Problem Set 7: Information Economics

Adverse Selection

1. **Second-degree price discrimination** A monopolist must deal with a buyer with quasilinear utility function

\[ u(q, t, \theta) = v(q, \theta) - t, \text{ where } v(q, \theta) = \theta q - \frac{1}{2} \theta^2; \]

\( q \) is the quantity of the monopolist’s good that she purchases and \( t \) is the total amount of money (“tariff”) she pays the monopolist for the \( q \) units. The monopolist believes there is a probability \( \lambda \) that \( \theta = \theta_L \equiv 6 \) and a probability \( 1 - \lambda \) that \( \theta = \theta_H \equiv 10 \), where \( \lambda = 50\% \). Call the buyer of type \( \theta_L \) (respectively, \( \theta_H \)) the “low-value” (“high-value”) buyer. The monopolist’s total cost of producing \( q \) units is \( c(q) = q \) (so his MC = 1 for each unit). NOTE: \( \lambda \) also can be interpreted as the proportion of his customers that are of type \( \theta_L \).

a. **Benchmark case** To set a benchmark, calculate the contracts \( \hat{C}_L \equiv (q_L, t_L) \) and \( \hat{C}_H \equiv (q_H, t_H) \) that the monopolist would offer the two types if there were complete information, i.e., if he could distinguish the two types. Illustrate the equilibrium using the two types’ demand curves, showing the tariff each must pay.

b. **Asymmetric information case** Now suppose incomplete information, so the monopolist cannot distinguish the buyers’ types. To maximize his profit the monopolist offers a pair of contracts \( C_L \equiv (q_L, t_L) \) and \( C_H \equiv (q_H, t_H) \), in an effort to screen the two types via self-selection. Write the monopolist’s expected-profit maximization problem, including the IC and IR constraints his contracts must satisfy.

c. Find the monopolist’s optimal screening contracts under incomplete information.

d. Illustrate the optimal contracts using the two types’ demand curves, showing the tariff each must pay. Interpret the equilibrium as the monopolist offering a quantity discount. Explain the intuition why the low-valuing buyer is under-supplied.

e. Compare the equilibrium with the case when the monopolist has complete information about the buyers’ types. Explain why the high-value buyer gets some CS when there is incomplete information (an “information rent”). Relatedly explain why, if \( \lambda \) were sufficiently low, the monopolist would only serve the high-valuing buyer and leave her with no CS.

f. **Monopoly versus perfect competition** Competition can solve privacy problems. To illustrate, consider what would happen if there were free entry, hence many potential suppliers with MC = 1.

2. **Insurance** Using the geometry of the state preference approach, show:

a. If insurance is actuarially fair and individuals can buy as much insurance as they like, then any risk averse individual will fully insure.

b. If insurance is actuarially unfair (i.e., \( E z < 0 \)) and individuals can buy as much insurance as they like, then any risk averse individual will not fully insure.

c. Use the Rothschild-Stiglitz model to give one reason why, in the real world, some individuals cannot buy
as much insurance as they like at actuarially fair rates.

3. **A Rat Race** Here is another incarnation of a Rothschild-Stiglitz equilibrium.

   Suppose there are 2 types of workers, \( \theta = 1 \) and \( \theta = 2 \), with an equal number of each type. For each type, utility depends on wages \( w \) and the speed of work \( s \). Specifically

   \[
   u(w, s, \theta) = w - \frac{s^2}{2\theta}.
   \]

   Notice that \( \theta = 2 \) finds speed less costly.

   There are many firms competing for workers. Each firm runs exactly one assembly line at a chosen speed \( s \). The Marginal Product of a worker of type \( \theta \) at a firm with speed \( s \) is given by

   \[
   MP(s, \theta) = s + \theta,
   \]

   so type \( \theta = 2 \) workers are more productive. Note that in this example \( s \)—which can be any non-negative real number—is chosen by the firm the worker joins; it is not subject to worker discretion (no moral hazard).

   No firm can distinguish workers’ types; and there is **team production**, so a firm cannot condition contracts on any given worker’s output. Thus, any firm will pay each of its workers the average product of its work force.

   **a.** Illustrate in \( w - s \) space (with \( w \) on the vertical axis) the indifference curves of each type worker. Do they exhibit the single crossing property?

   **b.** Draw a firm’s zero profit lines if (i) all the workers in the firm are of type \( \theta = 1 \); (ii) all the workers in the firm are of type \( \theta = 2 \); (iii) 50% of the workers are of each type. NOTE (iii) corresponds to the “market-odds line” in the Rothschild-Stiglitz insurance model.

   **c.** As a benchmark, what would be the market outcome if firms could distinguish individuals’ types?

   **d.** Show that a pooling equilibrium cannot exist.

   NOTE: This would involve each firm picking the same speed \( s \) for its assembly line and paying each of its workers \( 1\frac{1}{2} + s \) (where \( 1\frac{1}{2} \) is the average skill in the population).

   **e.** Illustrate the unique separating equilibrium for this economy. Calculate the equilibrium wage \( (w) \) and speed worked \( (s) \) for each type. The equilibrium may be viewed as a “rat race” (see FOOTNOTE below).

   **f.** The equilibrium outcome is not Pareto efficient. Give the intuition why the \( \theta = 2 \) workers are willing to get involved in a rat race.\(^1\)

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\(^1\)In a rat race the chances of getting the cheese increase with the speed of the rat, although no additional cheese is produced. In our example, unlike the rat race, workers produce more output at faster speeds; but, like the rat race, the private return from additional speed exceeds the additional output produced. Further, as in the rat race, the individual worker is goaded on by the knowledge that at slower speeds he must share his output with workers of lesser ability; or, he is spurred on by the knowledge that at faster speeds he will share the output of workers of greater ability. For more on rat races and other sad stories, you may want to see Akerlof, “The Economics of Caste and of the Rat Race and Other Woeful Tales,” *QJE*, Nov. ’76.
4. **Lemons** There are 10,000 buyers and 8,000 sellers of used cars. Each seller has one car, which is either of high quality $H$ or low quality $L$. The valuations for each buyer $b$ and each seller $s$ are:

$$
V_b^L = 800 \quad V_b^H = 1300 \\
V_s^L = 500 \quad V_s^H = 1000
$$

Each seller knows the quality of his car, but buyers cannot distinguish quality at the time of purchase. $\lambda$ equals the fraction of low-quality cars in the population. (All this is common knowledge.)

a. Suppose $\lambda = 3/4$. What will the price of cars be? Illustrate the equilibrium using a demand-and-supply diagram. Is the equilibrium efficient?

b. Repeat part a., now assuming $\lambda = 1/4$. Are used cars a Giffen good in this case (i.e., quantity demanded goes up when price goes up)? Explain.

5. **Education as a signal** Consider again the classroom example of education as a signal, but now suppose $\lambda = 1/4$ rather than $3/4$. (All other parameters are as given in class.)

a. Show that there exists a separating equilibrium.

b. Show that the separating equilibrium is not even “constrained Pareto optimal,” i.e., there is a simple policy that would result in a Pareto improvement even if the government is as uninformed as each principal. HINT: Compare a high-ability worker’s utility in the separating equilibrium with her utility in the pooling equilibrium.