is the one-period (either real or nominal) net interest rate. If R_t and Z_t are real returns, then $X_t \equiv U_t'$; if R_t and Z_t are nominal returns, then $X_t \equiv U_t'/P_t$ for the cash-in-advance model studied here. By the definition of conditional covariance (denoted $Cov_t(\cdot)$), the difference between the conditional expected return on the risky asset and the certain return (that is, the conditional risk premium) can be expressed as

$$E_t(Z_{t+2}) - R_t^2 = -\frac{Cov_t(Z_{t+2}, X_{t+2})}{E_t(X_{t+2})} \;. \tag{27}$$

Of course, for real returns this is just the consumption based CAPM. But for nominally denominated assets, (27) implies nominal yields are priced as if agents are concerned about the volatility of U_t'/P_t. An implication of (27) is that while an environment of constant consumption implies a zero real term premium, the nominal term premium will be nonzero if inflation is stochastic.

To see that this property is indeed present in the behavior of the nominal term premium, note equations (22)–(24) imply

$$sign[tp(s)] = sign\; [Cov_s\, \{[1 + \xi(s')],\, [n_1\,(s')]\}] \tag{28}$$

Suppose states are positively autocorrelated so that $tp(s) < 0$ and consider the consequences of a realization of $\xi(h)$ in period $t + 2$. By definition, this implies a high value of U_{t+2}'/P_{t+2}. Furthermore, because of positive serial correlation, the state in period $t + 1$ was, on average, high (that is, $\xi(h)$ prevailed) so that the one-period nominal interest rate in period $t + 1$ was low. Hence, the realized return on the rollover strategy in period $t + 2$ is low, implying a negative covariance between the return and U'_{t+2}/P_{t+2}. From (27), the (nominal) conditional risk premium on the rollover strategy will be positive and the conditional nominal term premium will be negative since $tp(s)$ is defined as the negative of a risk premium.

4. EMPIRICAL TESTS AND RESULTS

As demonstrated in the preceding two sections, the model implies the following restrictions on the data:

$$Var\ (n_1) > Var\ (n_2) \tag{R1}$$

$$Corr\ [fp_t,\ \Delta n_{1,t+1}] > 0 \tag{R2}$$

$$Corr\ [n_{1,t},\ \xi_t] > (<)\ 0 \text{ as } Corr\ (\xi_t,\ \xi_{t-1}) < (>)\ 0 \tag{R3}$$

$$[f_{1,t}] - E\ [n_{1,t+1}] < (>)\ 0 \text{ as } Corr\ (\xi_t,\ \xi_{t-1}) > (<)\ 0\ . \tag{R4}$$

The data used to test these restrictions were obtained from the following sources. The returns on three- and six-month Treasury Bills were used as one- and two-period nominal interest rates respectively. These were obtained from the first month of each quarter as listed in various publications of the *Federal Reserve Bulletin.* Quarterly data on real consumption (excluding durables) and the GNP price deflator were obtained from the National Income and Product Accounts.[4] The sample period was 1959:1 to 1984:4.

Note that (R3) and (R4) depend critically on the serial correlation properties of $\xi(s)$ which, because of the presence of γ (agents' relative risk aversion) in its definition, is an unobservable variable. In order to test the latter two restrictions, artificial series for $\xi(s)$ were generated by combining observations on $(g_t,\ \rho_t)$ with the following assumed values for γ (0.05, 0.25, 0.5, 1.5, 2.0, 3.0, 5.0, 7.0, 10.0)[5] using the approximation in equation (9). The implied sample mean, standard deviation, and first-order serial correlation for the generated series are presented in Table 1. As shown, $\xi(s)$ exhibits positive autocorrelation for all values of γ. Hence the correlation in (R3) should be negative and the forward rate should, on average, be less than the future one-period spot rate.

The values for the relevant sample moments implied by (R1)–(R4) are presented in Table 2. While the model is supported by the data for (R1), (R2), and (R3), the observed behavior of the term premium as reflected in (R4) is dramatically at odds with the prediction of the model. Specifically, the model implies that, on average,

[4]Per capita consumption was not used due to the presence of large outliers in the quarterly population series (published by the Department of Commerce, Bureau of Economic Analysis), as noted by Watson (1986). Since, for the most part, population follows a deterministic trend, the use of actual consumption should at most bias the estimate of the mean of ξ.

[5]This range for γ was chosen since it contains all reasonable estimates for relative risk aversion. See the discussion in Mehra and Prescott (1985).

TABLE 1

MOMENTS OF GENERATED SERIES (ξ)

γ	.05	.25	.50	1.0	1.5	2.0	3.0	5.0	7.0	10.0
E (ξ	−.012	−.014	−.016	−.020	−.020	−.027	−.035	−.051	−.066	.089
sd (ξ)	.007	.007	.006	.007	.008	.010	.015	.025	.035	.051
$Corr$ (ξ_t, ξ_{t-1})	.82	.81	.75	.53	.33	.23	.17	.17	.19	.20

the return from purchasing a sequence of 3-month T-Bills should be *greater* than holding a six-month T-Bill until maturity; over the sample period the sequence strategy generated a *lower* average return than that received on a six-month T-Bill.

