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ABSTRACT

There is much debate about the usefulness of the neoclassical growth model for assessing the macroeconomic impact of fiscal shocks. We test the theory using data from World War II, which is by far the largest fiscal shock in the history of the United States. We take observed changes in fiscal policy during the war as inputs into a parameterized, dynamic general equilibrium model and compare the values of all variables in the model to the actual values of these variables in the data. Our main finding is that the theory quantitatively accounts for macroeconomic activity during this big fiscal shock.
1. Introduction

World War II is the largest fiscal shock in the history of the United States, and it also represents the most significant economic boom in U.S. history. Between 1941 and 1944, real per capita GNP rose 46 percent. Many economists agree that wartime government spending contributed to this economic boom. But beyond this general level of agreement, there is significant debate about the impact of World War II—and more generically the impact of other large fiscal shocks—on the economy. A major point of disagreement is the appropriate theoretical framework for quantitatively studying the impact of large fiscal shocks, such as World War II. Some economists argue that the standard neoclassical growth model is a useful tool for accounting for World War II, while others argue that accounting for the impact of large fiscal shocks requires major departures from the neoclassical framework.\(^1\)

The disagreement about the usefulness of the neoclassical model for understanding World War II partially reflects the fact that there is no comprehensive, quantitative analysis of the impact of the World War II shock on the U.S. economy using the neoclassical model. Consequently, there are several open questions: What are the neoclassical model’s successes and failures in accounting for the World War II economy? What are the impacts of the other large World War II shocks, such as the very large changes in taxes, in government investment, and in the draft? How does uncertainty about the war affect the model’s ability to account for the macroeconomy? World War II is the biggest macroeconomic shock to hit the U.S. economy and therefore provides a unique opportunity to test the neoclassical model.

This paper addresses these questions within a quantitative, dynamic general equilibrium framework. We construct a neoclassical growth model tailored to study World War II by including four types of shocks that were important during the war: (1) government spending, (2) income taxes, (3) the draft, and (4) productivity shocks. We conduct a sequence of quantitative experiments that investigate how well the model accounts for the major macroeconomic variables: output and its components, hours worked, and factor returns during World War II.
We develop a perfect foresight version of the model for heuristic purposes, and we then develop a stochastic version that serves as the primary model for studying the wartime economy. We analyze the stochastic model with all of the shocks to determine the conformity of the model to the data, and we then conduct stochastic experiments that include one shock at a time to isolate the contribution of each shock to the wartime economy. We also use the stochastic model to test whether the expectation of a postwar Depression has an important effect on wartime economic activity. Our stochastic analysis also evaluates how plausible changes in expectations over the exogenous shocks affect the results. In this regard we provide a new Monte Carlo–based approach to choosing probabilities over the states of a model when there is insufficient information to precisely choose those probabilities. This new procedure can be used in any model that has a stationary Markovian representation.

We find that real GNP, investment, consumption, labor supply, and the returns to capital and labor from the model are very similar to those in the data; the model captures the large increase in real output and hours worked, the declines in consumption and investment, and the movements in factor prices. Perhaps the most striking finding is that the deviations between the model and the data during World War II—a period of enormous shocks—are about the same size as those reported in other studies analyzing the postwar period, when the shocks are much smaller. Regarding the relative importance of the different wartime shocks, we find that the most important shock by far is the large increase in government spending; the large changes in the draft and in income tax rates have comparatively smaller effects. We also find that the results are robust to a wide range of uncertainty about the state variables in the model.

The paper is organized as follows. Section 2 reviews the literature and defines the scope of our analysis. Section 3 summarizes the shocks that we feed into the model. Section 4 summarizes the economic restrictions and regulations adopted in World War II. Section 5 presents the model. Section 6 presents the specification of the exogenous processes and parameter values. Section 7 conducts the quantitative analysis by comparing the model to the data. Section 8 discusses our findings in light of the issues raised in the literature. Section 9 summarizes the sensitivity analysis we conduct. Section 10 concludes.
2. Literature Review and the Scope of Analysis

We are unaware of any comprehensive assessment of the impact of World War II on the U.S. economy, particularly any studies that systematically address all of the open questions about the neoclassical model’s ability to account for the wartime economy. Specifically, there are questions about the conformity of the model with all of the key variables in the growth model: the World War II boom in output and labor, the components of output (consumption and investment), and changes in pre- and post-tax factor prices and returns. These are all open questions because none of the studies cited below simulate the model’s response to the World War II shock to compare the model’s outcomes to the data. This paper will address these questions, which are summarized below.

There are a number of questions about labor supply and post-tax wages. Mulligan (1998), Baxter and King (1993), and Burnside et al. (2004) all question the ability of the model to account for labor supply and after-tax factor price changes during World War II and/or during other episodes of higher military spending and higher taxes. Mulligan focuses his analysis on World War II. He conjectures that the neoclassical model will have a difficult time accounting for the large increase in World War II labor supply, as he shows that after-tax wages and returns to capital during the war are not particularly high. Mulligan concludes that the model may require “patriotism,” modeled as preference shifts, to jointly account for wartime labor supply and factor prices. Baxter and King (1993) raise a similar concern, questioning whether the model can account for the boom in labor supply given the large tax increases that occurred during World War II. Burnside et al. (2004) focus on the impact of post–World War II fiscal shocks on labor supply and after-tax wages. They argue that the standard neoclassical model (the one we use here) cannot quantitatively account for the impact of post–World War II fiscal shocks on real wages and labor supply, and that habit formation and investment adjustment costs are required. We will therefore compare labor supply, wages, and capital returns from the model to their data counterparts to address these questions.

There are also questions about pre-tax wages. Rotemberg and Woodford (1992) focus on the impact of military spending shocks on pre-tax real wages. They note that
the neoclassical model (with constant returns to scale production and perfectly competitive product and factor markets) can only account for higher pre-tax real wages during World War II with a shift in the production function, through either capital accumulation and/or a technological shift. This leads them to conjecture that the model requires significant, time-varying markups to account for pre-tax wages. We will therefore compare pre-tax real wages from our competitive model to actual pre-tax real wages to address this issue.

There are questions about changes in the composition of output between consumption and investment during large fiscal shocks. Blanchard and Perotti (2002) use a vector autoregression (VAR) to analyze the impact of post–World War II military spending shocks. They do not focus their analysis on any particular theoretical model, but argue that their VAR results regarding the decrease in private investment in response to military spending shocks are a challenge for most theories. We will therefore compare the division of output between investment and consumption to address questions about the response of the components of output to the war shock. Another reason to examine the division of output is because the composition of expenditure changed significantly during the war. Employment in motor vehicles and housing fell substantially, while employment in other sectors, such as steel and chemicals expanded considerably. These compositional changes at the industry level may potentially shift the distribution of output at the aggregate level between consumption and investment.

These questions fall under the umbrella of the broad issue that we address here: what are the quantitative successes and failures of the neoclassical model for understanding the World War II macroeconomy? We address this question (and by implication, the other questions cited above) as follows. We first identify the time series of the wartime shocks, we then feed the shocks into a parameterized neoclassical model and compute the equilibrium, and then graphically compare the actual time series of each endogenous variable (output and its components, labor, and factor prices) to their model counterparts for the years 1941–1946.

We focus our analysis on World War II because it is the most important and striking fiscal shock in the history of the United States as the shocks are by far the largest and are also very likely to be exogenous. These features of World War II provide a unique environment
for testing the neoclassical model. Other military episodes, such as the Vietnam War or the defense buildup during the Carter and Reagan administrations, are much smaller in terms of changes in spending, taxes, and the draft. These other episodes are also interesting to study but are beyond the scope of this paper.4

3. World War II Shocks

This section summarizes the shocks that we include in the model. There are three types of government policy (fiscal) shocks: government consumption and investment, income tax rates, and the draft. We call these shocks the fiscal state of the economy. In addition to these fiscal shocks, we also include productivity shocks. Figure 1 displays all six shocks.

The first series in Figure 1 is our measure of real government consumption and is total government spending less government investment and military compensation, all real. The source of the government spending data is the U.S. Department of Commerce (1975, Series F167; 1986, Tables 1.2, 3.7A, and 3.8A; and 1987, Table B12). The second series in Figure 1 is our measure of real government investment in plant and equipment that is used in the production of goods and services. This investment is total investment by federal, state, and local government less investment in military equipment and structures. Much of the government investment in this period was in government-owned, privately operated capital. This capital investment in equipment and factories substituted for private investment. We divide the series for government consumption and investment by the population over 16 from U.S. Department of Commerce (1975, Series A39) and by the growth rate of technology, which we estimate at 2 percent per year. To put the expenditure series in interpretable units, we also divide them by nonmilitary output in 1946, where nonmilitary output is GNP less military compensation.

The figure shows that both categories of government spending rose significantly over the course of the war. At the beginning and end of the war, government consumption is about 20 percent of trend nonmilitary output. At its peak, government consumption rises to almost 50 percent of trend nonmilitary output. Nonmilitary government investment doubles between 1941 and 1942, reaching almost 9 percent of trend nonmilitary output.
The third and fourth series in Figure 1 are estimates of labor and capital income tax rates from Joines (1981, Series MTRL1 and MTRK1). The most striking feature is that tax rates rose significantly. Labor taxes almost doubled, rising from about 10 percent to about 19 percent. Capital tax rates rose about 50 percent, from about 40 percent to more than 60 percent. The fifth series in Figure 1 shows the fraction of the working-age population in the military. The fraction rises from about 1 percent before the war to more than 12 percent at the peak of the war. These data clearly show that all of these elements of fiscal policy—government spending, labor income tax rates, capital income tax rates, and the number of individuals drafted—rose substantially during the war.

