Problem Set 6: Classic Models of Oligopoly

Quantity Competition

Questions 1-4 below all involve the same setup: There are J firms in an industry. All produce identical products. The market inverse demand curve is

\[ P(Q) = 1 - Q, \]

where \( Q = \sum_{j=1}^{J} q_j \). There are constant returns to scale so \( c_j(q_j) = c_j q_j \) for each firm \( j \); notice firms may have different marginal costs.

1. Cournot duopolists Suppose \( J = 2 \).
   a. Derive and graph each firm’s reaction function.
   b. Find the Cournot equilibrium (i.e., \( q_j^C, j = 1, 2 \)), and the equilibrium profit of each firm. (Assume both firms produce in equilibrium.)
   c. You’ll observe that firm 2’s equilibrium output and profit decrease when firm 1’s marginal cost decreases. What’s the intuition for this? HINT: What happens to 2’s residual demand curve?

2. Cournot convergence theorem Now suppose there are \( J \) identical firms, each with marginal cost \( c \). Let’s look for the symmetric Cournot equilibrium in which each firm produces the same output \( q^C \).
   a. Find each firm’s Cournot equilibrium output \( q^C \) (it will be a function of \( J \)). Also calculate the equilibrium price and each firm’s equilibrium profit (as functions of \( J \)).
   b. What happens to the equilibrium price as \( J \to \infty \)?

3. Stackelberg equilibrium Return to the duopoly case of Question 1. But now suppose the game is dynamic, with firm 1 the Stackelberg leader. Also suppose that \( c_1 = c_2 = 0 \).
   a. Calculate the Stackelberg equilibrium, and each firm’s equilibrium profit. Graphically illustrate the equilibrium as a tangency between the Stackelberg leader’s highest attainable isoprofit curve and the Stackelberg follower’s reaction curve. Is there a first-mover advantage in the Stackelberg game?
   b. Sunk costs and barriers to entry Here is a variation on the basic Stackelberg game due to Spence and Dixit. In part a., firm 1 accommodates firm 2’s entry. Now suppose there is a fixed cost of entry, \( F = .04 \), so for each firm \( i \),

\[
\pi_i(q_i, q_j) = \begin{cases} 
q_i(1 - q_i - q_j) - F & \text{if } q_i > 0 \\
0 & \text{if } q_i = 0.
\end{cases}
\]

   b. Draw firm 2’s new reaction curve and find the new Stackelberg equilibrium. Does firm 1 accommodate or deter entry in the equilibrium? Explain the intuition in terms of quantities being strategic substitutes.
   c. Illustrate the relation between firm 2’s average cost curve and its residual demand curve in the equilibrium.
4. **Fishing for profits: A puzzle**  You may have noticed that there is a striking similarity between the Tragedy of the Commons question on the last problem set and Question 2 above. Indeed, basically, the fishing problem and the Cournot oligopoly problem just involving changing the names of the characters (mathematically, they are “isomorphic”). To see the similarity, think of the Cournot oligopolists as fishing for profits by producing output for a common market (the market = the common property resource that all the firms are fishing from). Here is a more complete “key” for translating from one problem to the other; the first item in each row gives the object in the fishing problem (Question 4 in PS#5) and the second item gives the corresponding object in the oligopoly problem (Question 2 above):

- lake ↔ market
- fisherman \((i)\) ↔ oligopolist \((j)\)
- number of fishermen \((I)\) ↔ number of oligopolists \((J)\)
- labor \((\ell_i)\) ↔ output \((q_j)\)
- total labor \((L)\) ↔ total output \((Q)\)
- \(AP(L)\) ↔ \(P(Q)\) (hence average price).

Indeed, if I had chosen \(P(Q) = 1000 - Q\) for the inverse demand curve (same as the \(AP(L)\) in the fishing example) and \(c = 10\) for the marginal cost of each oligopolist (same as the MC of each fisherman) then \(q^C(J)\) would equal \(\ell^*(I)\) in the fishing problem.

