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Agency Costs, Housing Production and Business Cycles

Abstract

This paper analyzes the role of uncertainty in a credit channel model that includes a housing sector. The framework is that of a multi-sector growth model where frictions arise due to the asymmetric information between credit lenders and housing developers. We include time varying uncertainty in the technology shocks that affect housing production. We show that the shocks to the housing production sector imply a quantitatively big role for uncertainty over the housing and business cycle. Moreover, our model can account for most of the salient features of housing stylized facts, in particular, the housing price is more volatile than output.

- *JEL Classification: E4, E5, E2, R2, R3*
- Keywords: *agency costs, credit channel, time-varying uncertainty, residential investment. co-movement, home production, multisector models*

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1 Introduction

Discussions in macroeconomics over the last year have veered from the modern topics of Calvo pricing and structural Phillips curves to those from a more distant past: bank runs, the collapse of long-established Wall Street firms, and the government's role in financial markets. These current events have once again demonstrated the influence that financial markets in general and the housing sector specifically have in the economy. While our focus here is not on monetary policy, we do attempt to bridge the gap between data and theory by developing a general equilibrium model of housing that also incorporates lending under asymmetric information. These efforts do not represent new theories but the, hopefully useful, combination of existing models.[1]

In particular, this paper analyzes the role of uncertainty in a credit channel model that includes a housing sector. The specific framework is that of a multi-sector real business cycle model with house production (e.g. Davis and Heathcote, 2005) and a financial sector with lending under time-varying asymmetric information (e.g. Carlstrom and Fuerst, 1997, 2001; Dorofeenko, Lee, and Salyer, 2008). We model time varying uncertainty as a mean preserving spread in the distribution of the technology shocks affecting house production and explore how changes in uncertainty affect equilibrium characteristics.

Our aim in examining this environment is twofold. First, we want to develop a framework that can capture one of the main components of the current financial crises, namely, changes in the uncertainty associated with future events. In addition, it is hoped that the framework will demonstrate plausible qualitative features (e.g. an increase in uncertainty results in a fall in aggregate investment) which, at same time, have quantitatively meaningful effects on housing market and business cycle properties. The second component of our framework is that we want to cast this analysis of time-varying second moments in a model that is consistent with a broad

[1] Some of the recent works on housing and credit are by Iacoviello and Minetti (2008), Iacoviello and Neri (2008) who estimate new-Keynesian DGSE two sector model, Iacoviello (2005) who looks at the real estate collateral and monetary effects, and Aoki, Proudman and Vliegh (2004) who analyse house price amplification effects in consumption and housing investment over the business cycle.

array of stylized facts for the housing sector. These are: (i) Housing prices are more volatile than output, (ii) Residential investment is about twice as volatile as non-residential investment, (iii) Residential investment and non-residential investment are highly procyclical, and iv) Residential investment leads output whereas non-residential investment lags.[2]

To that end, we employ the Davis and Heathcote (2005) model which, as demonstrated by the authors, can replicate the high volatility observed in residential investment. By incorporating an explicit financial market into this model, we can also produce large movements in housing prices, a feature of the data that was missing in the Davis and Heathcote (2005) analysis. We show that housing prices in our model are affected by expected bankruptcies and the associated agency costs; these serve as an endogenous, time-varying markup factor affecting the price of housing. The volatility in this markup translates into increased volatility in housing prices. In addition to these cyclical features of the data, a marked feature of the housing sector has been the growth in residential and commercial real estate lending over the last decade. As shown in Figure 2, residential real estate loans (excluding revolving home equity loans) account for approximately 50% of total lending by domestically chartered commercial banks in the United States over the period October 1996 to July 2007. This feature alone serves as strong motivation for the construction of a model that combines housing decisions and lending activity.[3]

Our analysis finds that plausible calibrations of the model with time varying uncertainty produce a quantitatively meaningful role for uncertainty over the housing and business cycle. Specifically, we compare the impulse response functions for aggregate variables (such as output, consumption expenditure, and investment) due to a 1% increase in technology shocks to the construction sector to a 1% increase in uncertainty to shocks affecting housing production. We find that, quantitatively, the impact of uncertainty shocks is almost as great as that from technology shocks. This comparison carries over to housing variables such as the price of housing, the risk

[2] One other often mentioned stylized fact is that housing price is persistent and mean reverting (e.g. Glaser and Gyourko (2006). See Figure 1 and Table 4 for these cyclical and statistical features from the period of 1975 till the second quarter of 2007.

[3] Figure 3 shows the strong co-movement between the amount of real estate loans and house prices.

premium on loans, and the bankruptcy rate of housing producers. The model is not wholly satisfactory in that it can not account for the lead-lag structure of residential and non-residential investment. Still, we think the approach presented here provides a useful start in studying the effects of time-varying uncertainty on housing and housing finance.

2 Model Description

As stated above, our model builds on two separate strands of literature: Davis and Heathcote's (2005) multi-sector growth model with housing, and Dorofeenko, Lee and Salyer's (2008) credit channel model with uncertainty. For expositional clarity, we first briefly outline our variant of the Davis and Heathcote model and then introduce the credit channel with time varying uncertainty.

2.1 Production

2.1.1 Firms

The economy consists of two agents, a consumer and entrepreneur, and four sectors: an intermediate goods sector, a final goods sector, a housing goods sector and a banking sector. The intermediate sector is comprised of three perfectly competitive industries: a building/construction sector, a manufacturing sector and a service sector. The output from these sectors are then combined to produce a residential investment good and a consumption good which can be consumed or used as investment. Entrepreneurs combine this latter good with a fixed factor (land) to produce housing; this last sector is where the lending channel and financial intermediation plays a role.

Turning first to the intermediate goods sector, the representative firm in each sector is characterized by the following Cobb-Douglas production function:

$$x_{it} = k_{it}^{\theta_i} (n_{it} \exp^{z_{it}})^{1-\theta_i}$$

where $i = b, m, s$ (building/construction, manufacture, service), k_{it}, n_{it},and z_{it} are capital, household labor, and a labor augmenting productivity shock respectively for each sector, with θ_i being the share of capital for sector i.[4] In our calibration we set $\theta_b < \theta_m$ reflecting the fact that the manufacturing sector is more capital intensive (or less labor intensive) than the construction sector.

The production shocks in each sector exhibit stochastic growth as given by:

$$z_i = t \ln g_{z,i} + \tilde{z}_i$$

The vector of technology shocks, $\vec{z} = (z_b, z_m, z_s)$, follows an $AR(1)$ process:

$$\vec{z}_{t+1} = B \cdot \vec{z}_t + \vec{\varepsilon}_{t+1}$$

The innovation vector $\vec{\varepsilon}$ is distributed normally with a given covariance matrix Σ_ε.[5]

These intermediate firms maximize a conventional static profit function at t

$$\max_{\{k_{it}, n_{it}\}} \left\{ \sum_i p_{it} x_{it} - r_t k_t - w_t n_t \right\}$$

subject to equations $k_t \geq \sum_i k_{it}, n_t \geq \sum_i n_{it}$, and non-negativity of inputs. The usual first order relations are characterized by

$$r_t k_{it} = \theta_i p_{it} x_{it}, \qquad w_t n_{it} = (1 - \theta_i) p_{it} x_{it}$$

[4] Real estate developers, i.e. entrepreneurs, also provide labor to the intermediate goods sectors. This is a technical consideration so that the net worth of entrepreneurs, including those that go bankrupt, is positive. Labor's share for entrepreneurs is set to a trivial number and has no effect on output dynamics. Hence, for expositional purposes, we ignore this factor in the presentation.

