A NOTE ON MODELLING MONEY DEMAND IN GROWING ECONOMIES

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ABSTRACT

A prominent feature of US data is the lack of cointegration between nominal interest rates and M1 velocity. Yet, most general-equilibrium monetary models that have been used for empirical analysis have imposed cointegration between these two series. This paper presents as an alternative a money-in-the-utility function model which does not imply cointegration even though a well-defined stationary monetary equilibrium exists.

I. INTRODUCTION

It is now a common practice in applied monetary economics to construct general equilibrium models whose parameters can be chosen (i.e. calibrated) so that the equilibrium behaviour of the model matches critical features of the data. Prominent within this approach are the use of cash-in-advance models and models in which real balances directly enter preferences. Recently, the latter framework has been employed with greater frequency owing to two observations. First, theoretically, the money-in-the-utility function (MIUF) approach is analogous to a model in which money reduces transactions costs associated with

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purchases. Second, empirically, the CIA model typically leads to the counterfactual implication that velocity is constant (see, for example, Hodrick, et al. (1991)); hence the MIUF model permits a richer model of money demand.

The characteristics of money demand in the MIUF framework will, however, be greatly dependent upon the choice of functional form for preferences; this dependence was examined within the context of the optimal quantity of money recently by Mulligan and Sala-i-Martín (1997). Also, Farmer (1997) was able to introduce sunspot equilibria in a monetary model by the judicious choice of preferences over real balances. Both papers illustrate that it is critical to understand fully the implications which are imposed by the adoption of a particular utility function. This paper attempts to further that understanding by examining the homogeneity assumption commonly employed in MIUF models. Specifically, it is typically assumed that utility is homogeneous in real balances and consumption; some examples in the literature which have employed this assumption are Danthine and Donaldson (1986), Danthine et al. (1987), Finn et al. (1990), Eckstein and Leiderman (1992), Bental and Eckstein (1997) and Walsh (1998). Critically, imposing this assumption implies that nominal interest rates and velocity are cointegrated. We demonstrate in Section II that this restriction is not supported by the data. We then, in Section III, propose an alternative utility function which does not impose cointegration between these variables even though a well-defined stationary monetary equilibrium exists. The equilibrium characteristics of the model are further discussed in that section and some concluding comments are offered in Section IV.

II. ARE NOMINAL INTEREST RATES AND VELOCITY COINTEGRATED?

We first establish that the answer to this question is no. Short-term interest rates are measured as the (gross) yield on 3-month Treasury Bills, while velocity is defined as the ratio of nominal consumption of nondurables and services to the monetary aggregate M1.1 (All series are from the St. Louis Federal Reserve Bank database FRED; the sample period is 1960:1–1998:4.) Using the natural logs of both series, we examine their

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1Since M1 is more liquid than M2, it fits more into the transaction role of money consistent with the MIUF approach and that is why we used this narrow monetary aggregate. Also, we compute consumption velocity because in the MIUF model without storage, market clearing requires consumption to be equal to the endowment. We examined the time-series properties of the income velocity of money and did not find it to be cointegrated with 3-month T-Bill yields, either. For brevity, we do not report these results.

testing the null of no cointegration between velocity and 3-month T-Bill rates

<table>
<thead>
<tr>
<th>Trend assumptions</th>
<th>Eigenvalue</th>
<th>Likelihood ratio</th>
<th>5% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No trend</td>
<td>0.0426</td>
<td>6.64</td>
<td>15.41</td>
</tr>
<tr>
<td>Linear deterministic trend</td>
<td>0.0890</td>
<td>19.95</td>
<td>25.32</td>
</tr>
</tbody>
</table>

Cointegration properties by employing the cointegration test procedure suggested by Johansen and Juselius (1990). Theorem 1 reports the results of this procedure for alternative trend specifications. A lag length of 4 is used which is appropriate for quarterly series. Changing the lag length and trend specification did not alter the cointegration results.

The maximum eigenvalue tests clearly fail to reject the null of no cointegration between 3-month T-Bill yields and velocity. The next section presents a recursive general equilibrium model which is consistent with this finding.

