Pre-Contractual Investment without the Fear of Holdups: The Perfect Competition Connection

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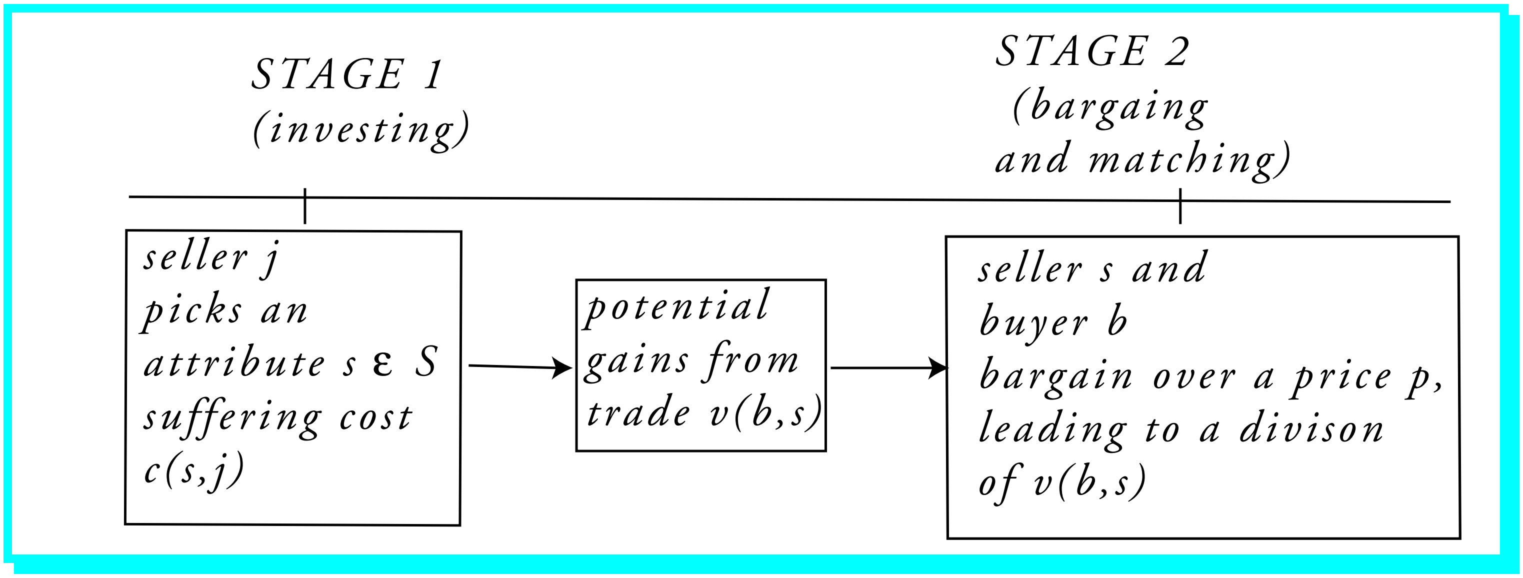
A STANDARD HOLDUP EXAMPLE

A 2-player, 2-stage game with perfect information

The Players: A buyer i with some attribute b and a seller j who must invest in some attribute $s \in S$.



If b is a very tough bargainer $\Rightarrow p < c(s,j) \Rightarrow$ seller faces a potential **holdup problem**.

If seller anticipates this, he'll **underinvest**.

NOTE: Can generalize example to 2-sided investment: buyer also picks his attribute $b \in B$.

MOTIVATING OBSERVATION: Many pre-contractual investments made without any fear of holdups. Examples: Get education *before* entering job market; build house *before* putting up for sale. Why? Trust in ex post competition.

