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EXCHANGE RATE VOLATILITY: THE ROLE OF REAL SHOCKS AND THE VELOCITY OF MONEY

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The effects of stochastic output shocks on the behavior of exchange rates and nominal price levels is studied within the context of a two-country, cash-in-advance model. The analysis of this model, in contrast to the existing cash-in-advance literature, demonstrates that exchange rates can be more volatile than price levels even though agents' elasticity of substitution between foreign and domestic goods is greater than one-half. This possibility arises when output shocks are autocorrelated and are due to revisions in expectations that affect the terms of trade and/or the velocity of money.

I. INTRODUCTION

A recent development in the theory of exchange rate determination has been the use of a cash-in-advance, two-country, recursive general equilibrium model to study the consequences of monetary and real disturbances for exchange rate behavior. As discussed in Lucas [1982] and Svensson [1985a], this framework provides a tractable synthesis between monetary and financial theory: In particular, the demand for money—and, hence, the behavior of the price level—depends on the future expected stream of endogenously determined liquidity services that money provides. Thus, money is treated in accordance with general asset pricing theory. In addition, using the cash-in-advance framework rather than placing money in the direct utility function means that the transactions role of money is modeled explicitly. As demonstrated below, different sequences of transactions imply different equilibrium characteristics for, in this context, exchange rates.

The use of a cash-in-advance (or liquidity) constraint in a recursive equilibrium model of exchange rates initially was proposed by Stockman [1980]. However, he could not provide an explicit solution under uncertainty due to technical complexities inherent in his framework. Lucas [1982] and Svensson [1985a], on the other hand, were able to solve for and characterize the general equilibrium solution in a two-country setting. They did so by positing that agents in both countries had identical preferences *and* held identical portfolios in which the underlying assets entitled the owner to the future stream of the nominal value of the endowment and monetary transfers of each country. Lucas's [1982] analysis was limited, however, in that he restricted his

*Vanderbilt University. I am indebted to Thomas Cooley, Harris Dellas, Steve LeRoy, and an anonymous referee for many insightful comments. In addition, Richard Sweeney provided helpful editorial assistance.

Economic Inquiry
Vol. XXVII, July 1989, 387–409

study to equilibria in which the liquidity constraint was always binding—i.e., the income velocity of money was constant. Svensson [1985a] relaxed this assumption and analyzed the effects of transitory and permanent monetary disturbances on exchange rate behavior.

The original motivation of Stockman's [1980] model, however, was to provide an equilibrium model of exchange rates with flexible prices—as opposed to Dornbusch's [1976] sticky-price model—that could explain deviations of exchange rates from that value predicted by purchasing power parity. This was done by demonstrating the role that changes in the terms of trade due to *real* (i.e., output) shocks have on exchange rate movements. This channel was present in Svensson's model but was ignored in his analysis. To remedy this deficiency, this analysis investigates the effects of real shocks—in the form of random growth rates of the endowment—on exchange rate movements within a recursive equilibrium setting. Particular attention is given to the volatility of exchange rates vis-a-vis price level movements. An important characteristic of the model used is that, under the proviso that within-period exchange of goods is impossible, the terms of trade are affected by both current and expected real shocks.[1] This latter factor is important because it expands the range of permissible values for the elasticity of substitution between foreign and domestic goods that generates predictions consistent with those observed in the data. That is, two phenomena exhibited in exchange rate data are (i) a contemporaneous positive correlation between real and nominal exchange rates and (ii) greater volatility of (nominal) exchange rates than that of the underlying price levels (see Stockman [1987]). As pointed out by Obstfeld and Stockman [1985], explaining this behavior using a constant-velocity, cash-in-advance model such as that used by Lucas [1982] requires the elasticity of substitution between foreign and domestic goods to be less than unity to explain phenomenon (i) and less than one-half to explain phenomenon (ii). However, such inelastic values are not consistent with most empirical estimates of import and export price elasticities lying in the region of unity. Furthermore, it is widely believed that these estimates are biased downward. (See the discussion in Shoven and Whalley [1984], and the references cited there.) This paper demonstrates that if shocks are autocorrelated, then the informational content of a supply shock can generate exchange rate behavior consistent with phenomena (i) and (ii) even when the elasticity of substitution is greater than unity.

1. As discussed later in the text, the restriction that within-period exchange of goods is impossible is implied by either imposing a cash-in-advance constraint on acquiring both goods and assets or having the goods market precede the asset market. If the asset market precedes the goods market (as in Lucas [1982]), it still is possible to relax the restriction on the elasticity of substitution between foreign and domestic goods and have the exchange rate exhibit greater volatility than the price levels. However, this behavior is due entirely to velocity changes as discussed in section VI.

Before presenting the model, some comments on the importance and consequences of alternative specifications of the cash-in-advance framework are in order. The normal setup, used by Lucas [1982] and Svensson [1985a; 1985b], is to assume two distinct markets: an asset market in which barter is allowed and a goods market in which all purchases must be financed out of current nominal balances—i.e., agents face a cash-in-advance constraint in the goods market. With two distinct markets, two alternative market sequences exist: After the state of the world is revealed, marking the beginning of a time period, the asset market may precede the goods market (as in Lucas [1982]) or vice versa (Svensson [1985a; 1985b]). Because the latter sequence implies that agents cannot revise their money holdings after learning of the state of the world, individuals have a precautionary demand for money in the latter sequence that is absent in the former sequence. However, assuming identical agents—implied by the perfectly pooled equilibrium used by both Lucas and Svensson—the precautionary demand for money is not manifest in the *equilibrium* behavior of real balances. (To be precise, the functional equations that determine the equilibrium real-balance function are the same in both market sequences. This is demonstrated in both Svensson [1985b] and Salyer [1986].) Consequently, the sequence of markets is irrelevant for the equilibrium behavior of the price level.[2]

However, the sequence of markets *is* important for the behavior of exchange rates since it defines quite different transactions roles for money. That is, if the asset market precedes the goods market, an agent is permitted to exchange foreign and domestic currencies before visiting the goods market during that period. This is tantamount to allowing same-period exchange between foreign and domestic goods. The alternative sequence also allows the exchange of foreign and domestic currencies in the asset market, but the goods market is not visited until after the new state of the world is revealed. This sequence, consequently, implies an inherent friction in monetary economies in which the indirect exchange of goods via money imposes an informational uncertainty that is absent if direct exchange is permitted. For the majority of this paper it is assumed that the goods market precedes the asset market, although the ramifications of the alternative sequence are discussed.