In addition to these fiscal shocks, we also include productivity shocks. The last series in Figure 1 shows detrended total factor productivity (TFP). We measure TFP using a capital share of 0.38 (we include both government and private capital) and a labor share of 0.62. Output used in the calculation of TFP is GNP less military compensation, which we refer to as nonmilitary output. The source of the GNP data is the U.S. Department of Commerce (1986, Table 1.2). The capital stock is the sum of private and public capital used in producing nonmilitary output; we exclude military equipment and structures. The source of the capital stock data is U.S. Department of Commerce (1987, Tables A6, A9, A15, A17, and A19). The labor input in the calculation of TFP is nonmilitary manhours reported in Kendrick (1961, Table A-X). We detrend TFP at its average growth rate of 2 percent per year. Detrended TFP rises 13 percent between 1941 and 1945. Several factors, including significant increases in research and development spending (R&D), the development of management science procedures and operations research practices, and substantial government infrastructure investment, plausibly raised TFP above trend during this period.

Regarding R&D spending, Mowery and Rosenberg (2000) document that real Federal R&D expenditures rose from $83 million in 1930 to $1.31 billion (in 1930 dollars) in 1945. This large increase in spending, which was concentrated among leading firms in science-based industries and research universities, plausibly led to significant productivity advances in a variety of industries. R&D grants were primarily managed by the Office of Scientific Research and Development, which entered into research partnerships with many leading universities.
and corporations, including 75 contracts with MIT. In conjunction with these grants, members of the scientific community were mobilized to recommend, guide, and participate in scientific research. Mowery and Rosenberg note that this advent of federal funding of R&D was the precursor of postwar federally funded and subsidized R&D programs.

Mowery and Rosenberg (2000) note that these R&D expenditures raised productivity in a number of industries, including airframes, shipping, radar, microwave technology, and fertilizer. Similarly, Davies and Stammers (1975) report that other industries with significant advances included air travel, synthetic rubber, oxygen steel, titanium, jet propulsion, silicones, urethanes, polythanes, chemotherapy, polymers, insecticides, nylon, and teflon. Sir Edward Bullard (1975) reports large advances in electronics and instrumentation.

Davies and Stammers (1975) report another significant source of World War II productivity advance was the significant development of management science and operations research practices by industrial scientists. These practices led to increased efficiency in factory output and more broadly in organizations.

In addition to higher public and private R&D spending, Field (2003) cites other factors that raised productivity during the war, including significant government infrastructure investment in roads, highways, bridges, and airports during this period. Moreover, he notes that the very high levels of private R&D spending of the 1930s likely continued to have productivity spillovers into the 1940s.

In addition, Alchian (1963), among other economists, have argued that wartime learning-by-doing raised productivity. While quantitatively accounting for the contribution of all of these factors to aggregate productivity change is beyond the scope of this study, these factors taken together clearly suggest the plausibility of significantly higher productivity during the war. It is also possible that changes in capacity utilization could account for some of the change in the Solow residual during this period. To address this possibility, we will consider in our sensitivity analyses a variant of our model with variable capacity.
4. Wartime Economic Restrictions and Regulations

A number of economic restrictions and regulations were adopted in World War II. Major restrictions include nominal wage and price controls and rationing (through ration coupons) of meat, butter, gasoline, sugar and other nondurable goods, de facto rationing of some durable goods, such as autos and residential capital (produced in small quantities during the war); and Federal Reserve management and control of U.S. Treasury security markets that fixed the nominal interest rates on these securities between 3/8 of 1 percent for short-term Treasury debt to 2.5 percent for long-term Treasury debt.

It is unknown whether these restrictions had quantitatively important effects on the major macroeconomic aggregates during this period, as there is no work addressing this question within the growth model. Uncertainty about the impact of these restrictions is also due to the fact that economic agents found ways to get around at least some of these restrictions. For example, the trading of ration coupons and the emergence of black markets allowed households to at least partially offset the impact of rationing, and firms supplemented salaries with non-wage benefits to offset the impact of wage controls (see Rockoff 1984). Furthermore, an environment that captured these regulations and restrictions would differ considerably from the simple environment in the standard growth model. For example including the Federal Reserve’s interest rate fixing policy and the fact that the debt was largely held by regulated intermediaries would add significantly to the complexity of the model we explore later.6

Because these additional factors would complicate the analysis considerably and because we do not know what their impact would be, we abstract from them in our initial analysis and assess how well a neoclassical model without (i) coupon rationing and price and wage controls, without (ii) constrained investment in certain sub-sectors of the economy, and without (iii) control of the pricing of government debt can account for the U.S. data during World War II. If abstracting from these restrictions and regulations is quantitatively important, then the model will fail along one or more dimensions, and this potential failure will shed light on how the model should be modified to incorporate one or more of these restrictions.
5. Model Economy

We start with a standard neoclassical model and tailor it to study the impact of wartime shocks. The model includes government consumption, government investment in physical capital, and government payments to military personnel, it includes taxation of capital and labor income, and it includes the draft.

There is an infinitely lived representative family with two types of family members: "civilians" and "draftees." Both types of family members have identical preferences given by $U(c, l) = \log(c) + V(1 - l)$, where $V$ is a concave and continuously differentiable function. There are $N_t$ total family members in period $t$, with fraction $a_t$ who are in the military and fraction $(1 - a_t)$ who are civilians.

The family optimally chooses consumption of both types, which we denote by $c_c$ and $c_d$, respectively. The family also chooses private investment in physical capital, $i_p$, and civilian labor input, $l_c$, to maximize its lifetime utility. Labor input for family members in the military is exogenously fixed at $\bar{l}_d$. The family’s maximization problem is given by

$$
\max_{\{c_{ct}, c_{dt}, i_{pt}, l_{ct}, b_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1 - a_t)U(c_{ct}, l_{ct}) + a_tU(c_{dt}, \bar{l}_d) \right\} N_t
$$

subject to

$$
(1 - a_t)c_{ct} + a_tc_{dt} + i_{pt} + b_{t+1} = (1 - \tau_{kt})r_{pt}k_{pt} + (1 - \tau_{lt})w_t(1 - a_t)l_{ct} + \tau_{kt} \delta k_{pt} + R_t b_t + a_t w_t \bar{l}_d + T_t
$$

$$
k_{pt+1} = [(1 - \delta)k_{pt} + i_{pt}]/(1 + \gamma_n)
$$

$$
N_t = (1 + \gamma_n)^t
$$

where $k_{pt}$ is the beginning-of-period capital stock in period $t$, $r_{pt}$ is the rental rate paid for that capital, $w_t$ is the wage rate in $t$, $\tau_{kt}$ and $\tau_{lt}$ are proportional tax rates on capital income and labor income, respectively, in $t$, $R_t b_t$ is the value of matured government debt, $b_{t+1}$ is new government debt holdings, and $T_t$ are government transfers. All quantities are in per...
capita terms; the constant growth rate of the population is given by $\gamma_n$. The processes for $a_t$, $r_{pt}$, $w_t$, $\tau_{kt}$, $\tau_{lt}$, $R_t$, and $T_t$ are viewed exogenously by the family and are specified later.

There is a single physical good which is produced from a constant returns to scale technology. The technology is operated by a competitive representative firm, which hires private capital, public capital, and labor services. Output, which we measure as nonmilitary output, is given by

\begin{equation}
Y_t = z_t F(K_{pt}, K_{gt}, A_t L_{pt}),
\end{equation}

where $K_{pt}$ is the beginning-of-period private capital stock for the economy in $t$, $K_{gt}$ is the beginning-of-period public capital stock used by the private sector in $t$, $A_t$ is the level of labor-augmenting technology in $t$, and $L_{pt}$ is the total labor input in the nonmilitary sector in $t$. We assume that the level of labor-augmenting technology grows at the constant rate $\gamma_A$: $A_t = (1 + \gamma_A)^t$. The term $z_t$ is a productivity shock. We will specify the process for this shock and the others later in this section.

We include government capital in production because the federal government financed increases in industrial construction and producers’ durable equipment during World War II, including significant investments in the aircraft, automotive, and aluminum industries. Gordon (1969) estimates that government-owned, privately operated capital increased the manufacturing capital stock by 30 percent between 1940 and 1945. (See also Gordon 1970, Jaszi 1970, and Wasson, Musgrave, and Harkins 1970.) We denote government investment expenditures by $I_g$.

Government purchases of consumption goods are denoted by $C_g$, and government payments to military personnel are denoted by $N_t a_t w_t \tilde{I}_d$. Total government spending is the sum of the three expenditure items:

\begin{equation}
G_t = C_{gt} + I_{gt} + N_t a_t w_t \tilde{I}_d.
\end{equation}

Government capital evolves according to the following law of motion:

\begin{equation}
K_{gt+1} = (1 - \delta) K_{gt} + I_{gt}
\end{equation}
with $K_{g0}$ and the process for $I_{gt}$ given. We assume that private and public capital depreciate at the same rate $\delta$. We also assume that the government satisfies the present value budget balance. The government budget constraint is given by

$$B_{t+1} = G_t + R_t B_t - \tau_{lt} N_t w_t ((1 - a_t) l_{ct} + a_t l_{ct}) - \tau_{kt} (r_{pt} - \delta) K_{pt} - r_{gt} K_{gt} + T_t. \tag{9}$$

We close the model by specifying the functions that the family treats exogenously when solving its optimization problem in (1). Since firms are competitive, the rental prices for the factors of production are equal to their marginal products. Therefore, the rental rates in (2) and (9) and the wage rate in (2) are equal to the partial derivatives of the production function $F$ in (6) with respect to $K_p$, $K_g$, and $L_p$, respectively.