[5] In their analysis, Davis and Heathcote (2005) introduced a government sector characterized by non-stochastic tax rates and government expenditures and a balanced budget in every period. We abstract from these features in order to focus on time varying uncertainty and the credit channel. Our original model included these elements but it was determined that they did not have much influence on the policy functions that characterize equilibrium (although they clearly influence steady-state values).

where r_t, w_t, and p_{it} are the capital rental, wage, and output prices.

The intermediate goods are then used as inputs to produce two final goods, y_j, where $j = c, d$ (consumption/capital investment and residential investment respectively). This technology is also assumed to be Cobb-Douglas with constant returns to scale:

$$y_{jt} = \prod_{i=b,m,s} x1_{ijt}^{\rho_{ij}}, \quad j = c, d. \tag{1}$$

The input matrix is defined by

$$x1 = \begin{pmatrix} b_c & b_d \\ m_c & m_d \\ s_c & s_d \end{pmatrix},$$

and the shares of construction, manufactures and services for sector j are defined by the matrix

$$\rho = \begin{pmatrix} B_c & B_d \\ M_c & M_d \\ S_c & S_d \end{pmatrix}.$$

The relative shares of the three intermediate inputs differ in producing two final goods. For example, in the calibration of the model, we set $B_c < B_d$ to represent the fact that the residential investment is more construction input intensive.

The first degree homogeneity of the production process implies:

$$\sum_i \rho_{ij} = 1; \quad j = c, d$$

Market clearing in the intermediate goods markets requires

$$x_{it} = \sum_j x1_{ijt}; \quad i = b, m, s$$

With intermediate goods as inputs, the final goods' firms solve the following static profit maximization problem at t where the price of consumption good, p_{ct}, is normalized to 1:

$$\max_{x_{ijt}} \left\{ y_{ct} + p_{dt} y_{dt} - \sum_j \sum_i p_{it} x1_{ijt} \right\}$$

subject to the production functions (eq.(1)) and non-negativity of inputs.

The optimization problem then leads to the following relationships between the value of inputs and outputs:

$$p_{it} x1_{ijt} = \rho_{ij} p_{jt} y_{jt},$$

where

$$i = b, m, s; \quad j = c, d$$

Constant returns to scale implies zero profits in both sectors so we have the following relationships:

$$\sum_j p_{jt} y_{jt} = \sum_i p_{it} x_{it} = r_r k_t + w_t n_t$$

Finally, new housing structures, y_{ht}, are produced by entrepreneurs (i.e. real estate developers) using the residential investment good, y_{dt}, and land, x_{lt}, as inputs. For entrepreneur a, the production function is denoted $F\left(x_{alt}, y_{adt}\right)$ and is assumed to exhibit constant returns to scale. Specifically, we assume:

$$y_{aht} = \omega_{at} F\left(x_{alt}, y_{adt}\right) = \omega_{at} x_{alt}^{\zeta} y_{adt}^{1-\zeta}$$

where, ζ denotes the share of land. It is assumed that the aggregate quantity of land is fixed. The technology shock, ω_{at}, is an idiosyncratic shock affecting real estate developers. The technology shock is assumed to have a unitary mean and standard deviation of $\sigma_{\omega,t}$. The standard deviation,

$\sigma_{\omega,t}$, follows an $AR\,(1)$ process:

$$\sigma_{\omega,t+1} = \sigma_0^{1-\chi} \sigma_{\omega,t}^{\chi} \exp^{\varepsilon_{\sigma,t+1}} \tag{2}$$

with the steady-state value $\sigma_0, \chi \in (0,1)$ and $\varepsilon_{\sigma,t+1}$ is a white noise innovation.[6]

Each period, the production of new housing is added to the depreciated stock of existing housing units. Davis and Heathcote (2005) exploit the geometric depreciation structure of housing in order to define a stock of effective housing units, denoted h_t. Given the lack of aggregate uncertainty in new housing production, the law of motion for per-capita effective housing can be written as:

$$\eta h_{t+1} = x_{lt}^{\zeta} y_{dt}^{1-\zeta} \left(1 - ac_t\right) + \left(1 - \delta_h\right) h_t$$

where δ_h is the depreciation on effective housing units, η represents the population growth rate (the same for households and entrepreneurs), and ac_t represents the agency costs due to bankruptcy of a fraction of real estate developers.[7] The last factor is critical and is discussed in more detail in the discussion of the lending channel presented below (see eq. (3) below).

2.1.2 Households

The representative household derives utility each period from consumption, c_t, housing, h_t, and leisure, $1 - n_t$. Instantaneous utility for the household is defined by the Cobb-Douglas functional form of

$$U(c_t, h_t, 1 - n_t) = \frac{\left(c_t^{\mu_c} h_t^{\mu_h} \left(1 - n_t\right)^{1-\mu_c-\mu_h}\right)^{1-\sigma}}{1 - \sigma}$$

[6] This autoregressive process is used so that, when the model is log- linearized, $\hat{\sigma}_{\omega,t}$ (defined as the percentage deviations from $\bar{\sigma}_0$) follows a standard, mean-zero AR(1) process.

[7] Davis and Heathcote (2005) derive the law of motion for effective housing units (with no agency costs) and demonstrate that the depreciation rate δ_h is related to the depreciation rate of structures. As mentioned in the text, it is not necessary to keep track of the stock of housing structures as an additional state variable; the amount of effective housing units, h_t, is a sufficient statistic.

where μ_c and μ_h are the weights for consumption and housing in utility, and σ represents the coefficient of relative risk aversion. The household maximizes expected lifetime utility as given by:

$$E_0 \sum_{t=0}^{\infty} (\beta\eta)^t U(c_t, h_t, 1-n_t)$$

Each period agents combine labor income with income from assets (capital, housing, land and loans to the banking sector, denote b_t) and use these to purchase consumption, new housing and investment. These choices are represented by the budget constraint:

$$c_t + \eta k_{t+1} + \eta p_{ht} h_{t+1} = w_t n_t + (r_t + 1 - \delta_\kappa) k_t + (1-\delta_h) p_{ht} h_t + p_{lt} x_{lt} + (R_t - 1)\, b_t$$

where δ_k and δ_h are the capital and house depreciation rates respectively and R_t is the return on bank deposits.[8] Note that loans to the banking sector are intra-period loans and, because financial intermediation eliminates all idiosyncratic risk as discussed below, the equilibrium interest on these loans will be unity, i.e. $R_t = 1$.

