1. The hold-up problem

a. The unique SPNE is that 1 plays “Don’t Invest,” while 2 plays “TB.” The idea is this: If Fisher Invests then GM will act as a Tough Bargainer and appropriate all the GFT! Fisher is “robbed” in the sense that it will not even be able to salvage its Investment cost. Notice that it is crucial that the game is dynamic: Because the cost of Fisher’s plant becomes a sunk cost after it is built, Fisher will find it hard to threaten not to produce for GM unless GM plays fair. Because the plant is right next to GM’s plant, there is no strong competition that would force GM to be nice to Fisher; Fisher would be locked into a bilateral monopoly relationship with a much bigger and more powerful player. The lesson is that, while it is easy to be tough (i.e., to fully appropriate) when there is perfect competition for your product, it is tough to be tough in the absence of competition for your product.

b. The possibility of an underdevelopment trap The rich country may fear that the poor country will nationalize its foreign investment projects, after the investments have been made. To avoid the hold up, it may not invest in the poor country at all.

2. Generalized backward induction First solve for the NE in each of the 2 possible games at date 2: One finds, just as a coincidence, that 1 plays d in both games A and B, while 2 plays U in both games A and B. So, the payoffs will be (3,7) if game A is played and (5,9) if game B is played. Use these numbers to fold back the game tree into the first stage. Solving for the NE in the “folded back” payoff matrix for the first stage, one finds that there are 2 possible SPNE: In

SPNE #1: 1 plays M in the first stage and always plays d in the second stage (regardless of the outcome in the first stage); 2 plays F in the first stage and always plays U in the second stage (regardless of the outcome in the first stage). In SPNE #2: 1 plays F in the first stage and always plays d in the second stage; 2 plays M in the first stage and always plays U in the second stage.

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3. The Tragedy of the Commons

a. The FONC for i is

\[
\frac{\partial \pi_i}{\partial \ell_i} = 1000 - 2\ell_i - L_{-i} - c = 0. \tag{1}
\]

At i’s optimal choice given \(L_{-i}\), his MPB equals zero or, equivalently, i’s (private) marginal product of labor given \(L_{-i}\) equals his marginal cost \(c\). Each i will solve (1) for \(\ell_i\) as a function of \(L_{-i}\); this is i’s best response
function, alias reaction function:

\[ \ell_i(L_{-i}) = \frac{990 - L_{-i}}{2} \]  \hspace{1cm} (2)

In the symmetric NE, \( L_{-i} = (I - 1)\ell_i \); substituting this into \( i \)'s reaction function and solving for \( \ell_i \) shows that in the symmetric NE \( \ell^* = \frac{990}{I(I + 1)} \).

The optimum is illustrated graphically in the FIGURE below, where \( MP_i(\ell_i) \) is \( i \)'s (private) marginal product of labor curve given \( L_{-i}^\ast \). Notice the similarity to the optimum for a Cournot oligopolist: Each fisherman, taking \( L_{-i}^\ast \) as given, finds his “residual AP(L) curve.” Then he takes the marginal of this curve—which equals his marginal product of labor given \( L_{-i}^\ast \)—and operates where his marginal product of labor equals his marginal cost.

b. The FONC for maximizing \( G \) is

\[ G'(L) = 1000 - 2L - c = 0, \] \hspace{1cm} (3)

i.e., at the social optimum the (social) marginal product of labor equals marginal cost: \( MP(L) = MC \). Hence

\[ L = 495. \]  \hspace{1cm} (4)

Contrast this to the symmetric NE, where the total hours fished is given by

\[ L^* = I \ell^* = \frac{I}{I + 1}990. \]

So if \( I = 1 \) then \( L^* = \hat{L} \). But if \( I > 1 \) then \( L^* > \hat{L} \). To illustrate using the benefit functions, if others fish \( L_{-i}^\ast \) hours, then for any one marginal fisherman:

\[ MPB_i(\ell_i) = \frac{\partial \pi_i}{\partial \ell_i} = 990 - 2\ell_i - L_{-i}^\ast, \]

\[ MSB_i(\ell_i) = \frac{\partial G(\ell_i + L_{-i}^\ast)}{\partial \ell_i} = 990 - 2\ell_i - 2L_{-i}^\ast \]

\[ M - EXT_i(\ell_i) = MSB_i(\ell_i) - MPB_i(\ell_i) = -L_{-i}^\ast \]

So, in terms of our four specimens, the tragedy of the commons is an example of a negative marginal externality. The inefficiency of the NE follows from Pigou’s Theorem.

Notice, as \( I \to \infty \), \( L^* \to 990 \)—moving further and further away from \( \hat{L} \). That is, the more fishermen harvesting the lake—each independently choosing his quantity,—the more the lake is over-fished as the negative externality becomes larger. In the limit there are absolutely no GFT from fishing since \( Q(990) - 990c = 0 \): In the limit, there
is “free entry” until no one earns any profit from fishing! See FIGURE below: $G(L)$ equals the shaded area because $G(L) \equiv Q(L) - c L = AP(L) \times L - c L$. Notice $G(990) = 0$

0. Solving for $t$ shows $t^* = \frac{495 I - 1}{I}$. Notice the tax is zero when $I = 1$ (the only efficient case) and approaches 495 as $I \to \infty$ (the “free entry” case).

d. Privatization as a solution Interestingly enough, the answer depends on whether or not the owner can control the quantity of fishing—as well as the price.