Five of the exogenous variables in the model have already been discussed: conscription ($a_t$), the tax rate on capital income ($\tau_{kt}$), the tax rate on labor income ($\tau_{lt}$), government consumption ($C_{gt}$), and government investment ($I_{gt}$). The sixth exogenous variable is related to the state of the postwar economy, which we denote as $D_t$. The evolution of the six exogenous variables is governed by a state variable, $s_t$, which specifies a particular set of values for $a_t$, $\tau_{lt}$, $\tau_{kt}$, $C_{gt}$, $I_{gt}$, and $D_t$. The state variable $s_t$ is modeled as a Markov chain. This specification is used for both the deterministic and the stochastic versions of the model. If individuals have perfect foresight, the process $s_t$ is degenerate. In the stochastic economies we specify the transition probabilities over $s_t$.

An equilibrium for this economy consists of the following: allocations for households $c_{ct}$, $c_{dt}$, $l_{ct}$, $l_{pt}$, and $k_{pt}$; inputs for firms $K_{pt}$, $K_{gt}$, and $L_{pt}$; and sequences of prices $r_{pt}$, $r_{gt}$, $w_t$, and $R_t$ that satisfy the following conditions: (i) taking prices and exogenous policies for $a_t$, $\tau_{kt}$, and $\tau_{lt}$ as given, households maximize utility subject to constraints (2)–(3); (ii) taking prices as given, firms maximize profits period by period $Y - r_p K_p - r_g K_g - w L_p$; (iii) factor markets clear:

$$K_{pt} = N_t k_{pt} \tag{10}$$

$$L_{pt} = N_t (1 - a_t) l_{ct} \tag{11}$$
(iv) the resource constraint

\[ C_{pt} + I_{pt} + C_{gt} + I_{gt} = Y_t \]  

holds, where \( C_{pt} = N_t[a_t c_{ct} + (1 - a_t)c_{dt}] \) and \( I_{pt} = N_t i_{pt} \); and (v) (9) is satisfied.

To test the robustness of our results, we will also consider a version of the model in which there is variable capacity utilization. We will see that the results are not sensitive to this modification, and therefore we present the variable capacity model in McGrattan and Ohanian (2006).

6. Calibration

This section presents the functional forms and parameter values, and the specifications of the states for the exogenous processes and the associated transition probabilities.

A. Functional Forms and Parameter Values

Functional forms and parameter values are identical in both the deterministic and stochastic economies. Table 1 summarizes the values of all parameters, which we discuss in detail below. Preferences are given by

\[ U(c, l) = \log(c) + \psi(1 - l)^\xi / \xi, \]

which implies a compensated labor supply elasticity of \((1 - l)/[l(1 - \xi)]\). We choose a benchmark value of \(\xi = 0\), which implies log preferences over leisure. We later evaluate the robustness of our results by choosing an alternative value of \(\xi\) that yields a lower labor supply elasticity.

The parameter \(\psi\) is chosen so that the fraction of time allocated to nonmilitary work in the deterministic steady state is 26.6 percent, which is consistent with the observed U.S. average over the period 1946–1960. We also need to specify the exogenous hours requirement for those in the military — we assume that \(\bar{I}_d = 50/84\), which implies that soldiers work 50
out of their 84 discretionary hours per week. We found that the quantitative results are not sensitive to plausible changes in this value.

Given that there is both government and private investment, the production technology is

\[ zF(k_p, k_g, Al) = z(bk_p^\rho + (1 - b)k_g^\rho)^\frac{1}{2} (Al)^{1 - \theta}. \]

We assume that government capital and private capital are perfect substitutes (\( \rho = 1 \)). As discussed above, this is a reasonable assumption, as most of this government investment was in government owned, privately operated plant and equipment. The parameter \( b \) governs the relative productivities of government and private capital. Given that the capital are perfect substitutes, we assume that they are equally productive, which implies that \( b = 1/2 \). We chose \( \theta = .38 \), which is consistent with the U.S. share of income paid to capital during this period. The parameter \( \beta \) is chosen so that the capital-output ratio is consistent with the U.S. level during the war. The depreciation rate (\( \delta \)) for both government and private capital is 5.5 percent.

There are two productivity parameters; \( z \) is a productivity shifter that fluctuates from its average value, while \( A \) grows at a constant rate. The growth rates of trend technological progress (\( A \)) and the population (\( N \)) are set to their average values over this period: \( \gamma_A = 2.0 \) percent and \( \gamma_n = 1.5 \) percent. The stochastic process for \( z \) is described below.

**B. Specifying the State Vector and Transition Matrix**

This section specifies the state vector and the probabilities governing the transitions across those states. We first discuss the realizations of the shocks that comprise the states.

**The States**

Recall that we will feed in to the model the actual realizations of the exogenous variables. The exogenous variables are the categories of government spending, labor and capital income tax rates, the draft, and productivity. This procedure requires specifying a
separate state for each year of the war years (1941–1945), and requires specifying a state for the normal peacetime economy.

To do this, we define a state with 1941 values of the exogenous variables, a state with 1942 values, and so forth through 1945. We also need to construct a peacetime state. For the peacetime state, we use the 1946 values of the exogenous variables. This is also the peacetime state for all but one of the stochastic experiments. The specific values of the exogenous variables used in the experiments are plotted in Figure 1. One noteworthy feature of these data is that the peacetime state has income tax rates that are high relative to those during and before the war.

In one of our stochastic experiments, we will allow for the possibility of a postwar depression. Thus, in this one stochastic case there are two possible peacetime states: either the 1946 actual values of the variables, or a depression state. We consider the possibility of a postwar depression in this one stochastic experiment because wartime surveys show that individuals as well as professional economists placed some probability on the event that the economy would reenter the Great Depression after the war. In testimony to the U.S. Senate Special Committee on Post-War Economic Policy and Planning in May of 1944, four economists reported that they expected a depression.

Unfortunately, there is not sufficient survey information to completely calibrate the Depression state, as the Gallup surveys do not characterize the expected severity or nature of the Depression. We therefore choose a Depression state such that the level of productivity in the model in this state is equal to the actual average of productivity between 1930 and 1938, which is 14 percent below trend (defined to be the 1929 level times \((1 + \gamma_A)^t\)). This productivity-driven model of a Depression, based on data from the U.S. Great Depression, generates substantially lower employment and output, and is the simplest mechanism for generating a Depression in the model.
Transition Matrix for the States

The economy will transit across the states described above. We now discuss the transition probabilities across these states. For both the perfect foresight and stochastic economies we construct a Markovian state vector, \( S_t \), with a Markov transition matrix denoted as \( \Phi \), that governs the transitions across these states. The transition matrix in the perfect foresight economy has unity on the first lower diagonal of \( \Phi \) governing the appropriate year-to-year transitions, i.e., 1941 to 1942, 1942 to 1943, etc., and it has zeroes everywhere else.

To describe the transition matrix for the stochastic economy, first consider the simplest stochastic case in which there is no possibility of a postwar Depression. The elements of the state vector can take on six possible values; five values are the actual realizations of each of the state variables from 1941 to 1945, and the sixth value is the peacetime state, which is the 1946 realization for each of the state variables.

The challenge for any stochastic analysis of World War II is specifying the transition probabilities of this matrix. We first approached this issue by examining survey evidence on expectations about the war and the economy from Gallup and the other major polls. Unfortunately, the surveys do not provide enough information to estimate probabilities over all possible states in the Markov chain. We therefore chose a Monte Carlo procedure in which we generate transition probabilities randomly from a uniform distribution and keep those draws that generate wars that are empirically plausible—those that generate war frequency and war duration similar to actual U.S. wars. We do not place any other restrictions on the transition probabilities other than the war duration and frequency criteria that we detail below. This approach has the additional benefit of evaluating the robustness of the results to changes in expectations and provides a new tool that can be used in any Markovian model that can be rendered stationary.

The first step in our procedure generates candidate transition probability matrices by drawing random numbers from a uniform \([0,1]\) distribution, insuring that the probabilities sum to 1. We keep those transition matrices that generate an average duration of a war, the frequency of a war starting, and the average fraction of years in war that are between 70 per-
cent to 130 percent of their historical U.S. averages. These historical averages are 3.7 years, 4.1 percent, and 15.2 percent, respectively. We generated over 18,000 of these matrices that satisfied the frequency and duration criteria. Appendix A describes this procedure in detail.

We compute the equilibrium of the stochastic model for every transition probability matrix that satisfies these frequency and duration criteria. To summarize the results from all of these trials, we will plot for each date upper and lower bounds for each of the endogenous variables, along with the actual data. We will highlight the distribution of the values for these endogenous variables across the trials by shading the region between the minimum and maximum values. Given our focus on reporting bounds on the endogenous variables, we summarize the transition probabilities from all of the draws in a similar fashion by reporting maximum and minimum values for each of the transition probabilities in the matrix $\Phi$. We report these probabilities in Appendix A. The key probabilities are for the 5 war years, 1941–1945. The transition probabilities along the diagonal that are equal to 1 in the perfect foresight case range from 0 to nearly 1 in the stochastic model. The other transition probabilities for the war years, which are 0 in the perfect foresight case, range between 0 up to about 0.74 in the stochastic model. Thus, the transition probabilities that we use cover most of the possible probabilities in the transition matrix.