As an internal check of the model, an empirical methodology similar to that developed by Mehra and Prescott (1985) in their analysis of the equity premium was employed. Specifically, I combined estimates of the technological paramaters ($\xi(s)$, $\mathbf{II}$) obtained from the data with assumed values for parameters describing preferences (β, γ) in order to generate artificial unconditional expected values for the forward premia using the models in section 2. The test consisted of comparing these artificial moments to the sample moments. In order to obtain estimates of ($\xi(s)$, $\mathbf{II}$), the following two-state representation was used:

$$\xi(h) = \alpha + \delta ;$$
$$\xi(\ell) = \alpha - \delta .$$

It was assumed $\mathbf{II}$ was a symmetric matrix with elements (π, $(1 - \pi)$) where π is defined by $\pi(h, h) = \pi(\ell, \ell) = \pi$. This restriction implies the ergodic distribution is given by $p(s) = ½$; $s = h, \ell$.

The above two-state representation implies the mean, standard deviation, and first-order serial correlation of ξ are, respectively,

$$E(\xi) = \alpha ;$$
$$sd(\xi) = \delta ;$$
$$corr(\xi_t, \xi_{t-1}) = (2\pi - 1) .$$

TABLE 2

(RI)	$Var(n_1) = 9.38 \times 10^{-4}$; $Var(n_2) = 8.92 \times 10^{-4}$									
(R2)	$Corr[fp_t, \Delta n_{1,t+1}] = 0.1269$									
(R3)	Contemporaneous Correlation between Three-Month T-Bill Yields (n_1) and ξ									
γ	.05	.25	.50	1.0	1.5	2.0	3.0	5.0	7.0	10.0
$Corr(n_1, \xi)$	−.81	−.81	−.80	−.78	−.76	−.74	−.70	−.63	−.57	−.51
(R4)	$E[f_{1,t} - n_{1,t+1}] = .0032$ (standard deviation = .0011)									

TABLE 3

CONDITIONAL AND UNCONDITIONAL FORWARD BIAS (all values × 10^{-2})

	γ			
	$.05 \leq \gamma \leq 3.0$	5.0	7.0	10.0
$b(h)$	−.02	−.07	−.20	−.53
$b(l)$	−.02	−.08	−.20	−.53
$\bar{B}$	−.02	−.07	−.20	−.54

For a given value of γ, values of (α, δ, π) were chosen so that the moments from the two-state process matched the corresponding entries in Table 1. The remaining parameter describing preferences, β, was held constant at .9925. These values were used as inputs into the previous section's models in order to calculate $[n_1(s), n_2(s)]$ which, in turn, were used to compute the conditional and unconditional bias in the forward rate $[b(s), \bar{B}]$, where these are defined as $b(s) \equiv f_1(s) - E_s\,[n_1(s')]$; $\bar{B} = [\Sigma_s b(s)]/2$.

As shown by the values in Table 3, the positive autocorrelation of ξ does indeed generate a negative rolling premia (both conditional and unconditional) from the model which, as stated earlier, is at odds with the data.

5. CONCLUSION

This paper has examined whether the time series properties of one- and two-period nominal interest rates implied by a simple cash-in-advance model are consistent with that exhibited in the data. While the model is consistent with some broad empirical characterizations, it fails to explain the behavior of the term premium. The failure represents another serious challenge to the representative agent asset pricing paradigm not unlike that of Mehra and Prescott's (1985) discussion of the equity premium puzzle. There, using a standard asset pricing model, they demonstrated that because of the relatively low *variance* of consumption, the model was incapable of generating an equity premium as large as that observed when reasonable values of relative risk aversion were used. Here, it is the *autocorrelation* properties of consumption and inflation that generate predictions for the nominal term premium that are at odds with the data. Perhaps a next step in resolving these equity and term premia puzzles would be the use of more general functional forms in describing agents' preferences; for example a nonseparable utility approach as in Dunn and Singleton (1986) and Constantinides (1989) or a nonexpected utility functional like that discussed in Epstein (1988) and Weil (1989). While these modifications may represent a possibility for salvaging the representative agent asset pricing paradigm with respect to the pricing of real-denominated assets, it is doubtful whether they will help to explain nominal returns and, in particular, the nominal term premia. That is, this paper has demonstrated that positive nominal term premia can be obtained if and only if there is negative autocorrelation in the growth rate of the term U'_t/P_t. As Constantinides (1989) has shown, habit persistence can induce negative autocorrelation in the growth rate of U_t' even though the growth rate of consumption is positively autocorrelated. However, it is my conjecture that this induced negative autocorrelation in the marginal utility of consumption is not, under reasonable parameter specifications, large enough to offset the positive serial correlation of inflation present in the data. Hence, even in light of these alternative preference specifications, the sign of the nominal term premium remains unexplained.

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