Now comes the puzzle. Although the two problems are mathematically identical in the above respects, their conclusions—as far as efficiency goes—are diametrically opposed. In the fishing problem, as \(I \to \infty\), the inefficiency gets worse and worse, with fishing on the lake creating absolutely no GFT in the limit. By contrast, in the Cournot oligopoly problem, as \(J \to \infty\), the inefficiency gets smaller and smaller, with fishing for profits leading to maximum GFT in the limit. Why is there this dramatic difference in the conclusions? HINT: Calculate and graph any one oligopolist’s MPB, MSB, and M-EXT functions given others produce \(Q_{-j}^*\) units, assuming \(P(Q) = 1000 - Q\) and \(c = 10\). Compare with the corresponding functions for the fishing problem.

## Price Competition

5. **Bertrand equilibrium with differentiated products**

Suppose two firms, producing differentiated goods, compete through prices. So firm \(i\)'s set of possible strategies is

\[A_i = \{p_i : p_i \in [0, \infty)\},\ i = 1, 2\]

The demand for the good produced by firm \(i\) is

\[q_i \equiv D_i(p_i, p_j) = 1 - bp_i + dp_j,\]

with \(0 < d < b\). Each firm \(i\) has unit costs \(c\), so

\[\pi_i(p_i, p_j) = (p_i - c)(1 - bp_i + dp_j).\]

Note that \(\pi_i\) is concave in \(p_i\).

a. Derive each firm’s reaction function. Show that the two firms’ prices are strategic complements.

b. Find the Bertrand equilibrium (i.e., \(p_i^B, j = 1, 2\), and the equilibrium profit of each firm.

6. **A dynamic variation: The price leadership game**

Continuing with the above example, now suppose for simplicity that \(d = 1, b = 2, c = 0\). Also, now suppose that firm 1 can commit to his price before firm 2. So, he’s a “Stackelberg leader in prices.”

a. Find the equilibrium in the price leadership game. Illustrate it graphically as a tangency between firm 1’s highest attainable isoprofit line and firm 2’s reaction curve.

b. Is there a “first-mover advantage” or a “second-mover advantage” in this game? HINT: Compare firm 1 and firm 2’s equilibrium profits.
Supergames

Here’s a question on supergames to conclude this Problem Set. It’s a repeated-game variation on the simple entry game. The question is: Will repetition make it credible for the incumbent to Prey on entrants, to build up a reputation for toughness in order to discourage future entry?

7. Building a reputation for toughness Suppose player I, the Incumbent, plays the following entry game T successive times, each time with a different opponent; his opponent at time \( t (t=1,\ldots,T) \) is entrant \( E_t \). Each entrant \( E_t \) only plays the game once; and \( E_t \) knows what happened in all the previous plays, i.e., in the games with opponents \( E_{t'}, t'=1,\ldots,t-1 \). So this is a game involving perfect information, and it involves repeated play for the incumbent. Player I evaluates his payoffs for the “repeated game” using the discount factor \( \delta \in (0,1) \); i.e., one dollar in period \( t \) is only worth \( \delta^{t-1} \) to him today.

The story is that player I is a chain store monopolist with \( T \) stores in \( T \) different towns. In each town, there is the threat of entry from a firm who would operate only in that town. Threats of entry occur successively rather than simultaneously: entrant \( E_t \) only considers entering at time \( t \), \( t=1,\ldots,T \).

Intuitively, one might expect the incumbent to prey on early entrants, in order to discourage future entry in other markets. That is, he would want to build up a reputation for toughness. Let’s see if this intuition makes sense.

a. The chain store paradox Suppose \( T \) is large but finite. Find the unique SPNE for the “repeated game.” Do you see why the outcome is called the “chain store paradox”?

b. A credible threat Now suppose \( T \) is infinite. Show that—provided the incumbent’s discount factor \( \delta \) is sufficiently close to 1,—there is a SPNE for the supergame in which no one enters for fear of the incumbent preying on him. In particular:

- Find trigger-like strategies for the incumbent, say \( a^t_I \), and for each entrant, say \( a^t_I \) \( (t=1,2,\ldots) \), such that your profile of strategies forms a SPNE for the supergame provided the incumbent’s \( \delta \) is sufficiently close to 1.
- For what values of \( \delta \) is your strategy profile at SPNE?
- Check that your strategy profile is indeed a SPNE for your specified values of \( \delta \). That is, check it is a NE at each subgame.