We now turn to the stochastic experiment in which we allow for the possibility of a postwar Depression. We consider this extension because there is evidence that individuals and economists placed some probability on a postwar Depression. We treat this as an extension, rather than as a baseline feature of the stochastic model, because there is relatively little information that we can use to calibrate the severity of this Depression.

We use Gallup and Roper surveys of the time to model the expectations of a postwar depression. For example, in a Gallup survey of September 1941 respondents were asked, “Do you think we are likely to have a greater prosperity, or another depression after the present war?” Seventy-eight percent expected another depression. As we report in Table 1, we set the probability of entering the depression state, conditional on leaving the war, at 78 percent in 1941. We used the same or similar questions asked in Gallup and Roper polls for the other
years to set the transition probabilities for the postwar state. Appendix A provides further details.

C. The U.S. Data

We compare seven variables from the model to their counterparts in the U.S. data from 1941 to 1946. Real GNP, consumption, investment, two measures of hours worked, and two measures of factor productivity are compared to U.S. counterparts derived from the national income and product accounts (U.S. Department of Commerce 1986), the reproducible tangible wealth tables (U.S. Department of Commerce 1987), and data on manhours (Kendrick 1961).

We compare production plus military compensation in the model \((Y + wNa\bar{d})\) to real U.S. GNP. We compare private consumption in the model to U.S. personal consumption expenditures on nondurables, services, and the service flow from the stock of consumer durables.\(^{14}\) We compare private investment in the model to U.S. gross private domestic investment plus foreign net investment. For both the model and the data, we report GNP, consumption, and investment in per capita, detrended terms as we did for the government spending series in Figure 1. Specifically, we divide the series by the population over 16 and by the growth rate of technology, which we estimate at 2 percent per year.

We compare total per capita hours—nonmilitary plus military—in the model \(((1 - a)l_c + a\bar{d})\) to total U.S. manhours in Kendrick (1961) divided by the population over 16. Because other researchers have questioned the ability of the model to account for nonmilitary hours, we also compare the nonmilitary component in the model and data. The nonmilitary hours series is the same as that used to compute TFP. Both the actual and model per capita hours series are normalized by discretionary time, which is 12 hours per day.

Finally, we compare measures of factor productivities in the model and data. For capital, we compare nonmilitary output \(Y\) divided by total capital \(K_p + K_g\) to its counterpart in U.S. data. The output and capital measures are the same as that used in computing TFP. To put this ratio in more interpretable units, that is in the units of an after-tax return, we multiply both the U.S. and model series by the capital share times one minus the tax rate
on capital \((\theta(1 - \tau_k))\) and then subtract the depreciation rate \(\delta\). For labor, we compare nonmilitary output \(Y\) divided by nonmilitary hours \(L_p\) to its counterpart in U.S. data. In Appendix B, we also discuss several alternative measures of factor returns and how they compare to the model’s predictions.

7. Comparing the Model to the Data

We compare the perfect foresight simulation to the data by plotting the actual realizations of output, consumption, investment, labor, and factor productivities between 1941 and 1946 against the model realizations for these variables. We follow the same procedure for the stochastic simulations with the exception that we report the distribution of the outcomes for each model by constructing bounds with minimum and maximum values for each endogenous variable at each date, and the interior between these bounds is shaded according to the density of the distribution. Recall that the realizations of the state variables are identical for both the deterministic and stochastic economies. Specifically, all the experiments begin with the capital stock equal to its actual value, and the other state variables equal to their 1941 values. Similarly, the realizations for the state variables for 1942–1946 are also equal to their actual values.\(^{15}\) We begin with the perfect foresight economy because of its simplicity. We approximate the equilibrium in all cases using the finite element method, as the shocks are sufficiently large as to raise questions about the accuracy of a first-order approximation. (See McGrattan 1996.)

A. Results from the Perfect Foresight Model

Figures 2–4 show time series for the model and data between 1941 and 1946. The lines with open squares are the U.S. series, and the lines with the filled circles are the model series. The main finding is that the time series for the model and data are quite similar. The model captures the changes that occur over time in these variables, and captures much of the timing of these changes.

The first plot in Figure 2 is real detrended GNP in both the model and the data. Real GNP rises about 40 percent through the war, with a large decline occurring between
1945 and 1946. The second plot shows private consumption in both the model and the data. Model consumption shows almost no change, while U.S. consumption shows a decline of about 3 percent between 1941 and 1944. (The stochastic results will show that the very flat consumption pattern in this case is a consequence of the perfect foresight assumption, and we will also see that the deviation between the model and the data is smaller in the stochastic case.)

Most of the change in GNP is not due to private consumption. It is due to change in government spending and change in private investment. The third plot in Figure 2 shows that private investment in both the model and the data declines significantly through 1944, and then recovers after that.

Figure 3 has two plots, one for total hours and one for nonmilitary hours. Both the data and the model series are divided by the 1946–1960 U.S. average fraction of discretionary time at work. Thus, the figure shows hours of work relative to a postwar trend. In this perfect foresight simulation, predicted hours are high at the start of the war—higher than observed—because higher labor tax rates are perfectly anticipated. But by 1944, total hours in both the model and the data are close to 30 percent above trend. A significant drop in total hours in both the model and the data occurs between 1945 and 1946 with the large reduction in military employment.

A comparison of the two plots in Figure 3 reveals an interesting fact: despite the huge increase in military employment during World War II, nonmilitary hours in this period rose significantly above trend as well. Specifically, U.S. nonmilitary hours rose to 15 percent above trend. This is also the case for the model except that the timing is different: nonmilitary hours in the model are highest in 1941, while in the U.S. data they are highest in 1943. Despite the large increase in labor tax rates, the model hours are above trend during the war because the wartime tax revenues are significantly below the wartime government expenditures—and thus households anticipate high taxes after the war to pay off the government debt.

Figure 4 shows measures of factor returns. The first plot shows the time series for the
after-tax marginal product of capital given by \(100(1 - \tau_k)(\theta Y/K - \delta)\), where \(Y\) is nonmilitary output and \(K = K_p + K_g\) is capital used to produce nonmilitary output. We multiplied by the capital share \(\theta\) times one minus the tax rate \(1 - \tau_k\) and subtracted the depreciation rate in order to put it in the standard units of an asset return. The model predicts a return that is roughly 3.5 percent for the first half of the war, rising to almost 4 percent in 1944. The U.S. return rises initially, but the rates are very close to the model’s prediction. The second plot in Figure 4 is detrended nonmilitary labor productivity given by \(Y_t/[L_p(1 + \gamma_A)^t]\). As before, nonmilitary hours \(L_p\) are normalized; they are divided by the 1946–1960 average fraction of discretionary time in work. During the war, both the model and the data show a rise in productivity relative to the trend in TFP.

Note that these graphs display each of the variables individually; some authors use an alternative approach to comparing model and data by computing deviations in the first-order conditions. We do this for the household’s first-order condition that equates the marginal rate of substitution between consumption and leisure to the after-tax real wage in the model and the data, as this condition has recently attracted considerable attention in the literature. Deviations in this condition are thus a combination of deviations in consumption, hours worked, and the after-tax marginal product. The deviation in this condition is small; it averages 2.5 percent between 1941 and 1946 and reaches a maximum of 7.5 percent during this period.

This perfect foresight experiment is a useful heuristic device as it shows the response of the model endogenous variables when households have perfect foresight. With these results in mind, we now turn to the stochastic experiments.

**B. Results from the Stochastic Model**

Figures 5–7 show the variables from the stochastic model with all the shocks. This is our main experiment, which we will subsequently refer to as the *benchmark experiment*. Each graph presents the U.S. data and a shaded region representing the model predictions for all probability matrices satisfying the frequency and duration criteria described earlier. The shading indicates the relative mass of the realizations from the experiments, which are
approximately normally distributed. Thus, the darker shading toward the midpoint between the bounds indicates relatively more mass, and the lighter shading closer to the bounds indicates less mass.

The results have several noteworthy features. One is that the changes in output, consumption, investment, labor, and the marginal products of labor and capital are very similar between the benchmark model and the data; the actual data typically lie between the upper and lower bounds of the stochastic model, and in most cases there are small deviations between the highest and lowest model prediction in any year. For example, there is at most a 10 percent difference between the upper and lower bounds for nonmilitary hours during the war. The fact that the model does so well in predicting the productivities of capital and labor is particularly interesting given government regulations in the labor and capital markets. It appears that wage controls during the war apparently did not have a significant effect for aggregate labor productivity. Similarly, the interest rate control on government bonds apparently did not have a significant effect on aggregate capital productivity.

Another noteworthy feature of the results is that the general patterns of the variables in the stochastic model are similar to those in the perfect foresight model. The main difference between the perfect foresight and the stochastic case is that consumption declines in the stochastic case, compared to the very flat pattern in the deterministic case.