Section II presents the model where the growth rate of the endowment in the domestic economy follows a two-state Markov process while the foreign endowment growth rate and the growth rates of both currencies are constant.

2. It is worth noting that an additional difference between the Lucas [1982] and Svensson [1985a; 1985b] frameworks is the timing of monetary transfers: Agents receive the transfer at the beginning of the period in Lucas [1982], while they receive the monetary transfer after the goods market closes in Svensson [1985a; 1985b]. As demonstrated in Svensson [1985b], *this* difference does have implications for the equilibrium behavior of the price level when the monetary transfer is stochastic. The difference arises because, in Svensson's model, agents know the aggregate money stock available in the goods market for periods t and t+1. In Lucas's model, meanwhile, only the period t money stock is known. However, it should be stressed that once the timing of monetary transfers has been chosen, the choice of the sequence of markets is irrelevant for equilibrium price level characteristics.

Section III presents the equations determining equilibrium, and these are used to characterize the velocity of money in section IV. These results, in turn, are used to characterize exchange rate behavior in section V. Section VI analyzes exchange rate behavior if within-period exchange of goods is permitted. Section VII draws conclusions. The appendix contains much of the formal derivations and proofs.

II. INDIVIDUAL OPTIMIZATION

As stated in the introduction, the model used here is a variant of the cash-in-advance models presented in Lucas [1982] and Svensson [1985a]. This paper analyzes the behavior of exchange rates in a two-country world in which agents have identical preferences and hold identical portfolios consisting of mutual funds that give title to the stream of the nominal value of the endowment process and the monetary transfers in each country. This perfectly pooled equilibrium implies that the representative agent in each country consumes half of the endowment and holds half of the nominal money stock of each country in equilibrium.[3] Hence, equilibrium consumption levels are already known. The remaining problem is determining equilibrium prices and the velocity of money.

To characterize the behavior of equilibrium exchange rates, the following assumptions are made. First, to isolate the effects of real shocks, it is assumed that the monetary growth rate in each country is a constant, g. Second, the endowment growth rate in the foreign country is a constant, k, while that in the home country is random and can take on two values, $\alpha(h)$ (high) and $\alpha(l)$ (low). It is assumed that $\{\alpha_t\}$ follows a stationary Markov process.

It is worth noting that the assumption of stationarity in the *growth rate* of the endowment (as opposed to the *level* of the endowment) process is important for empirical and modeling purposes. Empirically, some have observed (e.g., Nelson and Plosser, [1982]) that first differences in the log of GNP are indeed stationary. Along modeling grounds, introducing serially correlated output quickly complicates the analysis of equilibrium—in particular, asset price behavior—since a current shock implies (as noted originally by Lucas [1978]) both an income effect and an information effect. Models which have assumed a continuous state space in levels (Lucas [1978] and Danthine and Donaldson [1986]) have found it necessary to restrict preferences—bounding relative risk aversion below unity—so as to sort out the implications of these two effects. On the other hand, restricting the state space to a two-state process in levels (as in LeRoy [1984]) generates a tractable model without restricting preferences but at the expense of severely limiting the informational role of a current realization. As illustrated in the appendix, a two-state process in growth rates does not restrict the relevance of new information nor is it necessary to restrict the range of relative risk aversion to characterize equilibrium. Consequently, the assumption leads to a rich structure of price-level and exchange-rate dynamics.

To maintain stationarity in the agent's maximization problem, the one-period utility function is assumed to be separable in both goods with each function a member of the constant relative risk aversion (CRRA) class of utility functions:

$$U(x,y) = \frac{x^{1-\gamma}-1}{1-\gamma} + \frac{y^{1-\gamma}-1}{1-\gamma} \quad \text{if} \quad \gamma \neq 1$$

$$U(x,y) = \ln x + \ln y \quad \text{if} \quad \gamma = 1$$

where x is consumption of the domestic good and y is the consumption of the foreign good. This assumption implies that, whereas the absolute level of saving will be both time dependent and state dependent, the savings-to-income ratio will be state dependent only (see Mehra and Prescott [1985] for a proof of this statement in a nonmonetary economy). In a representative individual cash-in-advance model, the decision of whether to exhaust money balances in the goods market is a savings decision and depends on the return characteristics of money. Hence, the implication of this assumption is that the ratio of real balances to output is state dependent only. (A proof is given in the appendix.) The inverse of this ratio is, of course, monetary velocity. Also, because γ measures both the elasticity of substitution between x and y and the elasticity of intertemporal substitution, the role of these elasticities in characterizing equilibrium is captured.

This paper employs the same timing of markets as does Svensson [1985a], whereas agents observe the state of the world at the beginning of the period [$\alpha(s)$; s=h,l]. Because equilibrium prices are functions of the state and it is assumed that agents know these functions along with the probability distribution of states, agents can infer current prices and, consequently, can make their portfolio and consumption decisions.

Individuals first visit the goods market where purchases of x and y are financed out of beginning-of-period domestic (M) and foreign (N) money holdings. That is, agents face a cash-in-advance constraint in acquiring both goods. Next, agents visit the asset market in which new domestic and foreign money, claims to the nominal value of the endowment process of each country (z_1 and z_2), and claims to the monetary process of each country (b_1, b_2) are purchased. (Subscripts 1, 2 denote domestic and foreign assets, respectively.) The dividends to z_i ($i = 1, 2$) are received in the asset market and are in the form of the cash proceeds from the current sale of x and y. The

3. The use of a perfectly pooled equilibrium is analytically convenient since it reduces a two-country environment to that of a representative agent model. However, a necessary condition for the existence of a pooled equilibrium is complete asset markets—a highly implausible assumption in a world that includes shocks to the value of human capital. Some investigations for the consequences of relaxing the assumption of complete asset markets on exchange rate behavior have been attempted recently (in particular, by Cole [1988]).

