PORTFOLIO CHOICE AND
MR. KEYNES’S SPECULATIVE DEMAND
FOR MONEY

1. Introduction: 3 assets and 2 interest rates

Standard IS-LM analysis assumes there is only one interest rate \( r \). This is very unrealistic. In the real world there are many interest rates, depending on the how long you lend your money and at what risk of default. To understand Mr. Keynes’s theory of the speculative demand for money (idle balances), we at least need to distinguish the interest rate on short-term lending \( r_{ST} \) and the interest rate on long-term lending \( r_{LT} \) because Mr. Keynes—unlike textbook Keynesians—mainly focused on \( r_{LT} \).

Hence we now assume an economy with three financial assets: in addition to demand deposits \( D \) paying no interest and time deposits \( T \) paying \( r_{ST} \), there are also long-term bonds \( b \)—which for simplicity we take to be perpetuities—paying \( r_{LT} \).

Let \( P_b \) be the current price of a bond, that is, the present discounted value of its future payments. For perpetuities, the present discounted value formula is especially simple:

\[
P_b = \frac{C}{r_{LT}},
\]

where \( C \) is the coupon on the bond, that is, \( C \) is the amount that the bond promises to pay each year forever.\(^1\) This formula reflects the inverse relation between the price of bonds and the long-term interest rate.

Long-term lending involves interest-rate risk: when you lock into a long rate this year, you are not sure what the long rate will be next year. Perhaps you have locked into lending at too low a rate? The lender’s expected rate of capital gains from long-term lending this year is the amount he expects his bonds to appreciate in value this year, per dollar put into bonds:

\[
g^e_L = \frac{P^e_L - P_b}{P_b},
\]

where \( P^e_L \) is the price of bonds the lender expects will prevail next year. In other words, using the inverse pricing formula, \( P^e_L = \frac{C}{r^e_{LT}} \), where \( r^e_{LT} \) is the long term interest rate the lender expects to prevail next year (that is, his best guess about the value of the long rate next year). His expected rate of return from long term lending equals his current interest income per dollar in bonds plus his expected rate of capital gains:

\[
R^e_{LT} \equiv r_{LT} + g^e_L.
\]

\(^1C\) is paid to the current owner. So if you buy the bond this year and keep it forever, you will get a check for \( C \) dollars each year. But if you only hold the bond this year and then resell it next year, you will get \( C \) dollars interest this year, but next year the \( C \) dollars of interest will go to the new owner.
2. Microeconomics: An Individual’s Portfolio Choice Problem

For simplicity, we will assume that individuals are risk neutral. That is, they choose between financial assets to try to maximize their expected rate of return. Unlike risk-averse individuals, risk-neutral individuals only hold the financial asset(s) with the highest expected rate of return; they do not diversify.

Now at the microeconomic level, consider an individual with $W$ dollars to lend. Think of $W$ as his total Wealth excluding the amount of money he needs to hold in his checking account for transaction purposes. The individual’s portfolio choice problem is to determine what mix of financial assets to hold his Wealth. In our case, there are only 3 financial assets, hence his portfolio choice problem reduces to how much of his $W$ dollars to put into $D$, how much to put into $T$, and how much to put into $b$.

2.1 Portfolio choice when $r_{ST} > 0$

Until subsection 2.3 below, we will simplify the individual’s portfolio choice problem even further by assuming $r_{ST} > 0$. Then he will not put any of his Wealth into $D$: $T$ offers a higher return. Thus, for now we can just focus on his choice between short-term lending $T$ and long-term lending $b$.

Define $r_{\text{min}}$ as the lowest value of $r_{LT}$ such that the individual would still be willing to lend long-term this year. Recall, if he lends long-term, his expected rate of return will be

$$R_{LT}^e \equiv r_{LT} + g_b^e.$$

On the other hand, if he lends short-term, his rate of return will be

$$R_{ST} \equiv r_{ST},$$

since short-term lending involves no interest rate risk (at the end of the year he will get back his original principal plus his interest; there is no chance of a capital loss, that is, his original principal shrinking in size). Consequently, $r_{\text{min}}$ is given by the value of $r_{LT}$ that satisfies the formula

$$R_{ST} = R_{LT}^e,$$

or equivalently,

$$\text{when } r_{LT} = r_{\text{min}} : \quad R_{ST} = r_{LT} + g_b^e \quad (1)$$

The idea is illustrated below.