The most striking finding is that the deviations between the model and the data during World War II are about the same size as in real business cycle (RBC) models during the peacetime period when the shocks are much smaller. For example, in the period 1941–1945, the deviations between actual GNP and the midpoint of the model’s predictions are as large as 4 percent. For investment measured relative to the trend in nonmilitary output, the largest deviation—which is roughly 30 percent—occurs in 1945 when both actual and predicted investments are low. (See Figure 2.) For total hours, the largest deviation is 7 percent. For labor productivity, the largest deviation is 3 percent. These deviations are quite small, considering the very large shocks that occurred during the war. To see this, we now compare the size of these model deviations during World War II to model deviations from studies that compute the equilibrium time path of RBC models during periods with
much smaller shocks.

Hansen and Prescott (1993) and Plosser (1989) are two prominent papers that conduct equilibrium path analyses during the relatively tranquil postwar period. Hansen and Prescott use an RBC model with multiple productivity shocks to study the 1990–1991 recession and the subsequent recovery. They compute the model between 1985 and 1992 and compare the variables from the model to the data at each date in this period. Labor deviations are as large as 4 percent, labor productivity deviations are as large as 3 percent, and deviations in the share of output allocated to investment are as large as 40 percent. Plosser uses an RBC model to study the 1954–1985 U.S. economy, and presents plots of growth rates of model variables compared to actual variables at each date. The model deviations in levels for labor are roughly as high as 10 percent for labor and 20 percent for the real wage. The deviations in both the Hansen-Prescott and Plosser analyses are as large as, or larger than, those in the model during World War II. This comparison shows that the level of success achieved in RBC models during the post–World War II period of small shocks is also achieved during World War II, when the economy is hit with shocks that are as much as two orders of magnitude larger. If anything, there would be a presumption that shocks of this magnitude, in a simple dynamic stochastic general equilibrium model that has been developed over the last 25 years to study the impact of small shocks, would generate much larger relative deviations than those reported here.

An implication of this finding is that the government programs of wage and price controls, rationing of nondurable and of some types of capital (autos and housing), and interest rate control of federal debt are not quantitatively important for understanding the major macroeconomic variables. In other words, the World War II macroeconomy behaved very much in line with the prediction of the optimal growth model in response to the shocks that are considered here.

The easiest way to understand the key economic forces at work in this analysis is to simulate the model economy by comparing the results from the benchmark model (with all the shocks) to the results from the model that omits one of the shocks. This comparison shows the contribution of each shock individually to the wartime economy. Specifically,
we compute the equilibrium with one exogenous variable held constant at its 1941 value throughout the war, and all the other shocks taking on their realizations each year as in the benchmark model. We do this for each of the shocks individually.\textsuperscript{18}

Figures 8–12 show the results of these experiments. Figure 8 is central to understanding the behavior of the World War II macroeconomy; it shows the equilibrium of the model with government spending held constant at its 1941 level through 1942–1945. The model variables with government spending fixed at its 1941 value look very different from those in the experiment in which government spending shocks are equal to their actual values for 1942–1945. Specifically, GNP in the benchmark model rises by more than 40 percent during the war, whereas GNP with government spending counterfactually fixed at its 1941 value rises only about 10 percent. Moreover, consumption and investment in the model with fixed government spending are systematically much too high, and labor is systematically much too low relative to the benchmark experiment. In other words, the model economy without the wartime government expenditure shocks, but all the other shocks, bears virtually no resemblance to the actual wartime economy or to the benchmark model. This experiment thus shows that government spending shocks are the major factor in accounting for macroeconomic changes during World War II.

We will next see that the other shocks are playing more modest roles in contributing to World War II macroeconomic changes, despite the fact that a number of these other shocks are very large. Figure 9 keeps the number of draftees constant throughout the war; relative to the benchmark model, there is a slight difference in hours worked and the marginal product of labor, otherwise the variables are extremely similar. Figure 10 keeps the labor income tax constant throughout the war; relative to the benchmark model, there is higher labor supply, a higher after-tax wage, and a slightly higher after-tax return to capital, while the remainder of the variables are quite similar. Thus, higher labor taxes reduced employment, ceteris paribus. Figure 11 keeps the capital income tax rate constant throughout the war; relative to the benchmark model, the after-tax return to capital is higher, and investment is also somewhat higher; the other variables are quite similar. Figure 12 keeps (detrended) productivity fixed at its 1941 value throughout the war. Relative to the benchmark model,
labor, consumption, and investment differ, with labor and investment significantly higher at the start of the war and lower at the end of the war. Thus, an important contribution of productivity is to affect the timing of the changes in the variables during the war.

To summarize, the government spending shock is the main driving force behind the wartime macroeconomy; productivity also has a significant effect, as does the labor tax rate, but the capital tax rate and the draft have relatively small effects.

Figure 13 shows the variables from the stochastic model which includes the possibility of a postwar Depression that never occurs. In the model with the Depression possibility, consumption is slightly lower and labor slightly higher, reflecting precautionary motives associated with the negative Depression state. Otherwise, the variables are similar. This finding, along with the findings that the bounds are fairly narrow, and that the stochastic and deterministic results are similar, suggests that the results are fairly robust to alternative specifications of uncertainty over the exogenous variables in the model.

8. Discussion

This section discusses our findings in light of the questions that have been raised about the ability of the neoclassical model to account for the impact of wartime fiscal shocks.

As discussed in Section 2, a number of authors have questioned whether the neoclassical model is consistent with the behavior of labor supply and factor prices during wars. For example, Mulligan (1998) argues that factor prices are too low to be consistent with higher labor supply during World War II: “empirical support... cannot be found because after-tax wages do not appear to be temporarily high during the war period. The primary force working against wage motives is the massive across-the-board income tax increases that occurred during the war” (p. 1071). However, Mulligan did not compute a general equilibrium model to assess the model endogenous variables in response to the wartime shocks. We computed the equilibrium in response to these shocks and found that the model is indeed consistent with these data. High government spending is the key reason why the model can simultaneously account for both wartime labor supply and after-tax factor compensation.
In particular, recall that Figure 8 shows government spending fixed at its 1941 level. This figure shows that in the absence of high government spending, the model generates labor supply and after-tax compensation that differ considerably from the data; model labor is too low, and the model after-tax wage is too high. Thus, the enormous resource drain of wartime government spending is the key factor that accurately generates high wartime labor supply without higher after-tax compensation in the model. To understand this result, note that at the peak of the war in 1944, government spending was about 80 percent of trend output. This means that if labor had not increased, private consumption would have dropped dramatically. Given concave utility, households in the model respond to this large resource drain by consuming less physical consumption and less leisure. Thus, understanding the conformity of the endogenous variables in the model with their data counterparts requires quantitatively accounting for the impact of the wartime shocks, as the responses of the endogenous variables in the model economy depend critically on the specific realizations of these shocks.

Rotemberg and Woodford (1992) raise questions about the neoclassical model’s ability to account for pre-tax real wages during wars and during other periods of large exogenous increases in government spending. The average deviation between the pre-tax marginal product of labor in the perfect foresight model and its analogue for U.S. data between 1941 and 1946 is \(-0.3\) percent, with the largest absolute deviation during this period equal to 2.6 percent. (See Figure 4.) The average deviation between the pre-tax marginal product in the perfect foresight model and the pre-tax U.S. nonfarm compensation per hour is \(-0.1\) percent, with the largest absolute deviation equal to 1.6 percent. (See Figure B1 in Appendix B.) Similarly, we find small deviations between the stochastic model’s predictions and the U.S. data. Thus, our view is that the neoclassical model does well in accounting for pre-tax real wages during World War II.\(^{19}\)

Blanchard and Perotti (2002) have questioned the ability of the theory to account for the division of output between consumption and investment in response to large fiscal shocks. In 1944, private consumption’s share of GNP in the perfect foresight model is 39 percent compared to 38 percent in U.S. data. In 1944, private investment’s share of GNP
in the perfect foresight model is 2 percent compare to 6 percent in U.S. data. The average deviations are even smaller for the stochastic simulations.

Burnside et al. (2004) find that the neoclassical model requires additional features, including habit formation in preferences and investment adjustment costs, to account for movements in labor during periods of post–World War II military buildups. This finding stands in contrast to our findings, which indicate that these additional features are not required. The differences in findings may be due to the fact that isolating the effects of increased military expenditures is very difficult in periods when the military shocks are relatively small. For example, the Carter-Reagan military buildup, which is one of the episodes analyzed by Burnside et al., is very small compared to World War II, and moreover it coincides with other significant nonmilitary changes, such as tax reform and deregulation. While Burnside et al. confront this difficult issue of identification in their analysis, it is very hard to know whether their estimates of the fiscal shock are unbiased, which is required for analyzing the impact of the shocks. In contrast, shock identification during World War II is much easier, as the military buildup swamps all other factors.

In summary, the findings show that the neoclassical model can account for the behavior of the main macroeconomic variables during World War II.

9. Sensitivity Analysis

In this section, we discuss three experiments that we ran to assess the robustness of our results. The first two focus on the robustness of the prediction for higher private labor input during World War II. First, we evaluate how high labor tax rates would have to be such that nonmilitary hours worked would not rise during the war. Second, we evaluate how low the labor supply elasticity would have to be such that, on average, nonmilitary hours worked remained at their trend level. We conduct both of these experiments in the simpler deterministic model. The third set of sensitivity experiments focus on the robustness of our predictions of the effects of fiscal shocks when we introduce variable capacity utilization in our model. Specifically we allow variation in the number of employed and the number of hours in the workweek, since both rose significantly during the war. We briefly describe the
results for this extended model here; further details can be found in McGrattan and Ohanian (2006).