The optimization problem leads to the following necessary conditions:

$$1 = \beta\eta E_t[(r_t + 1 - \delta_\kappa)\frac{U_1(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)}],$$

$$p_{ht} = \beta\eta E_t[\frac{U_2(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)} + (1-\delta_h) p_{ht+1} \frac{U_1(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)}],$$

$$w_t = \frac{U_3(c_t, h_t, 1-n_t)}{U_1(c_t, h_t, 1-n_t)}.$$

[8] Note that lower case variables for capital, labor and consumption represent per-capita quantities while upper case denote will denote aggregate quantities. Also, in addition to household's income from renting capital and providing labor, he also receives income from selling land to developers.

2.2 The Credit Channel

2.2.1 Housing Entrepreneurial Contract

The economy described above is identical to that studied in Davis and Heathcote (2005) except for the addition of productivity shocks affecting housing production.[9] We describe in more detail the nature of this sector and the role of the banking sector. It is assumed that a continuum of housing producing firms with unit mass are owned by risk-neutral entrepreneurs (developers). The costs of producing housing are financed via loans from risk-neutral intermediaries. Given the realization of the idiosyncratic shock to housing production, some real estate developers will not be able to satisfy their loan payments and will go bankrupt. The banks take over operations of these bankrupt firms but must pay an agency fee. These agency fees, therefore, affect the aggregate production of housing and, as shown below, imply an endogenous markup to housing prices. That is, since some housing output is lost to agency costs, the price of housing must be increased in order to cover factor costs.

The timing of events is critical:

1. The exogenous state vector of technology shocks and uncertainty shocks, denoted $(z_{i,t}, \sigma_{\omega,t})$, is realized.

2. Firms hire inputs of labor and capital from households and entrepreneurs and produce intermediate output via Cobb-Douglas production functions. These intermediate goods are then used to produce the two final outputs.

3. Households make their labor, consumption and savings/investment decisions.

4. With the savings resources from households, the banking sector provide loans to entrepreneurs via the optimal financial contract (described below). The contract is defined by the size of the loan (fp_{at}) and a cutoff level of productivity for the entrepreneurs' technology

[9] Also, as noted above, we abstract from growth and taxes.

shock, $\bar{\omega}_t$.

5. Entrepreneurs use their net worth and loans from the banking sector in order to purchase the factors for housing production. The quantity of factors (residential investment and land) is determined and paid for before the idiosyncratic technology shock is known.

6. The idiosyncratic technology shock of each entrepreneur is realized. If $\omega_{at} \geq \bar{\omega}_t$ the entrepreneur is solvent and the loan from the bank is repaid; otherwise the entrepreneur declares bankruptcy and production is monitored by the bank at a cost proportional to total factor payments.

7. Entrepreneurs that are solvent make consumption choices; these in part determine their net worth for the next period.

A schematic of the implied flows is presented in Figure 4.

Each period, entrepreneurs enter the period with net worth given by nw_{at}. Developers use this net worth and loans from the banking sector in order to purchase inputs. Letting fp_{at} denote the factor payments associated with developer a, we have:

$$fp_{at} = p_{dt}y_{adt} + p_{lt}x_{alt}$$

Hence, the size of the loan is $(fp_{at} - nw_{at})$. The realization of ω_{at} is privately observed by each entrepreneur – banks can observe the realization at a cost that is proportional to the total input bill. Letting μ denote the proportionality factor, the cost is therefore given by μfp_{at}.

With a positive net worth, the entrepreneur borrows $(fp_{at} - nw_{at})$ consumption goods and agrees to pay back $\left(1 + r^L\right)(fp_{at} - nw_{at})$ capital goods to the lender, where r^L is the interest rate on loans. Thus, the entrepreneur defaults on the loan if his realization of output is less then the re-payment, i.e.

$$\omega_t < \frac{\left(1 + r^L\right)(fp_{at} - nw_{at})}{fp_{at}} \equiv \bar{\omega}_t$$

i.e. the critical idiosyncratic technology shock $\bar{\omega}_t$ corresponds to the level of entrepreneurial's default level.

Before determining the optimal debt contract, it is convenient, as shown by Carlstrom and Fuerst (1997) to first define two functions which represent, respectively, the expected shares of housing production that go to real estate developer and lender, respectively. Denoting the *c.d.f.* and *p.d.f.* of ω_t as $\Phi(\omega_t;\sigma_{\omega,t})$ and $\phi(\omega_t;\sigma_{\omega,t})$,these are defined as:[10]

$$f(\bar{\omega}_t;\sigma_{\omega,t}) = \int_{\bar{\omega}_t}^{\infty} \omega\phi(\omega;\sigma_{\omega,t})\, d\omega - [1-\Phi(\bar{\omega}_t;\sigma_{\omega,t})]\,\bar{\omega}_t$$

and

$$g(\bar{\omega}_t;\sigma_{\omega,t}) = \int_{-\infty}^{\bar{\omega}_t} \omega\phi(\omega;\sigma_{\omega,t})\, d\omega + [1-\Phi(\bar{\omega}_t;\sigma_{\omega,t})]\,\bar{\omega}_t - \Phi(\bar{\omega}_t;\sigma_{\omega,t})\,\mu$$

Note that these two functions sum to:

$$f(\bar{\omega}_t;\sigma_{\omega,t}) + g(\bar{\omega}_t;\sigma_{\omega,t}) = 1 - \Phi(\bar{\omega}_t;\sigma_{\omega,t})\,\mu \tag{3}$$

Hence, the term $\Phi(\bar{\omega}_t;\sigma_{\omega,t})\,\mu$ captures the loss of housing due to the agency costs associated with bankruptcy. Note that that loss of output due to agency costs combined with the constant returns to scale production function implies that the value of housing output must exhibit a markup over factor costs. Denote this markup as $\bar{s}_t > 1$ which is taken as parametric for both lender and real estate developer. The optimal borrowing contract is defined by the pair $(fp_{at}, \bar{\omega}_t)$ that maximizes the entrepreneur's return subject to the lender's willingness to participate (all rents go to the entrepreneur).That is, the optimal contract is determined by the solution to:

$$\max_{\bar{\omega}_t, fp_{at}} \bar{s}_t fp_{at} f(\bar{\omega}_t;\sigma_{\omega,t}) \text{ subject to } \bar{s}_t fp_{at} g(\bar{\omega}_t;\sigma_{\omega,t}) \geqslant fp_{at} - nw_{at}$$

[10] The notation $\Phi(\omega;\sigma_{\omega,t})$ is used to denote that the distribution function is time-varying as determined by the realization of the random variable, $\sigma_{\omega,t}$. For expositional purposes, we suppress the time notation on the markup and net worth since these are treated as parameters in this section.