dividends to b_i $(i = 1, 2)$ are received at the beginning of the period and, hence, augment nominal balances carried over from the previous period. This scenario implies the following nominal constraints for a domestic agent, (with a corresponding set for a representative foreign agent:

$$W_t \geq M_t + e_t N_t + P_t x_t + e_t \varphi_t y_t + Q_{1t} z_{1t} + Q_{2t} z_{2t} + R_{1t} b_{1t} + e_t R_{2t} b_{2t} \tag{1}$$

$$M_{t-1} + b_{1t-1}\, g\, \overline{M}_{t-1} \geq P_t x_t \tag{2}$$

$$N_{t-1} + b_{2t-1}\, g\, \overline{N}_{t-1} \geq \varphi_t y_t \tag{3}$$

$$W_{t+1} = M_t + e_{t+1} N_t + z_{1t} (P_{t+1} \overline{x}_{t+1} + Q_{1t+1}) + e_{t+1} z_{2t} (\varphi_{t+1} \overline{y}_{t+1} + Q_{2t+1}) + b_{1t} (g \overline{M}_t + R_{1t+1}) + e_{t+1} b_{2t} (g \overline{N}_t + R_{2t+1}) \tag{4}$$

where

- W is nominal wealth measured in M units;
- e is the exchange rate measuring the price of foreign currency in terms of domestic currency;
- Q_j is the country j currency price of a claim to the endowment process in country j;
- R_j is the country j currency price of a claim to the monetary process in country j;
- P, φ are the domestic and foreign price levels, respectively;
- $\overline{x}, \overline{y}$ are endowments in the domestic and foreign countries, respectively; and
- $\overline{M}, \overline{N}$ are aggregate money stocks in the domestic and foreign countries, respectively.

Equation (1) expresses the budget constraint while equations (2) and (3) represent the liquidity constraints faced in the goods market. Equation (4) delineates the motion of nominal wealth as a result of asset choices made during the previous period.

Until this point, the agents' (domestic and foreign) maximization problems are analogous to those in Svensson [1985a].[4] As was demonstrated in that paper, and as is typical in a recursive equilibrium framework, these problems can be expressed as a dynamic programming problem in which the resulting Kuhn–Tucker conditions play two roles: (i) They represent necessary conditions for the agent's optimum, and (ii) when these conditions are evaluated at market-clearing quantities, they define the respective equilibrium price functions. For our purposes, we restrict our attention to that set of first-order conditions needed to determine equilibrium velocity, inflation rates, and exchange rates.[5]

First, the relative price of y in terms of x is denoted as ρ and defined by

$$\rho = e\varphi / P. \tag{5}$$

It is important to note that this relative price does not reflect actual trading opportunities due to the sequence of markets. That is, because agents must finance consumption during period t with beginning-of-period money balances, it is impossible to trade x for y directly during period t. This point is discussed in greater detail below in the derivation of the Kuhn–Tucker conditions. Letting λ denote the Lagrange multiplier for real wealth, μ and η the multipliers on the domestic and foreign liquidity constraints (all expressed in units of x), U_x and U_y the marginal utility of consumption of x and y, respectively, and $\pi_{ss'}$ the one-period transition probability of going to state s' conditional on currently being in state s, the appropriate Kuhn–Tucker conditions are

$$\mu = U_x - \beta \sum_{s'} \pi_{ss'} U_x' P/P' \geq 0 \tag{6}$$

$$\eta = \frac{1}{\rho}(U_y - \beta \sum_{s'} \pi_{ss'} U_y' \varphi/\varphi') \geq 0 \tag{7}$$

$$\rho = (\sum_{s'} \pi_{ss'} U_y' \varphi/\varphi') / (\sum_{s'} \pi_{ss'} U_x' P/P') \tag{8}$$

$$m - x \geq 0; \; \mu(m - x) = 0 \tag{9}$$

$$n - y \geq 0; \; \eta(n - y) = 0 \tag{10}$$

where $m = (M_{t-1} + b_{1t-1} g\overline{M}_{t-1})/P_t$ and $n = (N_{t-1} + b_{2t-1} g\overline{N}_{t-1})/\varphi_t$ and primes denote values next period.

Equation (6) demonstrates that if the liquidity constraint is not binding $(\mu = 0)$ domestic currency is being held as an asset so that the utility foregone from acquiring a unit of currency equals the expected utility gain that the

4. The current model and that of Svensson [1985a] are different in that here agents receive the monetary transfer at the beginning of the period. In Svensson [1985a], the monetary transfer is distributed at the close of the goods market. This distinction is not important for deriving the individual's first-order conditions but, as noted in note 2, does imply different equilibrium characteristics for real balances.

5. The remaining first-order conditions are capital asset pricing equations for the two mutual funds.

currency will provide next period. Equation (7) has a similar representation for foreign currency. Equation (8) highlights the critical role that money plays in our model: The relative price of the foreign good, i.e., the terms of trade, reflects the marginal rate of substitution between domestic and foreign *currency*. By contrast, in a barter economy or a cash-in-advance model in which the goods market follows the asset market, the terms of trade equal the marginal rate of substitution between foreign and domestic *goods*. Since direct, i.e., same-period, exchange of goods is not allowed when the goods market precedes the asset market, the relevant trade-off for the relative price of foreign goods is the marginal rate of substitution between *expected* domestic and foreign consumption currently represented by domestic and foreign real balances. The departure from the barter case can also be illustrated by using equations (6) and (7) to derive

$$\rho = U_y[\lambda + \mu]/U_x[\lambda + \eta]. \tag{11}$$

Hence, only if $\mu = \eta = 0$—both liquidity constraints are not binding—does the relative price of foreign goods equal the ratio of the current marginal utilities of consumption. Equations (9) and (10) represent the complementary slackness conditions for domestic and foreign currencies, respectively.

III. EQUILIBRIUM

The assumption of a perfectly pooled equilibrium ($b_{1t} = b_{2t} = z_{1t} = z_{2t} = 1/2$ for all t) implies that the equilibrium consumption vector for a domestic resident is $(1/2\,\bar{x}_t, 1/2\,\bar{y}_t)$ and for equilibrium money holdings is $(1/2\,\overline{M}_t, 1/2\,\overline{N}_t)$.