To see why Equation (1) characterizes $r_{\text{min}}$, observe that if $r_{LT}$ falls even a little below $r_{\text{min}}$, the individual will put all of $W$ into $T$ because $R_{ST} > R_{LT}^e$, so he would do no long-term lending. On the other hand, if $r_{LT} > r_{\text{min}}$, he will put all of $W$ into $b$ since bonds offer a higher expected return. Only when $r_{LT} = r_{\text{min}}$ will he be indifferent between short-term and long-term lending since both $T$ and $b$ yield the same expected return. Summarizing:
An individual’s optimal portfolio choice rule if $r_{ST} > 0$:

If $r_{LT} > r_{\text{min}}$, the individual just puts all his Wealth $W$ into bonds $b$ this year.

If $r_{LT} = r_{\text{min}}$, the individual is indifferent between the amount of his Wealth he puts into bonds $b$ and time deposits $T$ this year.

If $r_{LT} < r_{\text{min}}$, the individual just puts all his Wealth $W$ into time deposits $T$ this year.

The rule is illustrated in Figure 1.

![Diagram showing the rule for optimal portfolio choice](image)

Figure 1: Wealth put into $T$ rather than $b$ as a function of the current long rate, assuming $r_{ST} > 0$

Note: The above bang-bang (that is, all-or-nothing) feature of the individual’s optimal portfolio choice—unless $r_{LT}$ just happens to equal the individual’s $r_{\text{min}}$—follows from our assumption of risk neutrality. A risk averse individual would put some of his Wealth in $T$ even if $R_{ST} < r_{LT}^e$ in order to hedge against the interest rate risk involved in long-term lending. That is, he would hold a diversified portfolio rather than a portfolio concentrated on the one financial asset with the highest expected return.

### 2.2 How to calculate $r_{\text{min}}$

A little calculation will give us a neat formula for the individual’s $r_{\text{min}}$:

$$r_{\text{min}} = \frac{r_{LT}^e}{1 + r_{LT}^e} \times (1 + r_{ST}).$$

Notice his $r_{\text{min}}$ is a function of both his expectations $r_{LT}^e$ and the current short rate $r_{ST}$, hence we can write $r_{\text{min}} = r_{\text{min}}(r_{LT}^e, r_{ST})$. In particular, the individual’s $r_{\text{min}}$ goes up when either $r_{LT}^e$ or $r_{ST}$ goes up, so

$$r_{\text{min}} = r_{\text{min}}(r_{LT}^e, r_{ST}).$$

Here’s the derivation of the formula. Recall when $r_{LT} = r_{\text{min}}$:

$$r_{ST} = r_{LT} + g_b^e. \quad (1)$$

It is useful to have a little formula for $g_b^e$. Recall

$$g_b^e = \frac{(P_b^e - P_b)}{P_b} = \frac{P_b^e}{P_b} - 1.$$

The pricing formula for perpetuities implies:

$$P_b = \frac{C}{r_{LT}} \quad \text{and} \quad P_b^e = \frac{C}{r_{LT}^e}.$$
Plugging into the formula for $g'_b$, the $C$’s cancel yielding:

$$g'_b = \frac{r'_{LT}}{r''_{LT}} - 1.$$  

Using our little formula for $g'_b$, (1) becomes:

$$r_ST = r_{LT} + \left(\frac{r'_{LT}}{r''_{LT}} - 1\right).$$

Solving for $r_{LT}$ we find that when $r_{LT} = r_{\text{min}}$:

$$r_{LT} = \frac{r''_{LT}}{1 + \frac{r'_{LT}}{r''_{LT}}} \times (1 + r_ST).$$

So we have arrived at our formula for $r_{\text{min}}$!

**Example 1 (Speculative balances need not be idle):** Consider an individual with Wealth $W = $10,000 to lend. Suppose $r_ST = 1\%$ and the individual expects $r'_{LT} = 10\%$. Then plugging into the formula we find his $r_{\text{min}} = \frac{1}{1.1} \times (1.01) \approx 9.2\%$. That is, if the current long rate were below 9.2\%, the individual would not be willing to lend long-term this year, preferring to put all his Wealth $W$ into a time deposit $T$—in spite of the fact that his time deposit only pays him 1\% interest. The seeming paradox is explained by the fact that he expects the long rate to rise to 10\% next year; so if $r_{LT} < 9.2\%$, the individual will want to hold his money in $T$ speculating on the long rate increasing. In this sense his deposits in $T$ might be called “speculative balances” —even though they are not being held *idle* (that is, in $D$) because $r_ST > 0$.