The necessary conditions for the optimal contract problem are

$$\frac{\partial\left(.\right)}{\partial\bar{\omega}_t}:\bar{s}_t fp_{at}\frac{\partial f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{\partial\bar{\omega}_t}=-\lambda_t\bar{s}_t fp_{at}\frac{\partial g\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{\partial\bar{\omega}_t}$$

where λ_t is the shadow price of the entrepreneur's resources. Using the definitions of $f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)$ and $g\left(\bar{\omega}_t;\sigma_{\omega,t}\right)$, this can be rewritten as:[11]

$$1-\frac{1}{\lambda_t}=\frac{\phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{1-\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}\mu \tag{4}$$

As shown by eq.(4), the shadow price of the resources used in lending is an increasing function of the relevant Inverse Mill's ratio (interpreted as the conditional probability of bankruptcy) and the agency costs. If the product of these terms equals zero, then the shadow price equals the cost of capital production, i.e. $\lambda_t=1$.

The second necessary condition is:

$$\frac{\partial\left(.\right)}{\partial fp_{at}}:\bar{s}_t f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)=-\lambda_t\left[1-\bar{s}_t g\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\right]$$

These first-order conditions imply that, in general equilibrium, the markup factor, $\bar{s}_t$, will be endogenously determined and related to the probability of bankruptcy. Specifically, using the first order conditions, we have that the markup, $\bar{s}_t$, must satisfy:

$$\begin{aligned}\bar{s}_t^{-1} &= \left[\left(f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)+g\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\right)+\frac{\phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\mu f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{\frac{\partial f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{\partial\bar{\omega}_t}}\right] \\ &= \left[\underbrace{1-\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\mu}_{A}-\underbrace{\frac{\phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{1-\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}\mu f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}_{B}\right]\end{aligned} \tag{5}$$

First note that the markup factor depends only on economy-wide variables so that the aggregate

[11] Note that we have used the fact that $\frac{\partial f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{\partial\bar{\omega}_t}=\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)-1<0$

house price will be well defined. Also, the two terms, A and B, demonstrate that the markup factor is affected by both the total agency costs (term A) and the marginal effect that bankruptcy has on the entrepreneur's expected return. That is, term B reflects the loss of housing output, μ, weighted by the expected share that would go to entrepreneur's, $f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)$, and the conditional probability of bankruptcy (the Inverse Mill's ratio). Finally, note that, in the absence of credit market frictions, there is no markup. It is straightforward to show that equation (5) defines an implicit function $\bar{\omega}\left(\bar{s}_t, \sigma_{\omega,t}\right)$ that is increasing in $\bar{s}_t$.

The incentive compatibility constraint implies

$$fp_{at} = \frac{1}{\left(1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)\right)} nw_{at} \tag{6}$$

Equation (6) implies that the size of the loan is linear in entrepreneur's net worth so that aggregate lending is well-defined and a function of aggregate net worth.

The effect of an increase in uncertainty on lending can be understood in a partial equilibrium setting where $\bar{s}_t$ and nw_{at} are treated as parameters. As shown by eq. (5), the assumption that the price of capital is unchanged implies that the costs of default, represented in the function $D\left(\bar{\omega}_t, \sigma_{\omega,t}\right)$, must also be unchanged. With a mean-preserving spread in the distribution for ω_t, this means that $\bar{\omega}_t$ will fall (driven primarily by the term A). As a consequence, the lenders' expected return, $g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)$, will also fall since $g\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \approx \bar{\omega}_t$.[12] Given the binding incentive compatibility constraint

$$\bar{s}_t fp_{at} g\left(\bar{\omega}_t; \sigma_{\omega,t}\right) = fp_{at} - nw_{at} \tag{7}$$

the fall in the left-hand side induces a fall in fp_{at}. Hence, greater uncertainty results in a fall in housing production. This partial equilibrium result carries over to the general equilibrium setting.

The existence of the markup factor implies that inputs will be paid less than their marginal products. In particular, profit maximization in the housing development sector implies the fol-

[12] The proof of this approximation can be provided upon request.

lowing necessary conditions:

$$\frac{p_{lt}}{p_{ht}} = \frac{F_{xl}\left(x_{lt}, y_{dt}\right)}{\bar{s}_t}$$

$$\frac{p_{dt}}{p_{ht}} = \frac{F_{yd}\left(x_{lt}, y_{dt}\right)}{\bar{s}_t}$$

These expressions demonstrate that, in equilibrium, the endogenous markup (determined by the agency costs) will be a determinant of housing prices. The production of new housing net of agency costs is denoted $y_h = x_{lt}^{\zeta} y_{dt}^{1-\zeta}\left[1 - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right)\mu\right].$

2.2.2 Housing Entrepreneurial Consumption and House Prices

To rule out self-financing by the entrepreneur (i.e. which would eliminate the presence of agency costs), it is assumed that the entrepreneur discounts the future at a faster rate than the household. This is represented by following expected utility function:

$$E_0 \sum_{t=0}^{\infty} \left(\beta\eta\gamma\right)^t c_t^e$$

where c_t^e denotes entrepreneur's consumption at date t, and $\gamma \in (0,1)$. This new parameter, γ, will be chosen so that it offsets the steady-state internal rate of return to entrepreneurs' house production.

Each period, entrepreneur's net worth, nw_t is determined by the value of capital income and the remaining capital stock. Denoting entrepreneur's capital as k_t^e, this implies:[13]

$$nw_t = k_t^e\left[r_t + 1 - \delta_\kappa\right]$$

The law of motion for entrepreneurial capital stock is determined in two steps. First, new

[13] For expositional purposes, in this section we drop the subscript k denoting the individual entrepreneur.

capital is financed by the entrepreneur's value of housing output after subtracting his consumption:

$$\eta k_{t+1}^{e} = \bar{s}_t f p_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) - c_t^e$$

Then, using the incentive compatibility constraint, eq. (7), and the definition of net worth, this can be written as:

$$\eta k_{t+1}^{e} = k_t^e \frac{\left(r_t + 1 - \delta_\kappa\right)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} \bar{s}_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) - c_t^e$$

The term $\bar{s}_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) / \left(1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)\right)$ represents the entrepreneur's internal rate of return to housing output. Or, alternatively, it reflects the leverage enjoyed by the entrepreneur. Multiplying numerator and denominator by nw_t and again using the incentive compatibility constraint we have:

$$\frac{\bar{s}_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} = \frac{\bar{s}_t f p_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{nw_t}$$

That is, entrepreneurs use their net worth to finance factor inputs of value fp_t,this produces housing which sells at the markup $\bar{s}_t$ with entrepreneur's retaining fraction $f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)$ of the housing output. Given this setting, the optimal path of consumption implies the following Euler equation:

$$1 = \beta \eta \gamma E_t \left[\left(r_{t+1} + 1 - \delta_\kappa\right) \frac{\bar{s}_{t+1} \; f\left(\bar{\omega}_{t+1}; \sigma_{\omega,t+1}\right)}{1 - \bar{s}_{t+1} g\left(\bar{\omega}_{t+1}; \sigma_{\omega,t+1}\right)} \right]$$

Finally, we can derive an explicit relationship between entrepreneur's capital and the value of the housing stock using the incentive compatibility constraint and the fact that housing sells at a markup over the value of factor inputs. That is, since $p_{ht} F\left(x_{alt}, y_{adt}\right) = \bar{s}_t f p_t$, the incentive compatibility constraint implies:

$$p_{ht}\left(x_{lt}^{\zeta} y_{dt}^{1-\zeta}\right) = k_t^e \frac{\left(r_t + 1 - \delta_\kappa\right)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} \bar{s}_t$$

Again, it is important to note that the markup parameter plays a key role in determining housing prices and output.

