Exercise 1.1

Exercise 1.2 (a beneficial real externality)

a. $PB(q) = q - \frac{3}{2}q^2$, $EXT(q) = \frac{1}{2}q^2$, and $SB(q) = q - q^2$, where the last equation is the sum of the previous two. Differentiating shows: $MPB(q) = 1 - 3q$, $M-EXT(q) = q$ and $MSB(q) = 1 - 2q$, where again the last equation is the sum of the previous two. Graphically, it’s a case of Specimen 1. See RIGHT top.

b. The MSB of additional production (bees) is still positive at $q^*$ because additional production would increase the apple grower’s profit. But the beekeeper is only interested in his own profit, so he stops producing when $MPB = 0$, rather than when $MSB = 0$.

c. Since $MPB = -1/2$ at $\dot{q}$, $\ddot{q} = 1/2$. Graphically, this shifts the beekeeper’s $MPB$ curve upward so that it equals zero at $\dot{q}$.

d. Now: $PB(q) = \frac{3}{2}q - \frac{3}{2}q^2$, $EXT(q) = \frac{1}{2}q^2 - \frac{1}{2}q$, and $SB(q) = q - q^2$. Differentiating shows that now: $MPB(q) = \frac{3}{2} - 3q$, $M-EXT(q) = q - \frac{1}{2}$, and $MSB(q) = 1 - 2q$. Graphically, it’s now a case of Specimen 4.

e. Now the MSB of additional production is zero at $q^*$, that is, $MPB = MSB$ at $q^*$; so at the margin the beekeeper creates no additional externalities for his
neighbor. In appropriation terms, he fully appropriates the marginal social contribution of his bees.

**Exercise 1.3** The key is to take account of the budget constraint that someone must pay any subsidy given the beekeeper. So:

a. If \( q^\uparrow \), there is a positive marginal real externality for the beekeeper's neighbor AND a negative marginal pecuniary externality for taxpayers.

b. In particular, at \( \hat{q} \), the positive marginal real externality for his neighbor EQUALS the negative marginal pecuniary externality for taxpayers. So, the net marginal externality equals 0 at \( q = \hat{q} \).

**Exercise 1.4 (a beneficial pecuniary externality)**

Since \( \pi(q) = (1 - q)q - \frac{1}{2}q^2 = q - \frac{3}{2}q^2, \) \( PB(q) \) is the same as in Exercise 1.2. Similarly, since \( CS(q) = \frac{1}{2}q^2, \) \( EXT(q) \) is the same as in Exercise 1.2; so \( SB(q) \) is also the same. So the answer is the same!

**Exercise 1.5 (the efficiency of price-taking equilibrium)**

\[ PB(q) = \pi(q) = p^Wq - c(q) = \frac{1}{2}q - \frac{1}{2}q^2, \]
\[ EXT(q) = CS(q) = \frac{1}{2}q - \frac{1}{2}q^2, \]
\[ SB(q) = \pi(q) + CS(q) = q - q^2. \] Differentiating shows:
\[ MPB(q) = \frac{1}{2} - q, \]
\[ MSB(q) = 1 - 2q, \]
\[ M - EXT(q) = \frac{1}{2} - q. \] It's an example of Specimen 4: By increasing his production \( q \), the price-taking seller creates a positive and then negative marginal; pecuniary externality for consumers. [Why is it a pecuniary rather than real externality? Think of it this way. By increasing his production, the seller expands his customers' trading opportunities: they can buy more units at the price \( p^W = 1/2 \), which increases their consumers' surplus.] Since \( CS'(q) = 0 \) when \( q = q^* \), the seller fully appropriates his marginal social contribution, which suffices for the efficiency of his choice. It fits into Specimen 4: The picture looks very similar to FIGURE 1.4 in Chapter 1, except that the MPB and M-EXT curves happen to coincide in this example (I won't bother to graph it). The incentive scheme underlying the efficiency of price-taking: In a price-taking equilibrium, each individual's MPB is aligned with MSB.

**Exercise 2.1**

a.

\[ MRS = \frac{\partial u_i(q, m_i)}{\partial m_j} = v_i'(q_k) = 1 - q_i, \]

which is independent of \( m_i \). See FIGURE below.
b. Maximizing the sum of utilities over all feasible allocations is equivalent to maximizing $v_1(q_1) + v_2(q_2)$ subject to the feasibility condition $q_1 + q_2 \leq 1$—then distributing the 1 available unit of money any way one likes. The FOC’s show that $q_1 = q_2 = 1/2$. So the set of all PO allocations $x$ satisfy $x_1 = (1/2, m_1)$ and $x_2 = (1/2, m_2)$, with $m_1 + m_2 = 1$. See FIGURE below. The contract curve is a vertical line because the indifference curves are vertically parallel.