Regarding the counterfactual tax rate analysis, we conduct two specific tax experiments. In the first, we find a sequence of labor tax rates for the period 1941–1946 that imply no change in predicted nonmilitary hours. In the second, we repeat the experiment but vary capital tax rates instead of labor tax rates. We find that tax rates would have to be much larger than those reported by Joines (1981)—or any other estimates of U.S. tax rates—to keep nonmilitary hours worked during the war from rising. In Figure 14, we show the labor tax rates for this counterfactual experiment versus those estimated by Joines (1981) for the United States. The tax rates would have had to jump to 25 percent at the start of the war and continue to rise to close to 30 percent by 1943. These rates are significantly higher than the U.S. rates. When we vary capital tax rates, we find that even when we set them to 100 percent during the war, we cannot choke off the rise in predicted nonmilitary hours because the capital tax rates do not have a large enough effect on labor supply.

We turn next to the choice of labor supply elasticity. We find that the labor supply elasticity needs to drop by more than a factor of 8 relative to the log utility case used in the benchmark model to keep the average value of nonmilitary hours of work equal to its trend level. The elasticity for the log utility case is 2.75 percent compared to 0.32 percent in this experiment.

The elasticities used in these sensitivity experiments are much too low for an aggregate representative household model. To see this, consider replacing the log utility function in the prototype business cycle models studied by McGrattan (1994) with (13). We can set $\psi$ and $\xi$ so as to achieve the same steady-state hours worked and lower labor elasticities. In her benchmark case with technology shocks only and divisible labor, a labor elasticity of 0.5 generates a standard deviation of hours worked equal to 0.3; the standard deviation for U.S. hours is 1.52. (See McGrattan 1994, Table 1.) Similar results are found for her model with taxes. In that case, a labor elasticity of 0.5 generates a standard deviation of hours worked equal to 0.51—again, much lower than that in the data. For a labor elasticity of 0.32, the results are even more striking: the standard deviations of hours worked predicted
by the model are in the range of 0.22 to 0.38—significantly below the data. This implies that implausibly low aggregate labor supply elasticities are required in our model to choke off the World War II economic expansion.

The third set of experiments allows for variable capacity as in Kydland and Prescott (1991). This experiment allows for endogenous variation in the Solow residual. We therefore remeasure the Solow residual using a production function that has variable capacity utilization. We find that the predictions for the variable capacity model are very similar to those presented for our benchmark model. Because of this similarity, we report all results for the capacity utilization model separately in McGrattan and Ohanian (2006).

In summary, our sensitivity analysis shows that the results are robust to large changes in tax rates, in the labor supply elasticity, and also to the inclusion of variable capacity utilization.

10. Conclusion

We have found that standard neoclassical theory accounts for the large changes in macroeconomic variables during World War II. Specifically, the time paths of output, consumption, investment, labor input, wage rates, and returns to capital from the model in response to World War II government spending, draft, tax rate, and productivity shocks are very similar to those in the data. Perhaps the most striking finding is that the deviation between the model and the data during this period of very large shock is about the same size as or smaller than the deviations in similar models during the much more stable and less regulated postwar period. This is surprising because the view in the profession is that simple models perform poorly when confronted with shocks and policies that are far outside the norm. This view is summarized by David Romer, who remarked that “even in normal times, the best model is just a guide. If something extraordinary happens, like either Russia goes under or the stock market goes down by 20 percent, anyone with a modicum of common sense knows that the model’s not going to be a reliable guide” (Altman 2005).

The behavior of the U.S. macroeconomy during World War II is very similar to the
prediction of the optimal growth model, despite a number of nonmarket factors, such as rationing, price controls, and the Federal Reserve’s fixing of nominal interest rates on U.S. Treasury securities. It is widely acknowledged that the U.S. wartime economic boom was a key factor in winning World War II, and the government chose a fiscal mix of government consumption, investment, debt, and taxes that fostered an economic boom. Alternative policies with much higher tax rates (such as those considered in Section 9) were considered seriously by President Roosevelt and Congress during the war, but fortunately were not chosen. If high tax policies had been chosen, our analysis predicts that there would not have been a large output expansion. This prediction is consistent with the analysis of Cooley and Ohanian (1997), who found that the very high tax policies adopted by the United Kingdom during World War II prevented a wartime economic boom in Britain. Winning major wars requires big economic expansions; luckily, the United States chose a World War II policy that fostered the necessary increase in output to win the war.
Appendix A. Stochastic Simulations

This appendix describes how we generate the transition matrices ($\Phi$) for the Markov chains over the six states used in our stochastic simulations. There are a maximum of seven possible states: a state with 1941 values of exogenous variables, a state with 1942 values of exogenous variables, and so on up to 1945, a postwar state with no depression, and a postwar state with depression. The states for 1942–1945 are called the war states. To keep the exposition as simple as possible, we omit the depression state in the discussion that follows.

To choose the probabilities over these states, we randomly construct transition matrices and keep those that generate (i) the frequency of the outbreak of war, (ii) the average duration of war, and (iii) the fraction of years spent in war, that are between 70 percent to 130 percent of the United States pre–World War II averages for these statistics. (See Table 1.) Thus, households’ expectations regarding the frequency and duration of war are equal to the average values from previous U.S. wars. We place no other restrictions on these probabilities so that we can obtain a wide range of expectations and thus check the sensitivity of our results to differences in expectations.

We first generated candidate matrices by drawing each element of $\Phi$ independently from the same distribution (and then normalizing columns so they sum to 1). This approach was inefficient as it yielded very few Markov chains that satisfy the above criteria. A much more efficient procedure is to generate probabilities that place relatively more mass on the first off-diagonals of $\Phi$ so that the probability of transiting to the next phase of the war is sufficiently high as to obtain more draws that satisfy the above duration and frequency criteria.

In particular, we preset a parameter $\alpha \in [0, .5]$ and then use the following algorithm for $n = 1, \ldots N$, where $N$ is large:

- Initialize elements of the candidate transition matrix, $\Phi_n(i, j) = 0$
- For $j = 1, \ldots 5$,
  - Draw $\zeta \in \text{uniform}[0, 1]$ and set $\Phi_n(j + 1, j) = \alpha + .5\zeta$
  - Draw 5 more uniform random variables on $[0,1]$ for the remaining elements of column $j$, normalizing them so the probabilities sum to 1
- For $j = 6$,
  - Draw $\zeta \in \text{uniform}[0, 1]$ and set $\Phi_n(6, j) = .9 + .1\zeta$
  - Draw 5 more uniform random variables on $[0,1]$ for the remaining elements of column 6, normalizing them so the probabilities sum to 1
- Check to see if candidate $\Phi_n$ is within 70 percent to 130 percent of U.S. averages for the above criteria. If so, keep. Continue to next $n$. 
With \( \alpha = 0.5 \), the algorithm allows off-diagonals to range from 0 to 1, with probabilities over 0.5 more likely. To ensure that we span the probability space, we also repeat the procedure for values of \( \alpha \) (discretely) chosen between 0 and 0.5. The probabilities that we construct span most of the relevant probability space in the transition matrix. The following matrix shows the maximum probabilities across all matrices \( \Phi_k \) that were kept for simulations:

\[
\max_k \Phi_k = \begin{bmatrix}
0.63 & 0.52 & 0.65 & 0.54 & 0.65 & 0.04 \\
1 & 0.51 & 0.60 & 0.58 & 0.57 & 0.04 \\
0.57 & 1 & 0.53 & 0.74 & 0.53 & 0.04 \\
0.51 & 0.57 & 1 & 0.60 & 0.64 & 0.04 \\
0.54 & 0.62 & 0.61 & 1 & 0.58 & 0.04 \\
0.53 & 0.51 & 0.48 & 0.62 & 1 & 0.99 \\
\end{bmatrix}
\]

The minimum probabilities are \( \min_k \Phi_k(i, j) = 0 \) for all elements \((i, j)\) except element \((6,6)\), which has \( \min_k \Phi_k(6,6) = 0.93 \). Matrices with low values in element \((6,6)\) violate the criterion governing the duration of the war.

Following the generation of these transition matrices, we then compute the equilibrium for the model economy for each of the accepted transition matrices, feeding in the realizations sequentially from 1941 to 1945, and the postwar realization. Figures 5–13 show the maximum and minimum values for each of the endogenous variables, and the shading in the figure shows the relative mass of the distribution between the minimum and maximum values.

In one experiment, we allow for the possibility of a postwar Depression. Participants in a Gallup poll in September of 1941 were asked, “Do you think we are likely to have a greater prosperity, or another depression after the present war?” Seventy-eight percent of respondents said, “Another depression.” In June of 1942, Gallup poll participants were asked, “Which do you think the United States will have for the first two or three years after the war—depression or prosperity?” At that time, 43 percent said, “Depression.” In June of 1944 and again in May of 1945, a Roper poll asked, “Do you expect we probably will have a widespread depression within 10 years or so after the war is over, or do you think we probably will be able to avoid it?” Fifty-one percent said, “Depression” in 1944 and 44 percent in 1945.\(^{20}\) We use these wartime survey responses as the model expectations for the postwar state of depression for each year, respectively. (See Table 1.)
Appendix B. Alternative Measures of Factor Returns

In Section 7 we used marginal products constructed from U.S. data to assess the model’s predictions for returns to capital and labor. In theory, we can also use factor incomes per unit of input and, in the case of capital, returns on assets held by capital owners. For labor, we will see that the marginal product from the model accords well with measures of compensation per hour. For capital, we will see that the marginal product on average is consistent with returns based on capital incomes and stocks and with equity returns. We will explain why another measure of returns—based on debt assets—is not the appropriate measure for our model economy because of regulations and restrictions impacting the bond markets.