2.2.3 Financial Intermediaries

The Capital Mutual Funds (CMFs) act as risk-neutral financial intermediaries who earn no profit and produce neither consumption nor capital goods. There is a clear role for the CMF in this economy since, through pooling, all aggregate uncertainty of capital (house) production can be eliminated. The CMF receives capital from three sources: entrepreneurs sell undepreciated capital in advance of the loan, after the loan, the CMF receives the newly created capital through loan repayment and through monitoring of insolvent firms, and, finally, those entrepreneur's that are still solvent, sell some of their capital to the CMF to finance current period consumption. This capital is then sold at the price of $\bar{s}_t$ units of consumption to households for their investment plans.

3 Equilibrium

Prior to solving for equilibrium, it is necessary to express the growing economy in stationary form. Given that preferences and technologies are Cobb-Douglas, the economy will have a balanced growth path. Hence, it is possible to transform all variables by the appropriate growth factor. As discussed in Davis and Heathcote (2005), the output value of all markets (e.g. $p_d y_d, y_c, p_i x_i$ for $i = (b, m, s)$ are growing at the same rate as capital and consumption, g_k. This growth rate, in turn, is a geometric average of the growth rates in the intermediate sectors: $g_k = g_{zb}^{B_c} g_{zm}^{M_c} g_{zs}^{S_c}$. It is also the case that factor prices display the normal behavior along a balanced growth path: interest rates are stationary while the wage in all sectors is growing at the same rate. The growth rates for the various factors are presented in the following table (again see Davis and Heathcote (2005) for details):

Table 1: Growth Rates on the Balanced Growth Path

n_b, n_m, n_s, n, r	1
$k_b, k_m, k_s, k, c, y_c, w$	$g_k = \left[g_{zb}^{B_c(1-\theta_b)} g_{zm}^{M_c(1-\theta_m)} g_{zs}^{S_c(1-\theta_s)} \right]^{(1/(1-B_c\theta_b-M_c\theta_m-S_c\theta_s))}$
b_c, b_d, x_b	$g_b = g_k^{\theta_b} g_{zb}^{1-\theta_b}$
m_c, m_d, x_m	$g_m = g_k^{\theta_m} g_{zm}^{1-\theta_m}$
s_c, s_d, x_s	$g_s = g_k^{\theta_s} g_{zs}^{1-\theta_s}$
y_d	$g_d = g_b^{B_h} g_m^{M_h} g_s^{S_h}$
x_l	$g_l = \eta^{-1}$
y_h, h	$g_h = g_l^{\zeta} g_d^{1-\zeta}$
$p_h y_h, p_d x_d, p_l x_l, p_b x_b, p_m x_m, p_s x_s$	g_k

These growth factors were used to construct a stationary economy; all subsequent discussion is in terms of this transformed economy.

Equilibrium in the economy is described the vector of factor prices (w_t, r_t), the vector of intermediate goods prices, (p_{bt}, p_{mt}, p_{st}), the price of residential investment (p_{dt}), the price of land (p_{lt}), the price of housing (p_{ht}), and the markup factor $(\bar{s}_t)$. In total, therefore, there are nine equilibrium prices. In addition, the following quantities are determined in equilibrium: the vector of intermediate goods (x_{mt}, x_{bt}, x_{st}), the vector of labor inputs (n_{mt}, n_{bt}, n_{st}), the total amount of labor supplied, (n_t) ,the vector of inputs into the final goods sectors $(b_{ct}, b_{dt}, m_{ct}, m_{dt}, s_{ct}, s_{dt})$, the vector of capital inputs (k_{mt}, k_{bt}, k_{st}), entrepreneurial capital (k_t^e), household investment (k_{t+1}), the vector of final goods output (y_{ct}, y_{dt}) ,the technology cutoff level $(\bar{\omega}_t)$, the effective housing stock (h_{t+1}), and the consumption of households and entrepreneurs (c_t, c_t^e). In total, there are 24 quantities to be determined; adding the nine prices, the system is defined by 33 unknowns.

These are determined by the following conditions:

Factor demand optimality in the intermediate goods markets

$$r_t = \theta_i \frac{p_{it} x_{it}}{k_{it}} \quad (3 \; equations)$$

$$w_t = (1 - \theta_i) \frac{p_{it} x_{it}}{n_{it}} \quad (3 \; equations)$$

Factor demand optimality in the final goods sector:

$$p_{ct}y_{ct} = \frac{p_{bt}b_{ct}}{B_c} = \frac{p_{mt}m_{ct}}{M_c} = \frac{p_{st}s_{ct}}{S_{\ c}} \quad (3\ equations)$$

$$p_{dt}y_{dt} = \frac{p_{bt}b_{dt}}{B_d} = \frac{p_{mt}m_{dt}}{M_d} = \frac{p_{st}s_{dt}}{S_{\ d}} \quad (3\ equations)$$

Factor demand in the housing sector (using the fact that, in equilibrium $x_{lt} = 1$) produces two more equations:

$$\frac{p_{lt}}{p_{ht}} = \frac{\zeta y_{dt}^{1-\zeta}}{\bar{s}_t}$$

$$\frac{p_{dt}}{p_{ht}} = \frac{(1-\zeta)\, y_{dt}^{-\zeta}}{\bar{s}_t}$$

The household's necessary conditions provide 3 more equations:

$$1 = \beta\eta E_t[(r_t + 1 - \delta_\kappa)\frac{U_1(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)}],$$

$$p_{ht} = \beta\eta E_t[\frac{U_2(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)} + (1-\delta_h)p_{ht+1}\frac{U_1(c_{t+1}, h_{t+1}, 1-n_{t+1})}{U_1(c_t, h_t, 1-n_t)}],$$

$$w_t = \frac{U_3(c_t, h_t, 1-n_t)}{U_1(c_t, h_t, 1-n_t)}.$$

The financial contract provides the condition for the markup and the incentive compatibility constraint:

$$\bar{s}_t^{-1} = \left[(f(\bar{\omega}_t; \sigma_{\omega,t}) + g(\bar{\omega}_t; \sigma_{\omega,t})) + \frac{\phi(\bar{\omega}_t; \sigma_{\omega,t})\, \mu f(\bar{\omega}_t; \sigma_{\omega,t})}{\frac{\partial f(\bar{\omega}_t; \sigma_{\omega,t})}{\partial \bar{\omega}_t}} \right]$$

$$p_{ht}y_{dt}^{1-\zeta} = k_t^e \frac{(r_t + 1 - \delta_\kappa)}{1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t})}\bar{s}_t$$