### 2.3 Mr. Keynes’s microeconomic theory of $(\frac{M^d}{P})_{\text{spec}}$: Portfolio choice when $r_{ST} = 0$

So far we have assumed $r_{ST} > 0$. To arrive at Mr. Keynes’s theory of demand for idle speculative balances, we now need to relax this assumption. In this subsection we assume $r_{ST} = 0$, so holding one’s wealth in either checking $D$ or savings $T$ does not matter to the individual: either way he’ll have a zero rate of return on his Wealth. The formula for the individual’s $r_{\text{min}}$ remains unchanged. But now his optimal portfolio choice may include some idle balances if $r_{LT} \leq r_{\text{min}}$:

**An individual’s optimal portfolio choice rule if $r_{ST} = 0$:**

- If $r_{LT} > r_{\text{min}}$, the individual just puts all his Wealth $W$ in bonds $b$ this year since they have a positive expected rate of return, while both $D$ and $T$ have a zero return.
- If $r_{LT} = r_{\text{min}}$, the individual is *indifferent* between the amount of his Wealth he holds in bonds $b$, time deposits $T$, or demand deposits $D$ this year since they all have zero expected rate of return.
- If $r_{LT} < r_{\text{min}}$, the individual is indifferent between the amount of his Wealth $W$ he puts into time-deposits $T$ or demand deposits $D$ this year, but he does no long-term lending this year because its expected rate of return is negative (the rate of capital losses $-g'_b$ is large).
The rule is illustrated in Figure 2 below, drawn assuming the individual holds all his Wealth in his checking account when \( r < r_{\text{min}} \)—hence as idle speculative balances.

![Diagram showing speculative balances]

**Figure 2: If \( r_{ST} = 0 \), some people may hold idle speculative balances.**

**Example 2 (Speculative balances may be idle when \( r_{ST} = 0 \)):** Consider again the individual in Example 1 with Wealth \( W = 10,000 \) and expectations \( r_{LT}^e = 10\% \). Suppose now \( r_{ST} = 0\% \) rather than 1\%. Then plugging into the formula for \( r_{\text{min}} \) we find that now his \( r_{\text{min}} = \frac{1}{11} \approx 9.1\% \). That is, if the current long rate were below 9.1\%, the individual would not be willing to lend long-term this year, preferring to hold all his $10,000 of Wealth idle—in spite of the fact that idle balances pay him no interest. The seeming paradox is explained by the fact that he expects the long rate to rise to 10\% next year. So if \( r_{LT} < 9.1\% \), he prefers holding his money idle this year as Keynesian “idle speculative balances,” speculating on the long rate increasing next year. Since \( r_{ST} = 0 \), there is no lost interest in holding his cash idle in \( D \), rather than putting it into \( T \) where the bank may re-lend it. That is, in terms of portfolio choice, he’s indifferent between \( D \) and \( T \), but prefers either to \( b \). Indeed, even if he puts some of his Wealth into \( T \), since \( r_{ST} = 0 \) there is no particular reason the bank will lend all the money in \( T \) out—even on a short-term basis. Just like the individual, the bank has nothing to lose by holding some of the deposits in \( T \) as idle reserves.

### 3. Macroeconomics: Mr. Keynes’s escape from CQT

Now let’s move to the macro level. In general, different people may have different beliefs about the future long rate. One person may be very optimistic and expect the long rate to rise a lot next year, so for him \( r_{LT}^e \) is much larger than \( r_{LT} \); another person may be pessimistic and expect the long rate to fall next year, so from him \( r_{LT}^e < r_{LT} \). The optimistic individual will have a high \( r_{\text{min}} \) and, hence, may currently be holding lots of speculative balances gambling on the long rate increasing. By contrast, the pessimistic individual will have a low \( r_{\text{min}} \) and be glad to lock into the current long rate by lending long-term today before rates go any lower. Notice, like in Example 1, even the optimistic individual will not be holding any idle speculative balances if \( r_{ST} > 0 \): As long as \( r_{ST} > 0 \), he will want to put his speculative balances into a savings account \( T \) rather than into his checking account \( D \) which pays no interest; the bank will re-lend his savings deposits, so his speculative balances will not sit idle.

We now want to determine the equilibrium values of \( r_{ST} \) and \( r_{LT} \), given people’s expectations about the future. Let us denote these equilibrium values by \( r_{ST}^* \) and \( r_{LT}^* \). We will continue to assume everyone is risk neutral. Furthermore, for
simplicity, we will also restrict our attention to the special case in which

- everyone has the same expectations \( r_{LT}^e \), a given fixed number.