Generally, equilibrium real balances in the two countries are functions of both the state (i.e., the endowment growth rate) and the level of the endowment. However, as noted previously, the use of (CRRA) utility functions implies that these functions are homogeneous of degree one in output. Therefore, the income velocity of money, defined by the ratio of the endowment to real balances, is a function of the state only:

$$V(s) \equiv P\bar{x}/\overline{M}; \quad V*(s) \equiv \varphi\bar{y}/\overline{N} \tag{12}$$

where the asterisk denotes velocity in the foreign country.

Using (12), the one-period change in the price levels of both countries can be expressed in terms of velocity, i.e.,

$$P'/P = \{(1+g)/[1+\alpha(s')]\}\,[V(s')/V(s)] \tag{13}$$

$$\varphi'/\varphi = [(1+g)/(1+k)]\,[V*(s')/V*(s)]. \tag{14}$$

Equilibrium velocity is determined by replacing the one-period change in the price levels in (6)–(10) by using (13) and (14) and evaluating the resulting expressions at the equilibrium levels of consumption. Because of the use of CRRA utility functions, this procedure results in the following conditions:

$$\tilde{\mu}(s) \equiv \{1 - \beta\textstyle\sum_{s'} \pi_{ss'}\,[1 + \alpha(s')]^{1-\gamma}\, V(s)/[(1+g)\,V(s')]\} \tag{15}$$

$$0 < V(s) \le 1; \quad V(s) < 1 \quad \text{implies} \quad \tilde{\mu}(s) = 0 \tag{16}$$

$$\tilde{\eta}(s) \equiv \{1 - \beta\textstyle\sum_{s'} \pi_{ss'}\,(1+k)^{1-\gamma}\; V*(s)/[(1+g)\,V*(s')]\} \ge 0 \tag{17}$$

$$0 < V*(s) \le 1; \quad V*(s) < 1 \quad \text{implies} \quad \tilde{\eta}(s) = 0. \tag{18}$$

Given the equilibrium values of $V(s)$ and $V*(s)$ implied by (15)–(18), domestic and foreign inflation rates are determined by (13) and (14), respectively, while the terms of trade are derived from (8). Together, these values determine the equilibrium behavior of exchange rates.

IV. CHARACTERIZATION OF VELOCITY

Equations (15) and (16) determine monetary velocity in the domestic country while (17) and (18) determine the velocity of the foreign currency. A consequence of this independence and the nonstochastic behavior of both monetary and endowment growth rates in the foreign country is that the velocity of foreign currency always is unity. This is proven in the appendix. Consequently, the behavior of the foreign price level is given by the simple quantity equation

$$\varphi'/\varphi = (1+g)/(1+k)\,.$$

The following definition facilitates the analysis of domestic velocity.

$$[1+G(s)] \equiv (1+g)[1+\alpha(s)]^{\gamma-1}.$$

Note that $G(s)$ includes the effect of the change in the endowment and the change in the marginal utility of consumption. Furthermore, the monotonicity of $G(s)$ is determined by agents' relative risk-aversion parameter. Defining $f(\alpha) \equiv [1+G(\alpha)]$, then $f(\alpha)$ is either a constant, increasing, or decreasing function as γ is either equal to, greater than, or less than unity, respectively.

As demonstrated in the appendix, a necessary and sufficient condition for whether the cash-in-advance constraint is binding can be derived using (15) and (16). As also shown in the appendix, analyzing this condition characterizes the parameter values that determine whether the constraint is binding. This analysis implies the following:

Case I. Growth rates are serially uncorrelated and/or $\gamma = 1$: The constraint is always binding.

Case II. $\gamma < 1$: If the states are negatively (positively) serially correlated, the constraint is always binding in the high (low) endowment growth rate state.

Case III. $\gamma > 1$: If the states are negatively (positively) serially correlated, the constraint is always binding in the low (high) endowment growth rate state.

To understand these results and the importance of risk aversion and the serial correlations of states, it helps to recall the two motives for holding any asset: (i) the asset's expected return and (ii) the attempt to smooth the marginal utility of consumption across time. In the decision of whether to hold money as an asset when the endowment growth rates are random, these motives oppose one another for a given realization of α. That is, the high expected return state is the state in which there is most likely a deflation. But this is also the state in which the expected growth rate of the endowment is greater so that holding real balances as an asset increases the volatility of the marginal utility of consumption. Whether motive (i) or (ii) dominates depends on the magnitude of agents' relative risk aversion since this parameter is also the elasticity of marginal utility. If $\gamma = 1$, neither motive dominates so that money is not held as an asset in either state. If $\gamma < 1$, motive (i) dominates since the marginal utility of consumption inelastically responds to the high return on real balances. (Alternatively, agents' intertemporal elasticity of substitution—the inverse of γ—is high, implying that the high expected return from real balances induces agents to attempt to decrease present consumption in favor of future consumption.) Given the assumption of constant monetary growth, the expected return is greater in the state in which the expected endowment growth rate is larger. This, of course, is determined by the pattern of serial correlation. For example, in the case of negative serial correlation, the expected endowment growth rate is greater in the low-growth state so that agents may choose to hold nominal balances as an asset from t to $t+1$ in anticipation of a high expected return if $\gamma < 1$. If $\gamma > 1$, agents have an elastic marginal utility—a low elasticity of intertemporal substitution—and may attempt to hold money as an asset to smooth consumption (i.e., motive (ii) dominates). Hence, if the current endowment growth rate is high and states are negatively serially correlated, individuals may attempt to smooth their consumption by holding money as an asset in the high-growth state.

V. CHARACTERIZATION OF EXCHANGE RATE MOVEMENTS

As stated in section II, once the pattern of equilibrium velocity has been determined, the behavior of the terms of trade and of the exchange rate can be implied from equations (5), (8), (13), and (14).

Note that because the exchange rate is a function of the price levels, it is both time dependent and state dependent. Therefore, to maintain stationarity, we restrict our attention to the one-period change in the exchange rate given by the expression:

$$e(s,s') \equiv e_{t+1}(s')/e_t(s); \quad s' = (h,l). \tag{19}$$

From (8), (13), and (14) and using the result that velocity is always unity in the foreign country, the change in the exchange rate is

$$e(s,s') = [\theta(s')^{-\gamma}\, \mathrm{E}_s(\delta') / \mathrm{E}_{s'}(\delta'')]\ [\theta(s')V(s')/V(s)] \tag{20}$$

where

$$\theta(s') = (1+k)/[1+\alpha(s')] \qquad \text{and} \qquad \delta' = (1+\alpha_{t+1})^{1-\gamma}\, V_t/(1+g)\, V_{t+1}$$

(δ'' is δ' updated one period.)