III. THE MODEL

In order to study the behaviour of velocity and nominal interest rates, we construct an infinite-horizon exchange economy in which the level of the endowment, $x_t$, and money stock, $\bar{M}_t$, are nonstationary. The evolution of these series is represented as $x_{t+1} = \rho x_t$ and $\bar{M}_{t+1} = \mu \bar{M}_t$; uncertainty is introduced through the assumption that both growth rates are stochastic. The vector of growth rates, $s_t = (\rho_t, \mu_t)$, is assumed to follow a stationary first-order Markov process with transition function $F(s', s) = \Pr(s_{t+1} \leq s' \mid s_t = s)$ and support $S \subset \mathbb{R}^2$. Agents are assumed to be identical and at the beginning of each period receive the endowment, a lump-sum monetary transfer, $(\mu_t - 1)\bar{M}_{t-1}$ (the bar denotes the aggregate money stock), and the returns from one-period nominal bonds purchased in the previous period, $B_{t-1}(1 + n_{t-1})$. These revenues together with money balances acquired in the previous period are used for consumption, $c_t$, bond purchases and the revision of money balances. Hence, the nominal budget constraint in every period is:

$$P_t x_t + M_{t-1} + B_{t-1}(1 + n_{t-1}) + (\mu_t - 1)\bar{M}_{t-1} = P_t c_t + M_t + B_t \tag{1}$$

where $P_t$ denotes the nominal price level.

The standard augmented Dickey–Fuller tests were run to verify that both series are integrated of order 1. The unit-root tests fail to reject the null of a unit root for a lag length 4. The MacKinnon statistics for velocity and 3-month T-Bill yields are, respectively, $-1.27$ and $-2.26$, which are higher than their respective 5 per cent MacKinnon critical values: $-2.88$ and $-3.44$.

Consumption and asset choices are made in order to maximize expected utility represented by:

$$E_t \left\{ \sum_{j=1}^{\infty} \beta^{j-1} U \left( c_t, \frac{M_t}{P_t} \right) \right\}; \quad 0 < \beta < 1$$

(2)

with the one-period utility function assumed to be of the form:

$$U \left( c_t, \frac{M_t}{P_t} \right) = \begin{cases} 
  \frac{c_t^{1-\gamma} \left( M_t / P_t \right)^{1-\delta}}{1-\gamma} + \frac{1}{1-\delta} \ln c_t + \ln (M_t / P_t) & \text{for } (\lambda, \delta) \neq 1 \\
  \frac{\left[ c_t^{1-\gamma} (M_t / P_t)^{1-\delta} \right]^\theta}{\theta} & \text{for } \theta \neq 0 \\
  \varepsilon \ln c_t + (1-\varepsilon) \ln (M_t / P_t) & \text{for } \theta = 1.
\end{cases}$$

(3)

As noted in the introduction, this specification of preferences differs from that used in previous M1UF analyses in that, if $\gamma \neq \delta$, utility is not homogeneous in consumption and real balances. The importance of this departure is explored at length below.

Most studies have typically used the functional form

$$U \left( c_t, \frac{M_t}{P_t} \right) = \begin{cases} 
  \frac{[c_t^{1-\gamma} (M_t / P_t)^{1-\delta}]^\theta}{\theta} & \text{for } \theta \neq 0 \\
  \varepsilon \ln c_t + (1-\varepsilon) \ln (M_t / P_t) & \text{for } \theta = 1.
\end{cases}$$

(4)

Clearly, this specification of preferences implies $U(\cdot)$ is homogeneous of degree $\theta$ in consumption and real balances. In contrast, the preferences in (3) do not impose homogeneity when $\gamma \neq \delta$. This distinction is important because of the ramifications for velocity in a stationary equilibrium. That is, in a stationary monetary equilibrium the marginal rate of substitution between consumption and real balances will itself be a stationary function of the underlying state variables. (This will be demonstrated below.) As is easily verified, the marginal rate of substitution implied by the preferences in (4) is proportional to the inverse of velocity so that, in equilibrium, the level of velocity will also be stationary. As discussed in the previous section, this prediction is at odds with the observed characteristics of M1 velocity. Relaxing the restriction of homogeneity thus permits nonstationarity of velocity.