Since \( r_{LT}^e \) is fixed, if the current long rate \( r_{LT} \) goes below \( r_{LT}^e \), lenders believe it will revert to \( r_{LT}^e \) next year. Thus this assumption follows Mr. Keynes. Recall he thought lenders have a belief about a “normal” level for the long term interest rate. Whenever lenders see the current long rate \( r_{LT} \) fall below this normal rate, they believe it will soon return to normal; in the jargon, Keynes believed individuals have inelastic interest rate expectations. We identify lenders beliefs about what is the “normal” level of the long rate with their fixed expectations about next year’s long rate, \( r_{LT}^e \).

The tricky thing is that we need to determine 2 interest rates, not just one. Otherwise we could just look at the interest \( r^* \) at an IS-LM crossing. We will proceed in 3 steps. The first step will take care of this tricky problem: it will show that there exists a simple relation between \( r_{ST}^* \) and \( r_{LT}^* \). So if we know one equilibrium rate, we will know them both!

**Step 1: The relation between \( r_{ST}^* \) and \( r_{LT}^* \)**

My claim is that, in equilibrium, the expected rates of return from short-term and long-term lending must be equal, given lenders’ expectations. That is, \( R_{ST} = R_{LT}^e \), or graphically:

![Diagram showing the relation between \( r_{ST}^* \) and \( r_{LT}^* \)](attachment:image)

Why? Suppose \( R_{ST} > R_{LT}^e \). Then no one would lend long-term (recall everyone is risk neutral and has the same expectations). Hence firms could not get any long-term loans to finance their investments. They would offer to sell their corporate bonds for less, to attract lenders. But by the inverse relation, \( P_{b\downarrow} \Rightarrow r_{LT}^\uparrow \). So the long rate would rise until equality is reached between the two rates of return. On the other hand suppose \( R_{ST} < R_{LT}^e \). Then no one would want to lend short-term; banks would get no deposits in savings accounts \( T \) since bonds are more attractive. But then banks would be eager to attract deposits since they see they could take the deposits paying only \( r_{ST} \) and then lend the money out long-term, making an expected profit. To attract depositors, they would raise \( r_{ST} \) until again there was equality. Summarizing, if the rates of return from lending long-term and short-term are equal, either \( r_{ST} \) or \( r_{LT} \) will adjust. So, as asserted, in equilibrium \( r_{ST}^* = r_{LT}^* + g^e_b \).

For any given \( r_{ST}^* \), recall \( r_{min}(r_{LT}^e, r_{ST}^*) \) is precisely the long rate that equalizes the expected rates of return from long-term and short-term lending. So, another way to express our conclusion is that in equilibrium \( r_{LT}^* = r_{min}(r_{LT}^e, r_{ST}^*) \). That is, when everyone has the same \( r_{LT}^e \), the formula for an individual’s \( r_{min} \) is also the formula for the equilibrium value of
\( r_{LT} \) given \( r_{ST} \)! In particular:

\[
 r^{LT}_{LT} = \frac{r^{LT}_{LT}}{1+r^{LT}_{LT}} \times (1+r^{ST}_{ST}).
\]

(2)

To graph (2), define \( r_{\text{min}}^{0} \equiv \frac{r^{LT}_{LT}}{1+r^{LT}_{LT}} \), that is, the long rate that would prevail if the short rate were zero. The graph of (2) is shown in Figure 3 below. Notice from the graph that \( r_{\text{min}}^{0} \)

equals the vertical intercept, and that \( r^{LT}_{LT} = r^{ST}_{ST} \) only when both rates equal \( r^{e}_{LT} \). If \( r^{LT}_{LT} < r^{e}_{LT} \), the short rate is below the long rate because people expect capital losses from long-term lending; while if \( r^{LT}_{LT} > r^{e}_{LT} \), the short rate is above the long rate because people expect capital gains. These conclusions can be re-expressed in terms of the **term structure of interest rates**, that is, the graph in which the term to maturity of loans is put on the horizontal axis and the interest on loans is put on the vertical axis. The term structure is upward sloping when \( r^{LT}_{LT} < r^{e}_{LT} \) and it is downward sloping when \( r^{LT}_{LT} > r^{e}_{LT} \).

**Figure 3: The Graph of Equation (2)**

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**Step 2: The shape of the LM curve**

We will use an IS-LM diagram — with \( r_{LT} \) on the vertical axis — to determine \( r^{LT}_{LT} \). Then we will be done, since either Equation (2) or Figure 3 gives us the corresponding equilibrium value for the short rate.