If he only sets a price $p$ per hour of fishing the outcome will typically not be efficient. To illustrate, suppose $I = 1$. Notice $p$ acts just like a tax, increasing $i$’s marginal cost from $c$ to $c + p$. He will fish until his MPB = 0, so $\ell_i = 495 - p/2$. Hence the owner will set $p^* = 495$, which solves

$$\max_{p \geq 0} p[495 - p/2].$$

Notice this is larger than the Pigovian tax of zero when $I = 1$, so it will induce underfishing.¹ [You can check that the same conclusion applies for any $I$, although the outcome becomes almost efficient as $I \to \infty$—the free entry case when the seller faces PED.]

On the other hand, if the owner can control quantity as well as price, the outcome will be efficient because he can then fully appropriate the benefit of any $L$: For an $L$-hour lake—that is, if he restricts total fishing to $L$ hours—he can charge $p(L) \equiv [AP(L) - c]$ per person per hour (the zero-profit condition for fishermen who must compete for entry into the lake, each having constant marginal cost), yielding the owner total revenue $p(L)L$. He’ll set $L$ to solve

$$\max_{L \geq 0} p(L)L = [AP(L) - c]L = G(L).$$

Since he full appropriates the benefit of his lake—whatever $L$ he chooses, he chooses $L^* = L$, and

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¹This is an example of “double marginalization”: the owner takes the marginal of his total revenue function to determine $p$, while the fisherman takes the marginal of his payoff function given $p$ to determine $\ell_i$. Double marginalization leads to inefficiency, because each private optimizer does not take into account his effect on the other optimizer’s gain.
\[ p(L^*) = AP(L) - c = 495. \] Notice that the owner’s profit-maximizing price per fisherman per hour is the same as the efficient Pigovian tax in the limiting case of free entry, when fishermen do not appropriate any of the GFT. Also notice that there are redistributive consequences to privatization: fishermen are worse off than when they were commonly (and inefficiently) harvesting the lake—so they may resist privatization. It may not be a politically-feasible solution.

5. Rubinstein’s bargaining model

a. The subgame beginning at date 2 is identical to the ultimatum game in which player 2 has all the bargaining power. She offers \( x_2 = 0 \) and player 1 accepts. Working backward, Player 1, realizing what will happen in period 2 if an agreement is not reached in period 1, offers Player 2 the smallest possible share consistent with her saying YES, i.e., he offers \( x_1 \) satisfying

\[ x_1 = \delta, \]

where the RHS equals the present discounted value of player 2’s payoff if the bargaining goes into period 2. So, he offers \([x_1 = \delta]\) and player 2 accepts. Summarizing, in the unique SPNE the players’ strategies (i.e., complete contingent plans) are:

\( a_1^1 \): offers \( x_1 = \delta \) in period 1 and say YES to any offer in period 2.

\( a_2^1 \): say YES to any offer in period 1 that is not smaller than \( \delta \) and offer \( x_2 = 0 \) in period 2.

Compared to the ultimatum game, player 2 is better off being able to make a counter offer—unless \( \delta = 0 \), i.e., unless she is completely impatient. Indeed, the more patient she is, the closer she gets to reversing roles with player 1 in the ultimatum game; e.g., when \( \delta = 1 \), i.e., when she is not impatient at all, then she gets the whole pie right away since player 1 is forced to capitulate. In this game there is a second-mover advantage as long as \( \delta > 1/2 \).

b. A NE in which player 1 gets the whole pie:

\( a_1^2 \): offer \( x_1 = 0 \) in period 1 and say NO to any offer in period 2 except \( x_2 = 1 \).

\( a_2^2 \): say YES to any offer in period 1 and offer \( x_2 = 1 \) in period 2.

A NE in which player 2 gets the whole pie:

\( a_1^1 \): offer \( x_1 = 1 \) in period 1 and say YES to any offer in period 2.

\( a_2^2 \): say NO to any offer except \( x_1 = 1 \) in period 1 and offer \( x_2 = 0 \) in period 2.

Neither of these NE is subgame perfect. For example, in the first NE, it is not rational for player 1 to say NO to an offer of \( 1/2 \) in period 2. Similarly, in the second NE, it is not rational for player 2 to say NO to an offer of \( 1 - \epsilon \) in period 1, if \( \epsilon \) is sufficiently small.

c. There are 2 sorts of subgames: (i) when player \( i \) must make an offer, and (ii) when player \( i \) must respond to some offer of \( x \in [0, 1] \).

Let’s begin with the first sort of subgame. Given \( j \)’s strategy, the least \( i \) can offer without rejection (this period or any other period) is \( \delta/(1 - \delta) \)—which yields her the share \( 1/(1 - \delta) \). If she offers less then, given \( j \)’s strategy, \( i \) can expect to be offered a share \( \delta/(1 - \delta) \) next period (and every other period) by \( j \). So, since \( \delta \leq 1 \), her NE strategy of offering \( \delta/(1 - \delta) \) whenever it is her time to offer is indeed a best response to \( j \)’s strategy.

Now consider the second sort of subgame, a subgame following an offer of \( x \), for some \( x \in [0, 1] \). If she rejects, then she can demand \( 1/(1 - \delta) \) next period without rejection, which is the same as obtaining \( \delta/(1 - \delta) \) this period—given her discounting. Further, given \( j \)’s strategy, she cannot hope to obtain any more than \( \delta/(1 - \delta) \) in present-value terms. So, her NE strategy of rejecting \( x \) unless it is at least \( \delta/(1 - \delta) \) is indeed a best response to \( j \)’s strategy.