1. Let’s convene that UD means “play U if player 2 finds himself in the top information set and play D if he finds himself in the bottom information set.” With this convention, the matrix form is shown below. There are two Nash equilibria: NE #1: 1 plays 10, 10, 10, 10 and 2 plays DD. This one is not subgame perfect: 2’s threat to play D if he finds himself in the bottom information set is incredible: it’s not his best response if he finds himself there. The other NE is NE #2: 1 plays d and 2 plays DU. This one is subgame perfect; it’s the one found by backward induction.

<table>
<thead>
<tr>
<th></th>
<th>UU</th>
<th>UD</th>
<th>DU</th>
<th>DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>10,10</td>
<td>10,10</td>
<td>0,15</td>
<td>0,15</td>
</tr>
<tr>
<td>d</td>
<td>2,2</td>
<td>2,2</td>
<td>-1,-1</td>
<td>-1,-1</td>
</tr>
</tbody>
</table>

2. A non-traditional convergence theorem This example is worked out in chapter 7 of the lecture notes. In particular, regarding part d, you could say he is a “perfectly discriminating monopolist” in the limiting economy because he appropriates all the GFT; you could also say he is a “non-discriminating monopolist” because he does not price discriminate; you could also say he is a “perfect competitor” because he faces PED. In terms of our definitions, we’d call him a perfect competitor rather than a monopolist because he does not face a downward sloping demand curve.

3. Competition and the fear of holdups
   a. Maximizing $\beta s - s^2$, the FOC implies $\beta = 2s$. So $s^*(\beta) = 0.5\beta$.
   b. Maximizing $s - s^2$, the FOC implies $1 = 2s$. So $\bar{s} = 0.5$.
   c. Notice the seller will invest efficiently in period 1 iff $\beta = 1$, that is, iff he expects to be a very successful (tough) bargainer in period 2. In this event his PB = SB for all $s$ (full appropriation); so the efficient outcome is not surprising. If $\beta < 1$, the seller underinvests because his MPB for any skill level (namely $\beta s - 2s$) is strictly less than the MSB from any skill level (namely, $s - 2s$). The buyer gains some extra benefit from any additional investment by the seller (a positive pecuniary externality). The seller underinvests because he does not take this external benefit into account.
   d. In the extreme case when $\beta = 0$, the tough-bargaining buyer gets all the benefit from any period-1 investment by the seller. This is an extreme “hold up problem” from the point of view of the seller because he will not be able to salvage any of his period-1 investment cost. Anticipating being held up in period 2, he chooses $s = 0$ in period 1.
   e. Intense competition among the high-valuing buyers forces the price up to $p(s) = s$. See FIGURE (right). Since the seller fully appropriates the value of any $s$ (i.e., since his PB = SB for all $s$), he naturally invests efficiently.
4. A $q$-unit English auction

a. $P^* = 30$ and $GFT^* = 120 + 100 + 80 + 50 = 350$.

b. $GFT_{\text{without } i} = 120 + 80 + 50 + 30 = 280$. Hence $V_i - P^* = 70 = GFT^* - GFT_{\text{without } i}$. See FIGURE.

c. $PB = 120 < SB = 350$.

d. $P = 80$, which leads to a total revenue of 240. Notice only 3 shares will be sold, hence the outcome is not Pareto efficient: The total GFT will fall from 350 to 280, even though the seller’s PB will increase from 120 to 240.

e. For PED you need at least 5 highest-valuing buyers. For example $b = (120, 120, 120, 120, 120, 10)$. Then the intense competition between the 5 highest valuing buyers for the 4 available units will lead to a price of 120 — even if the seller sets $P = 0$. Not surprisingly, perfect competition (PED) leads to efficient trading—the 4 shares will be sold to the 4 highest-valuing buyers—since intense competition among these buyers permits the seller to fully appropriate the social value of his shares. There is no tension between efficiency and bargaining in this case.