The first term in brackets represents the movements in the terms of trade. It is important to note that these movements are a function of both current and expected shocks. The second term reflects the change in the price levels of the two countries. Again, movements in the price level of the domestic country reflect current and expected growth shocks through velocity changes.

An immediate result from equation (20) is that the exchange rate is constant if relative risk aversion is unity. That is, log utility implies constant unitary velocity—i.e., the elasticity of the general price level effects due to a growth shock is unity. However, the elasticity of the change in the terms of trade is also unity and offsets the price level changes exactly. The more interesting cases in which γ is nonunitary are detailed below.[6]

Independently Distributed Shocks

With independently distributed shocks, a realization provides no information about the future: Conditional and unconditional expectations are equal. As demonstrated previously, this implies that velocity is constant and equal to unity. Furthermore, changes in the terms of trade are not affected by changes in expectations—i.e., the expression for the change in the exchange rate simplifies to

$$e(s,s') = [\theta(s')^{-\gamma}]\ [(\theta(s')];\ s,s' = (h,l) \tag{21}$$

6. In what follows, only the cases of independence and positive autocorrelation are presented. Details for equilibrium behavior under negative autocorrelation are available from the author.

where the terms have not been combined to highlight the relative and general price level effects. The change in the exchange rate is completely determined by the current shock. That is,

$$e(h,h) = e(l,h) \equiv e(s,h)$$

$$e(l,l) = e(h,l) \equiv e(s,l).$$

Furthermore,

$$e(s,h) \gtreqless e(s,l) \quad \text{as} \quad \gamma \gtreqless 1.$$

Recall that γ reflects the elasticity of substitution between foreign and domestic goods and determines the magnitude of the change in the terms of trade due to a domestic output shock. A high growth rate always dampens domestic inflation while it also worsens the terms of trade. The general price level effect tends to appreciate the domestic currency (with an elasticity of 1) while the relative price change has a depreciating effect (with an elasticity of $-\gamma$). If $\gamma > 1$, the change in the terms of trade dominates the reduction in inflation and results in a situation of "immiserizing" growth.[7]

Taking logs,

$$\dot{e}(s,s') = (1-\gamma)\ \dot{p}(s,s')$$

where the dots denote percentage changes in variables and $\dot{p}(s,s')$ denotes the percentage change in the price levels [equal to $\ln \theta(s')$]. Consequently, when shocks are distributed independently, this model duplicates Lucas's [1982] exchange rate model in that (as noted by Obstfeld and Stockman [1985]) the exchange rate is more volatile than the underlying price levels if and only if $\gamma > 2$ or the elasticity of substitution between foreign and domestic goods is less than ½.

Positively Serially Correlated Shocks

If growth rates exhibit positive autocorrelation, then the expected value terms in the relative price effects—the one-period-ahead forecast of θ—changes whenever the state changes and, consequently, has important implications for both the movements and the volatility of the exchange rate. These terms are the conditional expected utility gain during period t+1 from a unit of domestic real balances purchased in the asset market at time t and, therefore, reflect the value of domestic real balances as an asset. Hence, the same factors that determined the pattern of velocity determine the qualitative relationship between these terms.

7. This comment refers to the behavior of the exchange rate only. Of course, growth is not immiserizing in terms of utility in this model.

With positive autocorrelation, a high endowment growth rate signals the high probability of a high growth rate next period implying a low expected inflation rate. Hence, the return motive for money is higher in the high growth rate state. However, the high growth rate also signals a high expected endowment and consumption level next period, and this tends to lower the demand for real balances. As in the discussion of determining velocity, the magnitude of relative risk aversion—i.e., greater or less than unity—determines whether the return motive dominates. These two subcases are examined below.

Relative risk aversion less than unity. With unity as the upper bound of relative risk aversion, the return motive dominates so that the expected marginal utility of domestic real balances increases in the high-output state. That is, $E_h(\delta') > E_l(\delta')$. (Because attention is restricted to a stationary equilibrium, the one-period-ahead forecast of δ is time independent, i.e., $E_s(\delta') = E_{s'}(\delta'')$ if $s = s'$.)

To study the implications for exchange rate movements, note that equation (20) implies that expectations—whether manifest directly through $E_s(\delta')$ or indirectly through $V(s)$—influence exchange rate behavior *only* in transition states. Under positive serial correlation, changes in expectations exacerbate the effects of current productivity shocks and this leads to increased volatility in the exchange rate demonstrated next.

First note that as in the case of independently distributed shocks, if $\gamma < 1$, general price level effects dominate relative price changes if the state does not change:

$$e(h,h) < e(l,l). \tag{23}$$

If the state changes, then the results that $E_h(\delta') > E_l(\delta')$ and $V(h) \le V(l)$ imply

$$e(l,h) < e(h,h) < e(l,l) < e(h,l). \tag{24}$$

Furthermore, the exchange rate can exhibit diverse volatility behavior relative to the underlying price levels due to the change in expectations. Again taking logs,

$$\dot{e}(s,s') = [-\gamma \ln \theta(s') + \dot{E}(s', s)] + [\ln \theta(s') + \dot{V}(s, s')] \tag{25}$$

where

$$\dot{E}(s',s) = \ln E_s(\delta')/E_{s'}(\delta'')$$

and

$$\dot{V}(s,s') = \ln V(s')/V(s).$$

To illustrate the implications, let $\pi = 0.75$, $\alpha(h) = 0.03$, $\alpha(l) = -0.03$, $\beta = 0.99$, $\gamma = 0.5$, and $k = g = 0$. The implied equilibrium values for the change in the exchange rate during transition periods are

$$e(l,h) = [0.997]\ [0.971] = 0.969$$

$$e(h,l) = [1.032]\ [1.031] = 1.064.$$

Note that realizing a high domestic output implies a *fall* in the relative price of imports while a low output realization implies an *increase* in the relative price of imports. That is, while realizing a high domestic growth rate increases the marginal rate of substitution component of the relative price, this change is more than offset by the increase in the expected utility gain from domestic balances. Hence, the combined result is an overall improvement in the terms of trade. The interpretation for the behavior of the exchange rate when the state goes from high to low is similar. The important implication is that in both instances, the real and nominal exchange rates move together and the percentage change in the nominal exchange rate exceeds that of the price levels even though the elasticity of substitution between domestic and foreign goods is greater than unity. Generally, a wide range of values of (β, $\alpha(l)$, $\alpha(h)$, π and $\gamma\epsilon(0, 1)$) are consistent with the above equilibrium characteristics.