The entrepreneur's maximization problem provides the following Euler equation:

$$1 = \beta\eta\gamma E_t \left[(r_{t+1} + 1 - \delta_\kappa)\frac{\bar{s}_{t+1}\ f(\bar{\omega}_{t+1}; \sigma_{\omega,t+1})}{1 - \bar{s}_{t+1}g(\bar{\omega}_{t+1}; \sigma_{\omega,t+1})} \right]$$

To these optimality conditions, we have the following market clearing conditions:

Labor market clearing:

$$n_t = \sum_i n_{it}, \quad i = b, m, s$$

Market clearing for capital:

$$k_t = \sum_i k_{it}, \qquad i = b, m, s$$

Market clearing for intermediate goods:

$$x_{bt} = b_{ct} + b_{dt}, \quad x_{mt} = m_{ct} + m_{dt}, \quad x_{st} = s_{ct} + s_{dt}.$$

The aggregate resource constraint for the consumption final goods sector (i.e. the law of motion for capital)

$$\eta k_{t+1} = (1 - \delta_\kappa) k_t + y_{ct} - c_t - c_t^e$$

The law of motion for the effective housing units:

$$\eta h_{t+1} = (1 - \delta_h) h_t + y_{dt}^{1-\zeta} \left(1 - \Phi\left(\bar{\omega}_t\right) \mu\right)$$

The law of motion for entrepreneur's capital stock:

$$\eta k_{t+1}^e = k_t^e \frac{(r_t + 1 - \delta_\kappa)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} \bar{s}_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) - c_t^e$$

Finally, we have the production functions. Specifically, for the intermediate goods markets:

$$x_{it} = k_{it}^{\theta_i} (n_{it} \exp^{z_{it}})^{1-\theta_i}; \quad i = b, m, s$$

For the final goods sectors, we have:

$$y_{ct} = b_{ct}^{B_c} m_{ct}^{M_c} s_{ct}^{S_c}$$

$$y_{dt} = b_{dt}^{B_d} m_{dt}^{M_d} s_{dt}^{S_d}$$

These provide the required 33 equations to solve for equilibrium. In addition there are the laws of motion for the technology shocks and the uncertainty shocks.

$$\vec{z}_{t+1} = B \cdot \vec{z}_t + \vec{\varepsilon}_{t+1}$$

$$\sigma_{\omega,t+1} = \sigma_0^{1-\chi} \sigma_{\omega,t}^{\chi} \exp^{\varepsilon_{\sigma,t+1}}$$

To solve the model, we log linearize around the steady-state. The solution is defined by 33 equations in which the endogenous variables are expressed as linear functions of the vector of state variables $(z_{bt}, z_{mt}, z_{st}, \sigma_{\omega t}, k_t, k_t^e, h_t)$.

4 Calibration and Data

A strong motivation for using the Davis and Heathcote (2005) model is that the theoretical constructs have empirical counterparts. Hence, the model parameters can be calibrated to the data. We use directly the parameter values chosen by the previous authors; readers are directed to their paper for an explanation of their calibration methodology. Parameter values for preferences, depreciation rates, population growth and land's share are presented in Table 2.

In addition, the parameters for the intermediate production technologies are presented in Table 3.

Table 2: Key Preference and Production Parameters

Depreciation rate for capital: δ_κ	0.056
Depreciation rate for effective housing (h): δ_h	0.014
Land's share in new housing: ζ	0.106
Population growth rate: η	1.017
Discount factor: β	0.951
Risk aversion: σ	2.00
Consumption's share in utility: μ_c	0.314
Housing's share in utility: μ_h	0.044
Leisure's share in utility: 1-$\mu_c - \mu_h$	0.642

Table 3: Intermediate Production Technology Parameters

	B	M	S
Input shares for consumption/investment good (B_c, M_c, S_c)	0.031	0.270	0.700
Input shares for residential investment (B_d, M_d, S_d)	0.470	0.238	0.292
Capital's share in each sector $(\theta_b, \theta_m, \theta_s)$	0.132	0.309	0.237
Sectoral trend productivity growth (%) (g_{zb}, g_{zm}, g_{zs})	-0.27	2.85	1.65

For the financial sector, we use the parameter values employed originally by Carlstrom and Fuerst (1997). In particular, the average spread between the prime and commercial paper rates is used to define the average risk premium (rp) associated with loans to entrepreneurs as defined in Carlstrom and Fuerst (1997). The bankruptcy rate (br) is given by $\Phi(\varpi, \sigma_\omega)$. We use the values from Carlstrom and Fuerst (1997): $br = 0.974\%$, $rp = 1.87\%$. In order to match Davis and Heathcote (2005) annual model, we adjust the following financial parameter accordingly

$$\begin{aligned} \Phi(\varpi, \sigma_\omega) &= 4br, \\ \frac{\varpi}{g(\varpi, \sigma_\omega)} - 1 &= rp, \end{aligned} \tag{8}$$

yielding $\varpi \approx -0.43$, $\sigma_\omega \approx 0.23$. Note that the risk premium can be derived from the markup share of the realized output and the amount of payment on borrowing:

$$\bar{s}_t \bar{\omega}_t f p_t = (1 + rp)(f p_t - n w_t).$$

And using the optimal factor payment (project investment), fp_t, in equation (6), we arrive at the risk premium equation in (8).

Finally, the entrepreneurial discount factor γ can be recovered by the condition that the internal rate of return to entrepreneur is offset by their additional discount factor:

$$\gamma \left[\frac{\bar{s}_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} \right] = 1$$

and using the mark-up equation for $\bar{s}_t$ in (5), the parameter γ then satisfies the relation

$$\gamma = \frac{g_U}{g_K} \left[1 + \frac{\phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{f'\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} \right] \approx 0.832$$

where, g_U and g_K are the growth rate of labor.[14]

Figure 1 and Table 4 show these cyclical and statistical features for the period from 1975 through the second quarter of 2007. The U.S. business cycle properties for various macro and housing variables are listed in table (4). As mentioned in the introduction, the listed housing stylized facts can be clearly seen in table (4): i) the house prices are much more volatile than output; ii) Residential investment is almost twice as more volatile than non-residential investment; iii) GDP, consumption, house price, non-residential - and residential investment all co-move positively; iv) and lastly, residential investment leads output by three quarters.

5 Results

5.1 Steady State Values, Second Moments and Lead - Lag Patterns

Table 5 shows some of the selected steady-state values from our model with financial friction. These steady state values differ a little from those without the friction. Nevertheless, as in Davis

[14] To derive the stationary model, all variables have thus to be detrended and written in terms of variables that are constant in the steady state.