Returning to the derivation of equilibrium, the optimal choices of $(c_t, M_t, B_t)$ implied by the agents’ maximization problem are characterized by the following necessary conditions:

$$\frac{U_{1,t}}{P_t} = U_{2,t} + \beta E_t \left( \frac{U_{1,t+1}}{P_{t+1}} \right)$$

(5)

$$\frac{U_{1,t}}{P_t} = (1 + n_t) \beta E_t \left( \frac{U_{1,t+1}}{P_{t+1}} \right)$$

(6)
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where $U_{i,t}$ represents the derivative of $U(\cdot)$ with respect to the $i$th argument, and $E_t$ denotes expectations conditional on information at time $t$. As is well known, these expressions represent the intertemporal marginal utility tradeoffs implicit in revising money and bond holdings. For example, the utility costs of foregoing current consumption in order to acquire an additional dollar is represented by the left-hand side of (5). The additional dollar increases utility through greater current real balances and increased consumption next period; these effects are reflected in the right-hand side of (5). At an optimum, these marginal costs and benefits must be equal. The interpretation of (6) for nominal bonds is analogous.

Using (6) to eliminate the expectational term in (5) yields:

$$
\frac{U_{2,t}}{U_{1,t}} = \frac{n_t}{1 + n_t}.
$$

That is, optimality requires the marginal rate of substitution between money and consumption to be equal to the present discounted value of the interest cost of borrowing a dollar. But, in addition to describing an individual’s optimum, (7) also has an important implication, alluded to above, for the characteristics of a stationary equilibrium. That is, if a stationary equilibrium exists, then nominal interest rates (i.e. the real opportunity cost of money) will be a stationary function of the exogenous state only (i.e. the vector of growth rates). Consequently, (7) implies that the ratio of marginal utilities in a stationary equilibrium will also be a (stationary) function of the exogenous state variables.

We use this insight to solve for a stationary equilibrium in this growing economy. First use the functional form in (3) and the market-clearing conditions, $c_t = x_t$ and $\bar{M}_t = \bar{M}_t$, to express (5) as:

$$
x_t^{-\gamma} = (\bar{M}_t / P_t) + \beta E_t [x_{t+1}^{-\gamma} (P_t / P_{t+1})].
$$

Based on the insight of (7) we conjecture that a stationary equilibrium can be defined in terms of the function $\lambda_t = \lambda(\rho_t, \mu_t) = (U_{1,t} / U_{2,t}) = \bar{M}_t / (x_t^{-\gamma} P_t)$. If this conjecture is correct, then the ratio of price levels, $P_t / P_{t+1}$, can be expressed in terms of consumption and monetary growth rates as:

$$
\frac{P_t}{P_{t+1}} = \alpha^{(1/\delta)}(\rho_t^{(1/\delta)} / \mu_t) (\lambda_{t+1} / \lambda_t)^{(1/\delta)}.
$$

Using this expression in (8) and rearranging yields the following functional equation in $\lambda(\rho_t, \mu_t)$:

$$
\lambda_t^{(1/\delta)} = \lambda_{t+1}^{(1-\delta)/\delta} + \alpha^{(1/\delta)} \beta E_t (\rho_t^{(1-\delta)/\delta} (\lambda_{t+1} / \mu_{t+1})).
$$

Proving the existence and uniqueness of equilibrium would, in general, consist of establishing that the above functional equation implies a contraction mapping in \( \lambda(\cdot) \).

The time-series characteristics of equilibrium velocity, \( \nu_t \), follow directly from the definition of \( \lambda(\cdot) \):

\[
\nu_t = \frac{P_t \epsilon_t}{M_t} = (1/\lambda_t)(\epsilon_t^{1-\gamma})(M_t/P_t)^{\delta-1}.
\]

(11)

The implication of (11) is that velocity will be nonstationary given the nonstationarity of consumption and real balances.

Intuition for the interaction between the specification of preferences and the behaviour of velocity can be enhanced by considering a nonstochastic version of this economy (i.e. both consumption and monetary growth rates are constant). In this setting, a steady-state equilibrium is characterized by a constant value \( \lambda \) which, as discussed above, implies that the marginal rate of substitution between consumption and real balances is constant. Because both consumption and real balances are growing in this economy, a constant value of \( \lambda \) implies that the marginal utilities of these terms must be changing (i.e. declining) at exactly the same rate. Note that the parameters \( \delta \) and \( \gamma \) represent the elasticities of the marginal utility of consumption and real balances respectively. Consequently, if \( \gamma \neq \delta \), then the growth rate of real balances can not be equal to that of consumption in a steady-state equilibrium.