Relative risk aversion greater than unity. If relative risk aversion is greater than unity, agents wish to hold domestic real balances to help smooth their marginal utility of consumption. That is, with positively autocorrelated growth rates, the expected utility gain from a unit of domestic real balances is greater in the low growth rate state even though this state is the low expected return state for real balances:

$$E_l(\delta') > E_h(\delta'). \tag{26}$$

This result and the previous result that $V(h) \geq V(l)$ implies

$$e(h,l) < e(l,l) < e(h,h) < e(l,h). \tag{27}$$

As another numerical example, let $\gamma = 1.5$ and keep all other parameter values unchanged. The values for the one-period change in the exchange rate during transition periods now become

$$e(l,h) = [1.063]\ [0.971] = 1.032$$

$$e(h,l) = [0.938]\ [1.03] = 0.961.$$

In this case, the changes in the marginal rate of substitution and the expected utility of domestic balances reinforce each other. Consequently, the exchange rate can display greater volatility than can price levels even though the elasticity of substitution between foreign and domestic goods—the inverse of γ—is greater than ½. (As mentioned in section I, the restriction ($\gamma > 2$) is a necessary condition for excess volatility in exchange rates in the cash-in-advance models discussed by Obstfeld and Stockman [1987].) Note that when $\gamma > 1$, the real and nominal exchange rates always move together.

VI. ALTERNATIVE SEQUENCE OF MARKETS[8]

The above definition of the terms of trade and subsequent discussion of exchange rate movements depend critically on the assumption that the goods market precedes the asset market. As stated in section II, if the reverse sequence is used, the terms of trade equal the marginal rate of substitution between foreign and domestic goods. Denoting the exchange rate in this case as $\tilde{e}$, the one-period change in this exchange rate is

$$\tilde{e}(s,s') = [\theta(s')^{-\gamma}]\ [\theta(s')\ V(s')/V(s)]. \tag{28}$$

Again, terms have not been combined to highlight relative (the first term in brackets) and general (the second term in brackets) price effects. The obvious difference between this expression and the previous one (equation (21)) is the lack of the expected-value terms in the relative price effects. (The presence of these terms reflected the inability to exchange foreign and domestic goods within the same time period or, more importantly, before new information was revealed.) With the goods market preceding the asset market, the only possible manifestation of the information effect of a current shock is through velocity changes.

If velocity is constant, then the behavior of this exchange rate is identical to that studied previously when shocks are distributed independently. If velocity is nonunitary, velocity changes again reinforce relative price changes. Specifically, if $\gamma < 1$,

$$\tilde{e}(l,h) < \tilde{e}(h,h) < \tilde{e}(l,l) < \tilde{e}(h,l) \tag{29}$$

while if $\gamma > 1$,

$$\tilde{e}(h,l) < \tilde{e}(l,l) < \tilde{e}(h,h) < \tilde{e}(l,h).^{9} \tag{30}$$

8. The exchange rate implied by the asset market preceding the goods market—the sequence used in section VI—is the same as that discussed in the section on "continuous currency trading" in Svensson [1985a]. In addition, Stockman and Svensson [1987] discuss the distinction between "beginning-of-period" (section V) and "end-of-period" (section VI) exchange rates.

9. These qualitative relationships can be verified by substitution using the previous definition of $\theta(s)$ and the expression for $V(s)$ as given by equation (A.13) for velocity in the state in which the constraint is not binding (see appendix).

It is worth noting that even using this definition of relative prices, an elasticity of substitution between foreign and domestic goods less than ½ no longer is necessary to have greater volatility in exchange rates than in price levels provided velocity is not constant. For example, consider the case of $2 > \gamma > 1$ and positive serial correlation and suppose that the state goes low to high. The resulting increase in velocity ($V(h) > V(l)$ in this case) offsets the deflationary effects of a high output growth rate. Consequently, the change in the relative price term can be twice (in absolute value) the change in the general price term even though the elasticity of substitution is greater than ½.

VII. CONCLUSION

As Svensson [1985a] stated in his conclusion, "I think there is a possibility that this and similar kinds of monetary asset-pricing models can be used as a new generation of work horses in international economics." This paper can be viewed as presenting another example of this new class of models. A next step would be to formulate empirical tests based on the monetary asset-pricing framework. In particular, are the parameter values that produce excessive volatility of exchange rates within the model empirically realistic? For example, Nelson and Plosser's [1982] finding that the log of U.S. GNP can be modeled as a random walk with drift implies the absence of serial correlation in real growth rates. As noted in the paper, the existence of serial correlation is necessary to predict excess volatility in exchange rates if one allows the elasticity of substitution between domestic and foreign goods to be greater than ½. However, Nelson and Plosser's findings are not necessarily contradictory to the theoretical predictions of this model if serially correlated monetary growth rates are introduced. To resolve these issues, a possible empirical methodology would be to proceed along the lines of Mehra and Prescott [1985]: Estimate the technological parameters for the serial correlation of real and monetary growth rates and determine whether these parameters, when combined with a set of parameters defining individual preferences (β, γ), can generate exchange-rate behavior consistent with the actual data. However, while the analytically tractable representative agent assumption may be appropriate in an empirical study of stock price behavior (as in Mehra and Prescott [1985], it may be too restrictive for a study of exchange rates.