Table 4: Business Cycle Properties (1975:1 - 2007:2)

Data: all series are Hodrick-Prescott filtered with the smoothing parameter set to 1600			
% S.D.			
GDP	1.2		
Consumption	0.69		
House Price Index (HPI)	1.9		
Non - Residential Fixed Investment (Non-Res)	4.5		
Residential Fixed Investment (Res)	8.7		
Correlations			
GDP, Consumption	0.83		
GDP, HPI	0.31		
GDP, HPI (for pre 1990)	0.21		
GDP, HPI (for post 1990)	0.51		
Non-Res, Res	0.29		
GDP, Non-Res	0.81		
GDP, Res	0.30		
GDP, Real Estate Loans (from 1985:1)	0.15		
Real Estate Loans, HPI	0.47		
Lead - Lag correlations	$i = -3$	$i = 0$	$i = 3$
$GDP_t, Non - res_{t-i}$	0.47	0.78	0.31
GDP_t, res_{t-i}	-0.27	0.20	0.32
$Non - res_{t-i}, res_t$	0.63	0.26	-0.27

Table 5: Steady - State Values: Ratios to GDP

Variables	Our model	D & H	Data (1948 - 2001)
Capital Stock (K)	1.96	1.52	1.52
Residential structures stock($P_d \times S$)	1.14	1	1
Private consumption (PCE)	0.767	0.639	0.638
Nonresidential investment (i_c)	0.183	0.139	0.135
Residential investment (i_d)	0.049	0.044	0.047
Construction ($b = p_b x_b$)	0.050	0.048	0.052
Manufacturing ($m = p_m x_m$)	0.242	0.247	0.328
Services ($p_s x_s + qh$)	0.708	0.706	0.615

and Heathcote (2005), our first moments (steady state values) match quite successfully to the data.

As shown in Table 6, various second moments from our model can also match the data relatively well. In particular, an increase in the volatility of σ_ω (uncertainty), we match the first housing stylized fact mentioned above: house prices being more volatile than output.

The standard deviations are generated in the same way as in Davis and Heathcote (2005). The volatility of σ_ω is taken low as 15% and high as 85%, i.e. $\varepsilon_\sigma \sim N(0, 0.15), \varepsilon_\sigma \sim N(0, 0.5)$ and $\varepsilon_\sigma \sim N(0, 85)$.[15] Table 6 shows the standard deviations with three volatility value of σ_ω for variables in question. When the variance of uncertainty (ε_σ) is low (15%), our financial friction model delivers essentially the same results as Davis and Heathcote (2005). As the volatility increases to 50% ($\varepsilon_\sigma \sim N(0, 0.5)$), the standard deviations for almost all variables (except for construction labor) increase. In particular, the standard deviation for house price increases from 0.45 to 0.875. The down side of this housing price volatility increase is that the standard deviation of non-residential to residential investment is too big in comparison to the data: $\frac{13.6}{0.875}$ to $\frac{5.04}{2.3}$. As we increase the volatility of uncertainty to 85%, we correctly match the housing price volatility. But this match, as in the case for the 50%, comes with the fact that the residential investment is too volatile in relation to non-residential investment.

Another set of second moments that are in question is the correlations. Table 7 shows the

[15] Due to the linearity of the variables between house price, ϖ, and σ_ω , we can always match the data for house price volatility if we increase volatile of σ_ω (e.g. $\varepsilon_\sigma \sim N(0, 0.85)$).

Table 6: Standard Deviations in ratio to GDP

Variables	Our model			D & H	Data
					(1948 - 2001)
Volatility of σ_ω	15%	50%	85%		
Output (GDP)	1.79	1.82	1.84	1.73	2.26
Private consumption (PCE)	0.544	0.562	0.588	0.48	0.78
Labor (N)	0.415	0.46	0.526	0.41	1.01
Nonresidential investment (i_c)	3.31	4.06	5.24	3.21	2.3
Residential investment (i_d)	5.42	13.6	**22.3**	6.12	5.04
House price (p_h)	0.45	0.875	**1.37**	0.4	1.37
Construction output (x_b)	3.82	6.79	10.4	4.02	2.74
Manufacturing output (x_m)	1.61	0.412	1.61	1.58	1.85
Service output (x_s)	1.04	1.07	1.13	0.99	0.85
Construction labor (n_b)	2.39	6.07	9.93	2.15	2.32
Manufacturing (n_m)	0.403	0.412	0.419	0.39	1.53
Service (n_s)	0.403	0.484	0.614	0.37	0.66
Construction Investment (i_b)	2.68	6.21	10	25.9	9.69
Manufacturing Investment (i_m)	1.11	1.12	1.12	3.23	3.53
Service Investment (i_b)	1.1	1.13	1.19	3.43	2.35

Table 7: Correlations

Variables $\varepsilon_\sigma \sim N(0,15)$	Our Model	D & H	Data (1948 - 2001)
Correlations			
(GDP, PCE)	0.96	0.95	0.8
(GDP, p_h)	0.60	0.65	0.65
(i_c, PCE)	0.9	0.91	0.61
(i_d, PCE)	0.25	0.26	0.66
(i_c, i_d)	0.01	0.15	0.25
(i_d, p_h)	-0.28	-0.2	0.34

correlations results for the case when $\varepsilon_\sigma \sim N(0,15)$. All variables co-move positively with the exception of house price and residential investment.

The last set of housing stylized facts that is in question is the lead - lag patterns of residential and non-residential investments. Table 8 shows the results. As in Davis and Heathcote (2005), we also fail to reproduce this feature of the data. Consequently, the propagation mechanism of agency costs model does amplify prices and other real variables, but does not contribute in explaining the lead-lag features.

Table 8: Lead - Lag Patterns: Annual Frequency

Variables $\varepsilon_\sigma \sim N(0,15)$	Our Model	D & H	Data (1948 - 2001)
$(i_c[-1], GDP[0])$	0.481	0.45	0.25
$(i_c[0], GDP[0])$	0.921	0.94	0.75
$(i_c[1], GDP[0])$	0.247	0.33	0.48
$(i_d[-1], GDP[0])$	0.113	0.19	0.52
$(i_d[0], GDP[0])$	0.358	0.44	0.47
$(i_d[1], GDP[0])$	0.345	0.14	-0.22
$(i_c[-1], i_d[0])$	0.287	0.07	-0.37
$(i_c[0], i_d[0])$	0.013	0.15	0.25
$(i_c[1], i_d[0])$	0.001	0.08	0.53

5.2 Dynamics: Impulse Response Functions

Equations in Section 3 determine the equilibrium properties of the economy. To analyze the cyclical properties of the economy, we linearize (i.e. take a first-order Taylor series expansion) of these equations around the steady-state values and express all terms as percentage deviations from steady-state values. This numerical approximation method is standard in quantitative macroeconomics. What is not standard in this model is that the second moment of technology shocks hitting the housing production sector will influence equilibrium behavior and, therefore, the equilibrium policy rules. That is, linearizing the equilibrium conditions around the steady-state typically imposes certainty equivalence so that variances do not matter. In this model, however, the variance of the technology shock can be treated as an additional state variable through its role in determining lending activities and, in particular, the nature of the lending contract.[16] Linearizing the system of equilibrium conditions does not eliminate that role in this economy and, hence, we think that this is an attractive feature of the model.

