1. **Basic game theory** Let’s begin with an easy question. Consider the following extensive-form game. Notice player 2 would like get the high payoff of 15; let’s see if he will be able to get it. Find the strategic (matrix) form for the game, and then find all its Nash equilibria. If any Nash equilibrium for the game is not subgame perfect, explain the incredible threat or promise.

![Game Tree Diagram]

2. **A non-traditional convergence theorem** Consider a single market TU economy with $n$ identical buyers and only 1 seller. Each buyer $i$’s valuation function for the non-money good is

$$v_i(q_i) = q_i - \frac{1}{2}q_i^2,$$

and the seller’s total cost function is

$$c(q) = \frac{1}{2}q^2.$$

a. Find the equation of the market demand function when there are $n$ buyers, $D(p, n)$.

b. Find the monopolist’s profit maximizing price and quantity if he cannot price discriminate. (Both the price and quantity will be a function of $n$, say $p(n)$ and $q(n)$.)

c. Calculate the efficient output as a function of $n$, say $q(n)$. What happens to the inefficiency of the seller’s actual output $q(n)$ as $n \to \infty$?

d. Calculate the elasticity of the demand function the seller faces for any given $n$ and output level $q$. Use this to explain the intuition for what happens (in terms of efficiency) as $n \to \infty$. In the limiting economy, is the seller a non-discriminating monopolist, a perfectly discriminating monopolist, a perfect competitor, or what? Briefly justify your answer.

e. Now explain the intuition in terms of the catallactic bee story, drawing the “blow up” around any consumer’s consumptive optimum in the limiting economy.
3. **Competition and the fear of holdups** Consider a 2-player, 2-stage perfect information game involving a buyer \( i \) and a seller \( j \). In period 1 the seller must decide on an amount to invest in acquiring skills; his total cost of skill level \( s \) is

\[
e(s) = s^2.
\]

Then, in period 2, the seller and buyer bargain over the price the buyer will pay the seller for a match. Assume the value to a buyer of a match with a seller having skill \( s \) is exactly \( s \) dollars. Also assume that period 2 bargaining results in a price \( p(s, \beta) \) that depends on both the seller’s skill \( s \) and the buyer’s relative bargaining ability \( \beta \). In particular,

\[
p(s, \beta) = \beta s,
\]

where the parameter \( \beta \) is in the interval \([0,1]\).

a. For any given \( \beta \in [0,1] \), calculate how much the seller will invest. That is, find the value of \( s \) that maximizes \( p(s, \beta) - e(s) \). Let’s call it \( s^*(\beta) \).

b. Find the value of \( s \) that maximizes social gain \( g(s) \equiv s - e(s) \). Let’s call it \( \hat{s} \).

c. For what values of \( \beta \) will the seller invest efficiently? For what values will he under-invest? Explain your findings by comparing his Private Benefit from investment with the Social Benefit.

d. The tension between efficiency and bargaining when \( \beta < 1 \) may be called a “fear of a holdup.” Explain why this terminology is appropriate for the extreme case when \( \beta = 0 \) (so the buyer is a very tough bargainer).

e. **What difference does competition make?** Now suppose there are at least two identical buyers \( i \) who will compete for a match with the seller in period 2. If the seller acquires skill \( s \) in period 1, any one of these buyers will be willing to pay at most \( s \) dollars for a match with the seller in period 2. The seller can match with at most one buyer.

For any period-1 investment choice \( s \geq 0 \) explain why, in period 2, the seller will be able to get a price \( p(s) \) equal to \( s \)—independent of any one buyer’s bargaining ability \( \beta \) (hint: use a competition argument; illustrate your answer with a demand-and-supply diagram). Explain why the seller invests efficiently when he expects intense ex post competition for his skill (hint: compare his PB from investment with the SB).
4. A *q*-unit English auction (Massi’s question) A seller has *q* units of a commodity for sale. For concreteness think of the seller as Google and the commodity as shares in Google. There are *I* bidders (*I* > *q*), each with unit demand. The seller asks each bidder *i* for a bid *b*(*i*) ∈ [0, ∞) and then arrays the *I* bids from the highest to the lowest, forming the vector

\[ b \equiv (b(1), b(2), \ldots, b(q), b(q + 1), \ldots, b(I)). \]

He then sells the *q* units to the *q* highest bidders for a price of *b*(q + 1) each. That is, each of the *q* highest bidders pays a price equal the *q* + 1st highest bid.1

Let \( V_i \in [0, \infty] \) equal buyer *i*’s willingness to pay for a share in Google. Just as in a 1-unit English auction, each buyer’s dominant strategy in a *q*-unit English auction is to bid his true valuation, that is, \( b_i = V_i \).

For concreteness suppose *q* = 4 and there are 6 potential buyers with bids (and hence valuations) given by

\[ b = (120, 100, 80, 50, 30, 10). \]

a. What will be the equilibrium price \( P^* \) of each share in Google? Illustrate your answer by drawing the Demand-and-Supply curves for shares in Google. What will the total gains from trade \( GFT^* \) equal in equilibrium; give a numerical answer and illustrate using your D-and-S diagram. NOTE: Assume the value of shares equals zero to the seller.

b. In the above English auction each potential buyer bids truthfully because his Private Benefit from participating in the auction equals the Social Benefit from his participation. Verify this assertion for the buyer with \( V_i = 100 \). That is, show that for this buyer,

\[ V_i - P^* = GFT^* - GFT_{\text{without } i}, \]

where \( GFT_{\text{without } i} \) denotes the gains from trade in a *q*-unit English auction without *i*’s participation. Illustrate your answer graphically, by drawing the market demand and supply curves with and without *i*.

c. Although all buyers fully appropriate their social contributions in the above English auction, the seller does not fully appropriate his social contribution. Show that the seller’s private benefit in equilibrium (i.e., his revenue \( qP^* \)) is strictly less than his social contribution. (Hint: The gains without the seller equals zero.)

Consider the modified English auction in which the seller announces that he will not sell to anyone bidding less than \( P \). \( P \) is common knowledge to all buyers once the seller announces it.

So the price of each share now will equal

\[ \max \{b(q + 1), P\}. \]

Anyone bidding less than \( P \) pays nothing and receives nothing. It can be shown that even with the introduction of a reservation price \( P \), the dominant strategy for each buyer *i* is still to bid truthful (i.e., \( b_i = V_i \)).

d. The tension between efficiency and bargaining re-appears Find the reservation price \( P \) that will maximizes the seller’s revenue when he knows that buyers’ valuations (hence bids) are as above, that is, \( (120, 100, 80, 50, 30, 10) \). Is the outcome of the auction still Pareto efficient?

e. The above profile of valuations is not perfectly competitive because the seller does not face PED. Specify a profile of valuations (hence bids) \( b = (b(1), b(2), \ldots, b(6)) \) that is perfectly competitive. Under the profile you have specified, why will the seller find it profit-maximizing to set \( P \) equal to zero, hence why will the outcome of the auction be efficient?

1The tie-breaker rule: If \( b(q) = b(q + 1) \), then the price equals \( b(q + 1) \); everyone who bids more than \( b(q) \) gets a unit for sure; while everyone who bids \( b(q) \) is given an equal chance of getting a unit at this price.