Recall the LM describes all combinations of \( r \) and \( Y \) that equilibrate the money market for a given real money supply. So we have to know what determines money demand in order to determine the shape of the LM. Following Mr. Keynes, we divide up total money demand into transaction demand, precautionary demand, and speculative demand.

First consider transaction demand. Recall that Mr. Keynes agreed with CQT that *transaction* balances are not very interest sensitive (unlike textbook Keynesians). So, following CQT and Mr. Keynes, assume that transaction demand is given by:

\[
 (\frac{M_{d}}{P})_{tr} = kY. \tag{3}
\]

You know that (3) leads to a vertical LM—if people hold no asset balances.

Since Keynes focused on speculative rather than precautionary balances, we will follow him and also assume

\[
 (\frac{M_{d}}{P})_{speculative} = 0. \tag{4}
\]

(Later in the course we will look at precautionary demand.)

Finally, let’s consider the speculative demand for money. Following Mr. Keynes we have already assumed that people have fixed expectations \( r^{e}_{LT} \). So when they see \( r_{LT} \) fall below \( r^{e}_{LT} \) this year, they expect it will revert to \( r^{e}_{LT} \) next year. As you know, this may give rise to a speculative demand for money.2
In particular, our analysis of individuals’ portfolio choice has shown that as long as \( r_{ST} > 0 \), no one will hold any money as speculative balances. Hence total money demand equals transaction demand as long as \( r_{ST} > 0 \) or, equivalently, as long as \( r_{LT} > r_{min}^0 \) (recall Equation (2)). This implies the LM curve is vertical until \( r_{LT} = r_{min}^0 \)—just as in CQT. But our analysis of individuals’ portfolio choice also showed that when \( r_{ST} = 0 \), no one will be willing to lend long-term below \( r_{min}^0 \)—speculating instead on the long rate rising next year. So, unlike CQT, the LM has a floor at \( r_{min}^0 \). Combining these observations shows that (3) plus our assumption that everyone has the same fixed \( r_{LT}^0 \) yields an LM that looks like the one in Figure 4 below.

**Step 3: Combining Keynes’s LM with an IS**

We are now essentially done. Combining our Keynesian LM with an IS will tell us the equilibrium long rate, and hence Equation (2) or Figure 3 will tell us the corresponding equilibrium short rate.

**Example 3:** Suppose the economy has been at full employment equilibrium for a long time at point \( A_1 \) in the figure below, with \( r_{LT} = r_{ST} = 10\% \). Since this has gone on for a long time, lenders have grown accustomed to getting 10% on their loans, hence \( r_{LT}^0 = 10\% \) also. Observe that this long-run equilibrium is consistent with Figure 3: \( r_{ST}^* = r_{LT}^* = r_{LT}^0 \).

If now there is a small real shock and the IS shifts down a little to \( IS_2 \), the economy will remain at full employment at point \( A_2 \): \( r_{LT} \) will go down enough to completely offset the real shock; and, (2) tells us that the short rate \( r_{ST} \) will also go down. Indeed \( r_{ST} \) will go down more than \( r_{LT} \) because people expect capital losses from long-term lending; and in equilibrium, the expected rates of return from short-term and long-term lending must be equal.

If instead there is a large real shock and the IS shifts down to \( IS_3 \), the economy will go into a liquidity trap recession at \( A_3 \): \( r_{LT} \) will go down to \( r_{min}^0 \approx 9.1\% \), which is not enough to offset the real shock; and, (2) tells us that the short rate \( r_{ST} \) will go down even more, indeed all the way to zero.

Notice that after each of the above real shocks the short rate fell below the long rate, so the term structure of inter-

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2By “money” we mean \( M1 \) (narrow money, hence \( D \)) rather than \( M2 \) (broad money, including both \( D \) and \( T \)). The narrower definition of money is more appropriate for IS-LM analysis.
est rates was upward sloping (which empirically is its usual shape). Consider instead a temporary increase in the IS—that is, an increase in attractive investment opportunities—to IS₀. Then the economy will move to A₀ with the long rate going up to r₀*. This rise in the long rate is needed to discourage investment demand given that the existing real money supply cannot finance any more real transactions. Further, (2) tells us that rₜₜ* will go up even more than rₗₜ*, so there will be an inversion of the term structure: people can get more by lending short-term than by lending long-term. How to explain it? Since rₗₜ* > rₗₜ*, people expect capital gains from long-term lending. So... (you should be able to fill in the rest).