We depart from Carlstrom and Fuerst (1997) by relaxing the *i.i.d.*: assumption for the housing sector technology shock. This is reflected in the law of motion for the standard deviation of the

[16] Specifically, ω_t is assumed to be log normally distributed. Hence, the linear approximation to the equations describing the financial contract will include the second moment of ω_t.

technology shock which is given in eq. (2); for convenience this is rewritten below:

$$\ln \frac{\sigma_{\omega,t+1}}{\sigma_0} = \chi \ln \frac{\sigma_{\omega,t}}{\sigma_0} + \varepsilon_{\sigma,t+1}$$

As in Carlstrom and Fuerst (1997), the standard deviation of the technology shock ω_t is, on average, equal to 0.207. That is, we set $\overline{\sigma}_\omega = 0.207$. The behavior of our model economy is analyzed by examining the impulse response functions of several key variables to a 1% innovation in the technology shocks to construction sector, z_b; and in uncertainty shocks, $\sigma_{\omega,t}$. These are presented in Figures 6-8. As the impulse response functions to 1% innovation in the technology shocks to construction sector, z_b; delivers the usual dynamics, we will only focus on the effects of uncertainty shocks.

We first turn to aggregate output and household consumption and investment. With greater uncertainty, the bankruptcy rate increases in the economy (this is verified in Figure 7), which implies that agency costs increase. The rate of return on investment for the economy therefore falls. Households, in response, reduce investment and increase consumption and leisure. The latter response causes output to fall. This is not the case, however, for investment - this is due to the increase in the price of capital (see Figure 7) and reflects the behavior of entrepreneurs. This behavior is understood after first examining the lending channel.

The increase in uncertainty affects, predictably, all three key variables in the lending channel: the price of capital, the risk premium associated with loans and the bankruptcy rate. As already mentioned, the bankruptcy rate increases. This result implies that the bankruptcy rate is countercyclical in this economy; in contrast, in the analysis by Carlstrom and Fuerst (1997) the bankruptcy rate was, counterfactually, procyclical.[17] Their focus was on the effects of innovation to the aggregate technology shock and, because of the assumed persistence in this shock, is driven

[17] In the Carlstrom and Fuerst (1997) model, a technology shock increases output and the demand for capital. The resulting increase in the price of capital implies greater lending activity and, hence, an increase in the bankruptcy rate (and risk premia). Here, greater uncertainty results in greater bankruptcy rates even though investment falls; since labor is also reduced, this produces countercyclical bankruptcy rates and risk premia.

by the change in the first moment of the aggregate production shock. Our analysis demonstrates that second moment effects may play a significant role in these correlations over the business cycle. Further research, both empirical and theoretical, in this area would be fruitful . Returning to the model, the increased bankruptcy rate implies that the price of capital is greater and this increase lasts longer in the high persistence economy. The same is true for the risk premium on loans.

Figure 8 reports the consumption and net worth of entrepreneurs in the economies. Entrepreneurs exploit the high price of capital to increase consumption: the lack persistence (i.e. one time shock) provides no incentive to increase investment. Since the price of capital quickly returns to its steady-state values, the increased consumption erodes entrepreneur's 'net worth. To restore net worth to its steady-state value, consumption falls temporarily. The magnitude of the effects of two different shocks, productivity shocks (z_{bt}) or uncertainty shocks (σ_t) ; on the aggregate variables suggests that shocks to uncertainty cannot be the dominant shock in the economy, but at the same time, they are not negligible as suggested in previous literature. Dorofeenko, Lee and Salyer (2008) finds that the uncertainty shocks, although qualitatively important, have quantitatively very small role in explaining the aggregate variables. In their paper, the effects of productive shocks dominate the effects of uncertainty shocks in the order of 100 times. In this paper, the order of effect is in the range of 2 to 10 times.

5.3 Some Final Remarks

The paper sets out to achieve two goals. First, we want to provide an adequate framework for analyzing current unfolding financial crisis: a framework where time varying uncertainty has nice qualitative features (e.g. an increase in uncertainty results in a fall in aggregate investment) and implies large quantitative exeffects on housing and business cycle properties. Second, we seek to provide a quantitative framework that could account for well known housing stylized facts.

Our primary findings fall into two broad categories. First, the shocks to the housing producing sector imply a quantitatively big role for uncertainty over the housing and business cycle. Second, our model can account for most of the salient features of housing stylized facts, in particular, the housing price is more volatile than output. The lead - lag pattern of residential and non-residential, however, is still not reconciled.

For future research, modelling uncertainty due to time variation in the types of entrepreneurs would be fruitful: Specifically, an analysis of two types of agents; a low risk agent whose productivity shocks exhibit low variance and a high risk agent with a high variance of productivity shocks. Because of restrictions on the types of financial contracts that can be offered, the equilibrium is a pooling equilibrium so that the same type of financial contract is offered to both types of agents. Hence the aggregate distribution for technology shocks hitting the entrepreneurial sector is a mixture of the underlying distributions for each type of agent. Our conjecture is that this form of uncertainty has important quantitative predictions and, hence, could be an important impulse mechanism in the credit channel literature that, heretofore, has been overlooked. It also anecdotally corresponds with explanations for the cause of the current credit crisis: a substantial fraction of mortgage borrowers had higher risk characteristics than originally thought.

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Data Appendix

- Loans: Federal Reserve Board, Statistics: Releases and Historical DataAssets and Liabilities of Commercial Banks in the U.S. - h.8. Seasonally adjusted, adjusted for mergers, billions of dollars. http://www.federalreserve.gov/releases/h8/data.htm

    1. Total Loans: Total loans and leases at commercial banks.
    2. Residential RealEstate Loans: Loans to residential sector excluding revolving home equity loans.
    3. Commercial Real Estate Loans: Loans to commercial sector excluding revolving home equity loans.
    4. Commercial and Industrial Loans (Business Loans): Commercial and industrial loans at all Commercial Banks.
    5. Consumer Loans: Consumer (Individual) loans at all commercial banks.

- Gross Domestic Product (GDP), Personal Consumption Expenditures (PCE), Aggregate of gross private domestic investment (Non-RESI), Residential gross private domestic investment (RESI) are all from the National Income and Product Accounts Tables (NIPA) at the Bureau of Economic Analysis

- House Price Index. (HPI). Constructed based on conventional conforming mortgage transactions obtained from the Federal Home Loan Mortgage Corporation(Freddie Mac) and the Federal National Mortgage Association (Fannie Mae). Source: The Office of Federal Housing Enterprise Oversight (OFHEO).