To express this algebraically, \( \% \Delta \lambda = \% \Delta U_1 - \% \Delta U_2 = 0 \). But, \( \% \Delta U_1 = -\gamma \% \Delta c \) and \( \% \Delta U_2 = -\delta \% \Delta (M/P) \). Consequently, in a steady-state equilibrium,

\[
\left( \frac{\gamma}{\delta} \right) = \frac{\% \Delta (M/P)}{\% \Delta c}.
\]

Since the growth rate of velocity is just the difference between the growth in consumption and real balances, the above expression implies that if \( \gamma < \delta \) then velocity will be increasing; velocity will decline if the inequality is reversed.

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3Note, however, that when \( \delta = 1 \), this task is simplified since (10) becomes \( \lambda_t = 1 + \alpha \beta E[\lambda_{t+1}/\mu_{t+1}] \). In this case, the assumption that \( \alpha \beta E[1/\mu_{t+1}] < 1 \) is sufficient (due to Blackwell’s sufficiency condition) to establish that (10) defines a contraction mapping in the function \( \lambda(\cdot) \) thus establishing the existence and uniqueness of equilibrium and verifying the original conjecture.

4In light of (11) the question is raised whether velocity is stationary with the cointegrating vector of \([\delta - 1, 1 - \gamma]\) associated with the logs of real balances and consumption. However, the definition of \( \lambda \) requires that there be a cointegrating vector \((-\gamma, \delta)\) between the log of consumption and the log of real balances. The uniqueness of equilibrium, moreover, implies that this vector be unique; hence, from (11), this implies that velocity is nonstationary.

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Nominal interest rates will be determined according to (7):

$$\frac{n_t}{1 + n_t} = \frac{1}{\lambda_t}. \quad (12)$$

or, rearranging

$$n_t = 1/(\lambda_t - 1). \quad (13)$$

Since $\lambda_t$ is stationary, the implication of (12) and (13) is that nominal interest rates and velocity will not, in general, be cointegrated; again, this prediction is consistent with the characteristics of the data presented in the previous section.

It is important to note that the absence of cointegration between nominal interest rates and velocity does not imply the absence of a stable money-demand relationship. Indeed, the equality between the discounted nominal interest rate and the marginal rate of substitution between real balances and consumption (see (7)) reflects precisely such a relationship. To see this, combine equations (7), (13) and the definition of $\lambda_t$ to yield:

$$\frac{(M_t/P_t)^{\delta}}{c_t^{\gamma}} = \frac{1 + n_t}{n_t}. \quad (14)$$

As stated in footnote 4, (14) implies a cointegrating vector for real balances and consumption with the residual from regressing real balances on consumption having a stable relationship with nominal interest rates.\(^5\)

IV. CONCLUSION

The assumption that real balances directly yield utility is a convenient one for applied monetary analysis, but the choice of functional form is critical in determining equilibrium characteristics. This paper has demonstrated that imposing homogeneity in consumption and real balances leads to the counterfactual prediction that nominal interest rates and velocity are cointegrated. It is an open question whether the specific functional form for the utility function proposed here, while consistent with the observed low-frequency characteristics of velocity and nominal interest rates, also matches the behavior of these series at business cycle frequencies. It is rarely the case that the modelling choice appropriate for one environment remains so in a different setting. It

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\(^5\) A slightly different characterization of the money-demand relationship in the model is captured in (11). This expression can be written as: $\nu_t^{\delta} = \lambda_t^{-\gamma} c_t^{\gamma (1 + \theta - \gamma)}$. Hence, regressing velocity and consumption will produce a stationary residual that is cointegrated (trivially) with nominal interest rates.

would perhaps be worthwhile to investigate whether the choice of the functional form for the utility function could shed some light into some of the existing asset pricing puzzles in monetary economics as documented in Finn, et al. (1990).

REFERENCES