A. Return to Labor

Figure B1 shows indices of the pre-tax marginal product of labor in our model and three different pre-tax real wage series: nonfarm compensation per hour, full-time equivalent compensation per hour, and manufacturing compensation per hour. All compensation rates are divided first by the deflator for GNP less military compensation and then by the growth trend for technology \((1 + \gamma_A)t\). Along with these compensation rates, we plot labor productivity for the model from Figure 4. Each series is normalized to the value of 100 in 1941.

The two broadest measures of aggregate real compensation per hour—nonfarm and full-time equivalent—are very similar to the marginal product of labor from the model. These compensation measures are the most reasonable comparison to the model marginal product, because they are economy-wide wage measures and because they are much less affected by changes in union bargaining power than is manufacturing compensation per hour. In particular, there were substantial declines in union bargaining power during World War II. Cole and Ohanian (2004) argue that manufacturing wages rose sharply in the mid- to late 1930s as a consequence of large increases in union bargaining power, and that this bargaining power declined sharply during the 1940s. During the war, wages were no longer set by collective bargaining, but rather by the National War Labor Board (NWLB), which routinely rejected negotiated wage settlements between firms and unions. After the war, the Taft-Hartley Act further reduced union bargaining power. Cole and Ohanian estimate that most of the increase in manufacturing wages generated by unions/cartels in the 1930s had vanished by 1947 as a result of the NWLB decisions. This finding is consistent with the decline in manufacturing compensation observed in Figure B1. It is reasonable to expect that introducing union bargaining, and distinguishing between manufacturing and non-manufacturing sectors, would allow the model to account for differences between manufacturing and non-manufacturing compensation. However, this is well beyond the scope of this analysis.

B. Return to Capital

In Figure 4, we used the marginal product of capital when comparing the return to capital in the data and model. Here, we discuss three alternative measures for the empirical counterpart of our model’s return: a measure of the return on capital found by dividing capital income by the capital stock, a measure of the return on equities, and a measure of
the return on bonds.

First, we can compare the model’s return on capital to some measure of capital income divided by the appropriate stock—both nominal since the Bureau of Economic Analysis (BEA) does not report real incomes. Before making these comparisons, we need to address several issues that arise when doing these calculations. One issue is data revisions: there have been significant revisions in nominal stock estimates across BEA reports, especially for structures. For example, for the period 1941–1946, the BEA estimates show an average level for the current-cost net stock of private nonresidential structures of $126.4 billion in the U.S. Department of Commerce (2002) and $70.4 billion in U.S. Department of Commerce (1987). Another issue that arises, especially in the case of World War II, is the possibility that factor payments by government to business are not allocative period-by-period. For example, in cases where the capital was government owned but privately operated, many contracts prespecified the returns on the investment. (See Braun and McGrattan 1993.) If capital returns are not allocative, then we can only compare average marginal products with average per-unit incomes. A related issue is how to attribute returns to assets, such as government assets, whose incomes are not included or imputed in the national income and product accounts (NIPA).

McGrattan and Prescott (2003), using recent BEA data, estimate a return to noncorporate capital of 4 percent over the period 1929–2000; over the period 1941–1946, the average return for their series is 4.95 percent. They impute a 4 percent return to government capital, which may be high for the World War II period. They also leave out the corporate sector because of issues with estimating returns to intangible capital. If we redo their calculation imputing a 0 percent return to government capital and adding in measured corporate income less tax (in the numerator) and measured corporate capital (in the denominator), then the average we compute is 4.5 percent over the period 1941–1946. As McGrattan and Prescott
point out, however, this is an overestimate of the true return because measured corporate income does include part of the income to intangible capital while measured corporate capital does not. This puts the estimate of the average closer to the average of the after-tax marginal product of capital, which is 3.55 percent for the U.S. data and 3.57 percent for the perfect foresight model.

Another possible comparison that can be made is the model’s return on capital and the return on corporate equities. When comparing the marginal product of capital with any equity return, the main deviation is in the volatility rather than the means. For example, averaging the inflation-adjusted total returns for the Standard and Poor’s (S&P) Composite reported in the Ibbotson Associates (2001) yearbook for the period 1941–1946, we find an average of 6.87 percent. If we adjust for taxes on dividends using McGrattan and Prescott’s (2003) tax rate on dividends times the S&P income return from Ibbotson Associates, then the average return is 4.1 percent. This slightly overestimates the actual equity return because of taxes on capital gains; however, these adjustments are difficult to calculate given the fact that only realized gains are taxed. If we abstract from taxes on capital gains, the average equity return is only 50 basis points higher than the model’s predicted return.

McGrattan and Prescott (2003, Figures 3–5) find that differences in average returns on bonds, stocks, and NIPA capital are not large except in the case of debt returns during the wartime period that includes World War II and the Korean War. Specifically, averaging inflation-adjusted total returns reported in the Ibbotson Associates (2001) yearbook for the period 1941–1946, we find corporate bonds earned −3.7 percent, long-term government bonds earned −3.5 percent, intermediate-term government bonds earned −5.1 percent, and U.S. Treasury bills earned −6.4 percent. These very low numbers for debt returns have been noted in earlier work as puzzling for neoclassical theory. (See, for example, Mulligan 1998.)

But these returns are not a good measure for the household intertemporal marginal rate of substitution, which is the relevant object in the model. This is because of several restrictions and regulations that impacted markets for debt assets. Federal government debt comprised about 5 percent of total household assets in 1945. Most Federal debt was held by the Federal Reserve system, commercial banks, insurance companies, pension funds, state and local governments, and government retirement accounts. Almost all of these institutions were restricted in terms of the assets they could hold; they typically could not hold equity, and their portfolios were held primarily in credit market instruments (debt) or cash. For example, life insurance companies, banks, savings institutions, pension funds, and state and local government retirement programs held between 50 and 65 percent of their assets in U.S. Treasury securities in 1945, compared to only about 5 percent of household assets in federal government debt. Nominal interest rates on Treasuries ranged between 3/8 of 1 percent on short-term debt and 2.5 percent for long-term debt. One policy which contributed to the Federal Reserve’s ability to fix nominal rates was by advancing reserves to banks at par for Treasuries. There was excess demand for long-term Treasury debt, and consequently some of this debt was allocated only to commercial banks. This interest rate control policy was abandoned in 1951 with the Treasury–Federal Reserve Accord, which laid the foundation of the independence of the Federal Reserve.

Of the small amount of federal debt that was held by households, roughly 67 percent was held in savings bonds. These savings bonds had their own set of idiosyncratic attributes:
there was no risk of capital loss, owners were insured against loss or theft, the securities were non-marketable, acquisition costs were very low (many savings bonds were acquired through payroll deduction), they were available in small denominations (as low as $25), and interest rates on some of these securities were higher than on long-term Treasury securities. Savings bonds were primarily accumulated by relatively low income households who found the low denominations and low acquisition costs attractive, and who in fact typically paid no taxes on the income from these securities. Low income households at this time had few other investment options, as the cost of holding a diversified equity portfolio was high, and minimum denominations for other forms of debt were typically much higher than for savings bonds. (See Board of Governors 1944.) To satisfactorily address U.S. debt returns during World War II would require modeling the details of regulations affecting the bond markets, which is well beyond the scope of this paper.

In summary, we have shown that the model’s predicted wage rate is consistent with standard measures of U.S. labor productivity and compensation per hour. We have shown that the model’s predicted return to capital is consistent with a measure of the return to NIPA capital based on the marginal product of capital; it is consistent on average with a measure of the return based on the ratio of capital income to the capital stock and with a measure of the return to corporate equities.
Notes

1By the neoclassical model, we mean the one-sector optimal growth model with a constant returns to scale technology with capital and labor, a standard law of motion for the capital stock, balanced growth preferences defined over consumption and leisure, a resource constraint that divides output between consumption, investment, and government spending, and perfect competition in all markets.

2A few studies have used the neoclassical model to address a limited set of questions about World War II. Braun and McGrattan (1993) use a stochastic model with World War II government expenditure shocks and focus on whether the model is consistent with pre-tax real wage changes during the war. However, their analysis omits all other shocks and does not provide a systematic test of the other variables in the model. Ohanian (1997) uses a perfect foresight neoclassical model for normative rather than positive purposes. He measures the welfare costs of the different war finance policies used in World War II and the Korean War. His analysis does not shed light on the present disagreement about the appropriate theoretical framework for analyzing large fiscal shocks.

3Rotemberg and Woodford (1992) analyze data on military expenditures and real wages from World War II, and also from other years. The World War II observations are likely the most informative because they are by far the largest expenditure shocks and also the most exogenous of the increases in military spending. They do not perform an assessment of the model for World War II.

4Analyzing these smaller episodes is also complicated because exogeneity of the spending, tax, and draft shocks may be harder to justify, and thus would require filtering out the exogenous component of the fiscal changes.

5Most of these R&D expenditures were outside of the Department of Defense. Real defense spending rose from $30 million in 1930 to $425 million in 1945, which implies that about 2/3 of these R&D expenditures were outside of defense. We have been unable to find measures of private R&D spending over this period, but it is almost certain that the sum of private and public R&D spending increased significantly during the war. In particular, the total number of scientists and engineers employed in the manufacturing sector almost doubled between 1940 and 1946. (See Mowery and Rosenberg 2000.)