APPENDIX

(1) *Restrictiveness of a Two-State Model in the Level of the Endowment*

To illustrate the reduced role of the informational content of a realization of the endowment when the level of the endowment follows a two-state process, the behavior of the real interest rate within such a model is compared with that implied by a model in which the growth rate of the endowment follows a two-state process. For illustrative ease, assume that preferences exhibit constant relative risk aversion and that the conditional probability of remaining in the same state is equal for both states:

$$\text{Prob}\,[x_{t+1} = x(h) \mid x_t = x(h)] = \text{Prob}\,[x_{t+1} = x(l) \mid x_t = x(l)] = P$$

where $x(h), x(l)$ denote the level of the endowment $[x(h) > x(l)]$ and

$$\text{Prob}\,[\alpha_{t+1} = \alpha(h) \mid \alpha_t = \alpha(h)] = \text{Prob}\,[\alpha_{t+1} = \alpha(l) \mid \alpha_t = \alpha(l)] = \pi$$

where $\alpha(h)$, $\alpha(l)$ denote the growth rate of the endowment.

The real interest rate in state i, $r(i)$, is defined by the equation

$$U'(i) = [1+r(i)]\,\beta\,E_i\,[U'_{t+1}]$$

where U' denotes the marginal utility of consumption evaluated at the endowment and E_i is the expectations operator conditional on state i. The expression implies that at the margin, the agent is indifferent between the loss in utility from the purchase of a bond and the expected utility gain that the return will provide next period.

If the endowment follows a two-state process, then

$$[1+r(h)]^{-1} = \beta\{P + (1-P)[x(l)/x(h)]^{-\gamma}\} > \beta\{P + (1-P)\,[x(h)/x(l)]^{-\gamma}\} = [1+r(l)]^{-1}. \tag{A.1}$$

That is, the expected marginal utility of consumption is always greater (less) than the current marginal utility of consumption if the current endowment is high (low). Hence, even though a high endowment signals a high probability of a high endowment next period, this information is less important than the current high income so that the interest rate falls.

If the growth rate of the endowment follows a two-state process, then

$$[1+r(h)]^{-1} = \beta\{\pi[1+\alpha(h)]^{-\gamma} + (1-\pi)[1+\alpha(l)]^{-\gamma}\} \tag{A.2}$$

and

$$[1+r(l)]^{-1} = \beta\{\pi[1+\alpha(l)]^{-\gamma} + (1-\pi)[1+\alpha(h)]^{-\gamma}\}. \tag{A.3}$$

Manipulation of these expressions implies

$$r(h)\ (>, =, <)\ r(l) \quad \text{as} \quad \pi\ (>, =, <)\ \tfrac{1}{2}$$

(i.e., positive, independent, negative serial correlation). Hence, the information provided by the current shock plays a critical role in determining the behavior of interest rates.

(2) *Homogeneity of the Real Price of Money*

In models in which the *level* of the endowment follows a stationary process, a stationary monetary equilibrium is defined in terms of real balances, i.e., $M_t/P_t = f(x_t)$ where x denotes the endowment (for example, see Danthine and Donaldson [1986]). Because the state in the present model is represented by the pair (x_t, α_t) where α denotes the current realization of the domestic endowment growth rate, then in general $M_t/P_t = f(x_t, \alpha_t)$. It must be demonstrated that the assumption of isoelastic preferences implies $f(\cdot)$ is homogeneous of degree one in x or, equivalently, that the real price of money has the representation $(1/P_t) = x_t f(\alpha_t)/M_t$. This is done by making the conjecture and showing that it is internally consistent with the model and, hence, verified (given the uniqueness of the equilibrium real balance function).

First, rewrite equation (6) as

$$(\mu_t/P_t) = (x_t^{-\gamma}/P_t) - \beta \mathrm{E}\, x_{t+1}^{-\gamma}/P_{t+1} \tag{A.4}$$

where it is assumed that $U(x)$ exhibits constant relative risk aversion (denoted by γ) and that, in equilibrium, agents' consumption equals the endowment. $\mathrm{E}_t(\ )$ denotes expectations conditional on information at time t.

Under the conjecture, $(1/P_t) = x_t f(\alpha_t)/M_t$ so that

$$(\mu_t/P_t) = x_t^{1-\gamma} f(\alpha_t)/M_t - \beta \mathrm{E}_t\, [x_{t+1}^{1-\gamma} f(\alpha_{t+1})/M_{t+1}]. \tag{A.5}$$

Given the motion of the endowment and the money stock and the assumption that α follows a Markov process, (A.5) can be written as

$$(\mu_t/P_t) = x_t^{1-\gamma}\{f(\alpha_t) - \mathrm{E}_t[(1+\alpha_{t+1})^{1-\gamma} f(\alpha_{t+1})]/(1+g)\}/M_t \equiv x_t^{1-\gamma} h(\alpha_t)/M_t\,. \tag{A.6}$$

Also note that equation (6) can be solved forward to yield

$$(1/P_t) = [\sum_{\tau=0}^{\infty} \beta^{\tau}\, \mathrm{E}_t(\mu_{t+\tau}/P_{t+\tau})]\, x_t^{\gamma}. \tag{A.7}$$

That is, the real price of money represents the expected stream of liquidity services that a dollar represents.

Using (A.6),

$$(1/P_t) = \{\sum_{\tau=0}^{\infty} \beta^{\tau}\, \mathrm{E}_t[x_{t+\tau}^{1-\gamma} h(\alpha_{t+\tau})/M_{t+\tau}]\}\, x_t^{\gamma}. \tag{A.8}$$

Because $M_{t+\tau} = M_t(1+g)^{\tau}$ and $x_{t+\tau} = x_t \prod_{i=1}^{\tau} (1+\alpha_{t+i})$,

this expression can be simplified to

$$(1/P_t) = (x_t/M_t) \sum_{\tau=0}^{\infty} \beta^{\tau}\, (1+g)^{-\tau}\, \mathrm{E}_t\{[\prod_{i=1}^{\tau}(1+\alpha_{t+i})] h(\alpha_{t+\tau})\} \tag{A.9}$$

$$\equiv x_t f(\alpha_t)/M_t\,.$$

Hence the conjecture is verified.