6We stress that the low returns on Treasury securities during the war is an interesting question in its own right. The fact that the Federal Reserve’s price fixing was successful, when other government price fixing schemes failed, is certainly noteworthy. We suspect, as do McGrattan and Prescott (2003), that the wartime regulations and restrictions may be central for understanding the low nominal interest rates on Treasuries. Additionally, the “disaster” models developed by Rietz (1988) and more recently by Barro (2005) may be important. Clearly more work is required to understand government debt returns during the war, but this is well beyond the scope of the paper.

7This model of the draft, which explicitly preserves the representative agent construct, differs from that in Ohanian (1997) in which some families were hit by the draft and others
were not. Ohanian preserves the representative agent construct by assuming separable consumption and leisure and by assuming that military labor income and private labor income are identical.

8Because preferences are separable between consumption and leisure, consumption for civilians and draftees will be equated.

9We include the possibility of transfers because it will let us examine how changing the quantity of debt issued by the government (by allowing a fraction of expenditures to be financed with lump-sum taxes) affects the results.

10We experimented with alternative specifications for the peacetime state by taking an average over multiple years in the post–World War II period. Our findings are robust across these alternatives because the values of the 1946 exogenous variables are very similar to their average values in the decades following the war.

11Testimony was given by Harold Moulton, president of the Brookings Institution; A. F. Hinrichs, acting commissioner of the Bureau of Labor; Matthew Noll of the American Federation of Labor; and Robert Nathan, representing the Commission for Economic Development.

12These statistics are calculated from the American Revolutionary War, the War of 1812, the Mexican War, the American Civil War, the Spanish American War, World War I, and World War II.

13Specifically, the maximum (minimum) value for a variable at each date is the maximum (minimum) across all realizations for that date.

14We subtract indirect business taxes for sales from both GNP and personal consumption expenditures. We impute a service flow for durables equal, in real terms, to 4 percent times the stock of durables.

15For our simulations, we ensure that the present value of peacetime tax revenues is sufficient to cover the government debt accumulated during the war.


17Relatively few papers in the RBC literature conduct actual time path analyses in which there is a comparison at various dates between model endogenous variables and actual variables; instead, most of the studies compare some subset of second moments from the model to the data.

18Some readers may be interested in trying to understand the results by decomposing the impact of the shocks into wealth effects and substitution effects, as in Barro (1981) and Hall (1980). This is complicated as the shocks are realized over different points in time, which generates sequences of wealth, intratemporal substitution, and intertemporal effects and intra- and intertemporal substitution effects that are not easy to understand. Our
alternative approach of evaluating the contribution of each shock provides a much easier way of understanding the factors that are driving these results.

19 There are some measurement differences between our analysis and those of Mulligan (1998) and Rotemberg and Woodford (1992), though they are not central for our findings. Our measure of the price deflator excludes military compensation, which is appropriate for our model. Mulligan deflates the wage using the CPI. Rotemberg and Woodford deflate World War II wages using the GNP deflator, which includes military compensation. Finally, we construct an after-tax wage using Joines’ (1981) estimate of the labor tax rate. Mulligan uses the Barro-Sahasakul (1986) tax rate that mixes tax rates on labor and capital income. Our results are robust to these differences in tax rates and most of the differences in price indices.

20 We did not have survey results for 1943 and therefore chose a probability of postwar depression intermediate to our estimates of 1942 and 1944.

21 All of our wage measures exclude estimates of wages of farm proprietors because, in general, it is hard to estimate the fraction of proprietor’s income that is labor income and, more specifically, because the relative price of farm output nearly doubled during World War II. Accounting for this enormous relative price change is beyond the scope of our one-sector model. It should be noted that the wages of farm employees are included in our full-time equivalent wage measure.

22 Some authors, including Mulligan (1998) and Rotemberg and Woodford (1992), have used average hourly earnings in 25 manufacturing industries as a wage measure for the manufacturing sector. This series is deficient in that it does not include non-wage compensation, which was significant in World War II, and it also does not cover the entire manufacturing sector.
References


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Table 1  
Parameter Values for Model Simulations

<table>
<thead>
<tr>
<th>Preferences</th>
<th>$\psi = 2.37, \xi = .0, \beta = .985, \bar{I}_d = 50/84$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology</td>
<td>$b = 1/2, \rho = 1, \theta = .38, \delta = .055$</td>
</tr>
<tr>
<td>Growth</td>
<td>$\gamma_n = .015, \gamma_z = .02$</td>
</tr>
</tbody>
</table>

Restrictions on Markov Chain$^\dagger$

- Average duration of war in $[2.6, 4.8]$ years
- Fraction of years a war is started in $[2.9, 5.3]$ percent
- Fraction of years in war in $[10.6, 19.8]$ percent

Probabilities of Postwar Depression$^{\dagger\dagger}$

<table>
<thead>
<tr>
<th>Year</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>in 1941</td>
<td>78%</td>
</tr>
<tr>
<td>in 1942</td>
<td>43</td>
</tr>
<tr>
<td>in 1943</td>
<td>47</td>
</tr>
<tr>
<td>in 1944</td>
<td>51</td>
</tr>
<tr>
<td>in 1945</td>
<td>44</td>
</tr>
</tbody>
</table>

$^\dagger$ Only relevant for stochastic simulations.

$^{\dagger\dagger}$ The probability is conditional on the war ending.
Figure 1. U.S. Government Spending, Tax Rates, Draft, and TFP, 1941–1946

Notes:
(1) Government spending series are real and detrended by dividing by the population over 16 and by the growth trend in technology (scaled so the 1946 real detrended level of GNP less military compensation equals 1).
(2) Total factor productivity is defined to be \( Y / (K^{1-\theta}L^\theta_p) \), where \( Y \) is real, detrended GNP less military compensation, \( K \) is real detrended nonmilitary capital stock, \( L^\theta_p \) is nonmilitary hours worked, and \( \theta = .38 \).
Figure 2. Real Detrended GNP, Private Consumption, and Private Investment, 1941–1946
(Benchmark Deterministic Model)

Note: Data series are divided by the 1946 real detrended level of GNP less military compensation.
Figure 3. Per Capita Total and Nonmilitary Hours of Work, 1941-1946
(Benchmark Deterministic Model)

Legend
U.S. Data
Model

Note: Hours series are divided by the 1946–1960 U.S. averages.
Figure 4. After-tax Return to Capital and Nonmilitary Labor Productivity, 1941–1946
(Benchmark Deterministic Model, All Series Constructed Using Marginal Productivities)

Legend
- U.S. Data
- Model

Note: Return to capital is equal to $100(1-\tau_k)(\theta Y/K-\delta)$.
Labor productivity is nonmilitary output divided by hours that are normalized by the 1946–1960 U.S. average.
Figure 5. Real Detrended GNP, Private Consumption, and Private Investment, 1941–1946 (Benchmark Stochastic Model)

Legend
U.S. Data
Probability of Value in Model

Note: Data series are divided by the 1946 real detrended level of GNP less military compensation.
Figure 6. Per Capita Total and Nonmilitary Hours of Work, 1941–1946
(Benchmark Stochastic Model)

Legend

U.S. Data

Probability of Value in Model

Note: Hours series are divided by the 1946–1960 U.S. averages.
Figure 7. After-tax Return to Capital and Nonmilitary Labor Productivity, 1941–1946
(Benchmark Stochastic Model, All Series Constructed Using Marginal Productivities)

Legend
U.S. Data
Probability of Value in Model

Note: Return to capital is equal to $100(1-\tau_k)(\theta Y/K-\delta)$.
Labor productivity is nonmilitary output divided by hours that are normalized by the 1946–1960 U.S. average.
Figure 8. U.S. Data and Stochastic Model Predictions, 1941–1946
(Model with Government Spending Constant)

Legend
U.S. Data
Probability of Value in Model

Note: See notes to Figures 5–7 for series normalizations.
Figure 9. U.S. Data and Stochastic Model Predictions, 1941–1946
(Model with Number of Draftees Constant)

Legend
U.S. Data
Probability of Value in Model

Note: See notes to Figures 5–7 for series normalizations.
Figure 10. U.S. Data and Stochastic Model Predictions, 1941–1946
(Model with Labor Tax Rate Constant)

[Graphs showing GNP, Consumption, Investment, Nonmilitary Hours, Return to Capital, Labor Productivity for years 1941 to 1946]

Legend
U.S. Data
Probability of Value in Model

Note: See notes to Figures 5–7 for series normalizations.
Figure 11. U.S. Data and Stochastic Model Predictions, 1941–1946 (Model with Capital Tax Rate Constant)

Legend

U.S. Data
Probability of Value in Model

Note: See notes to Figures 5–7 for series normalizations.
Figure 12. U.S. Data and Stochastic Model Predictions, 1941–1946
(Model with Technology Constant)

Legend

U.S. Data
Probability of Value in Model

Note: See notes to Figures 5–7 for series normalizations.
Figure 13. U.S. Data and Stochastic Model Predictions, 1941–1946 (Model with Postwar Depression State)

Legend
U.S. Data
Probability of Value in Model

Note: See notes to Figures 5–7 for series normalizations.
Figure 14. U.S. Tax Rate on Labor and Tax Rate Needed in Model for No Change in Nonmilitary Hours of Work, 1941–1946