(3) *Proof That the Foreign Currency Cash-in-Advance Constraint Always Is Binding*

Denote the two possible states as $s = 1, 2$ and suppose that this were not the case, i.e., that the constraint is not binding in some state, say, state 1. (The constraint must be binding in at least one state in equilibrium. If this were not the case. the shadow price of money would be zero in all states, implying an infinite nominal price level as can be verified by (A.7). From the cash-in-advance constraint, this would imply zero consumption so that the commodity market would not clear.) This would imply

$$\tilde{\eta}(1) = 0;\ V*(1) < 1;\ \tilde{\eta}(2) > 0;\ V*(2) = 1$$

so that equation (17) can be expressed as

$$1 - \beta(1+g)^{-1}(1+k)^{1-\gamma}\,[\pi_{11} + \pi_{12} V*(1)] = 0 \tag{A.10}$$

$$1 - \beta(1+g)^{-1}(1+k)^{1-\gamma}\,[\pi_{21}/V*(1) + \pi_{22}] > 0\,. \tag{A.11}$$

For the expression in (A.11) to be greater than that in (A.10), it is required that $\pi_{11} + \pi_{12} V*(1) > \pi_{21}/V*(1) + \pi_{22}$. But because $\pi_{11} + \pi_{12} = \pi_{21} + \pi_{22}$ and $V*(1) < 1$, by assumption the inequality is not satisfied.

(4) Derivation of the Necessary and Sufficient Condition for the Constraint to Be Slack in a State in Terms of G(s)

Given the definition of $G(s)$, equation (15) can be expressed as

$$\tilde{\mu}(s) = \{1 - \beta \sum_{s'} \pi_{ss'} [1 + G(s')]^{-1} [V(s) / V(s')]\} \geq 0; \; s, s' = h, l. \tag{A.12}$$

Using (A.12) and the complementary slackness conditions, we can derive restrictions on the parameters that must be satisfied for the assumption to be true. That is, if the constraint is slack in state 1, then $\tilde{\mu}(1) = 0$; $V(1) < 1$; $\tilde{\mu}(2) > 0$; $V(2) = 1$. Solving the appropriate expression from (A.12) for $V(1)^{-1}$ *yields*

$$V(1)^{-1} = \beta\pi_{12}[1 + G(1) - \beta\pi_{11}]^{-1}\{[1 + G(1)] + G(2)]\} > 1. \tag{A.13}$$

Substitution into $\tilde{\mu}(2)$ generates the inequality

$$[1+G(2)] > (\beta^2\pi_{12}\pi_{21})\,[1 + G(1) - \beta\pi_{11}]^{-1} + \beta\pi_{22}. \tag{A.14}$$

Combining (A.13) and (A.14) yields the necessary and sufficient—due to the equality in (A.13)—condition for the constraint to be slack in state 1:

$$\beta\pi_{12}[1 + G(1)][1 + G(1) - \beta\pi_{11}]^{-1} > [1 + G(2)]$$

$$> (\beta^2\pi_{12}\pi_{21})\,[1+G(1) - \beta\pi_{11}]^{-1} + \beta\pi_{22}.$$

Analyzing this condition determines the characteristics of the parameters when the constraint is slack in a state and generates the following propositions.

Proposition (1). If states are distributed independently ($\pi_{11} = \pi_{21}$), then the constraint is always binding.

Proof: Substitution for π_{12} and π_{21} yields the contradiction

$$\beta\pi_{22}\,[1+G(1)] > [1+G(2)]\,[1+G(1)-\beta_{11}] > \beta\pi_{22}[1+G(1)]. \tag{A.16}$$

Proposition (2). If $G(1)$, $G(2) > 0$, then the constraint is always binding.

Proof. From the left-hand term,

$\beta\pi_{12}[1 + G(1)]\,[1 + G(1) - \beta\pi_{11}]^{-1} \gtrless 1$ as $\beta\pi_{11}/(1 - \beta\pi_{12}) \gtrless [1 + G(1)]$ but $\beta\pi_{11}/(1 - \beta\pi_{12}) < 1$ since $1 > \beta$.

Therefore, $G(1) > 0$ implies $1 + G(2) < 1$, which is violated if $G(2) > 0$.

The next proposition is used to establish propositions (4a) and (4b).

Proposition (3). If (A.15) holds, then $[1 + G(1)] >(<)\; \beta$ if states are negatively (positively) serially correlated.

Proof: Comparing the left- and right-hand terms of (A.15),

$$\beta\pi_{12}\,[1 + G(1)]\,[1 + G(1) - \beta\pi_{11}]^{-1} > (\beta^2\pi_{12}\pi_{21})[1 + G(1) - \beta\pi_{11}]^{-1} + \beta\pi_{22}$$

if and only if

$$[1 + G(1)](1 - \pi_{11} - \pi_{22}) > \beta(1 - \pi_{11} - \pi_{22}).$$

If states are negatively serially correlated,

$$1 - \pi_{11} - \pi_{22} > 0$$

so that

$$[1+G(1)] > \beta.$$

If states are positively serially correlated,

$$1 - \pi_{11} - \pi_{22} < 0$$

so that

$$[1+G(1)] < \beta.$$

Proposition (4a). If (A.15) holds and states are negatively serially correlated, then the constraint is be binding in the low-growth state.

Proof: The left-hand inequality in (A.15) can be expressed as

$$[\beta(1 - \pi_{11})]/[1 + G(1) - \beta\pi_{11}] > [1 + G(2)]/[1 + G(1)].$$

It is easy to show that proposition (3) implies that the left term is less than unity so that $G(1) > G(2)$. As a corollary, if $G(1) > 0$ and (A.15) holds, then this implies that $G(2) < 0$.

Proposition (4b). If (A.15) holds and states are positively serially correlated, then the constraint is binding in the high-growth state.

Proof: The right-hand inequality can be rewritten as

$$\beta^{-1}[1 + G(2)] > \{\beta(1 - \pi_{11} - \pi_{22}) + \pi_{22}\;[1 + G(1)]\}/[1 + G(1) - \beta\pi_{11}].$$

The right-hand term is greater than unity if and only if $[1 + G(1)]/\beta < 1$, which occurs only under positive serial correlation of states. Therefore,

$$\beta^{-1}[1 + G(2)] > 1 > \beta^{-1}[1 + G(1)],$$

implying that $G(2) > G(1)$.

The monotonicity of $G(s)$ together with proposition (4) implies case II and III in the text. The proof of case I is analogous to that presented in the first part of the appendix—i.e., the constant G implies an always-binding constraint.

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