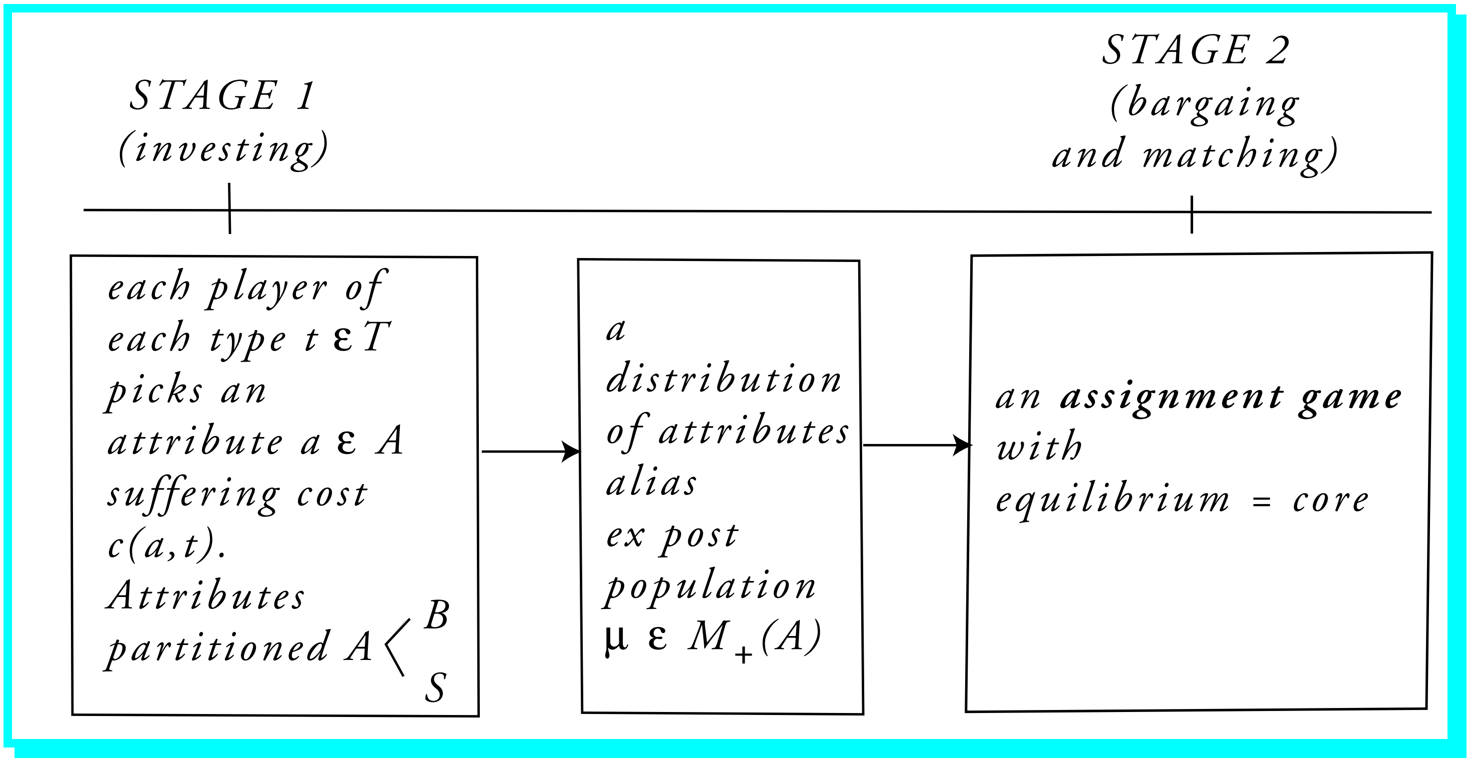
LITERATURE: Cole, Mailath, Postlewaite [2001a, 2001b] — henceforth CMP1 and CMP2 — and Felli and Roberts [2000].

MODEL SKETCHED

Like CMP2, the paper analyzes a 2-stage, perfect information game with a continuum of **player types** T, partitioned in a set of **buyer types** I and **seller types** J. Assume T is a compact metric space. A population is a positive Borel measure on T:

$$\mathcal{E} \in M_+(T).$$

NOTE: Using "distribution approach" rather than "names approach" to modelling a continuum of agents.



Assume A, B, and S are compact metric spaces.

LITERATURE ON CONTINUUM ASSIGNMENT MODEL: Gretsky, Ostroy, Zame [1992, 1999] — henceforth GOZ1 and GOZ2. GOZ2 uses very different notion of perfect competition than CMP2.

MY ORIGINAL PLAN: Apply GOZ2's results to the CMP2 model; see what difference it makes. Then found error in GOZ2. . .

THE EX POST ASSIGNMENT GAME

- To permit the possibility of some individuals remaining unmatched, let $B^0 \equiv B \cup \{\mathbf{0}\}$ and $S^0 \equiv S \cup \{\mathbf{0}\}$. An **assignment** is a measure

  $$x \in M_+(B^0 \times S^0).$$

  The assignment x is *feasible* for μ if ...

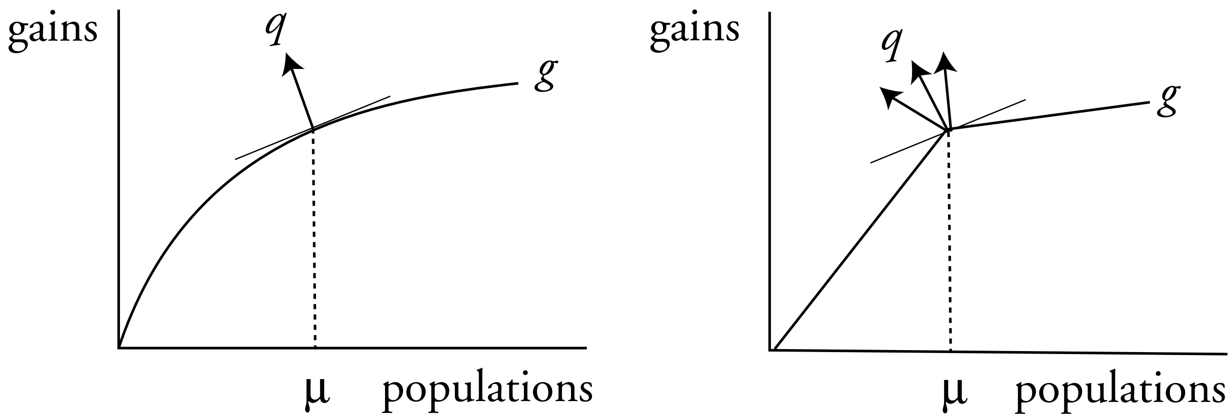
- a **value function** $v \in C_+(B^0 \times S^0)$, includes $v(b,s)$, $v(b,\mathbf{0})$, $v(\mathbf{0},s)$.

- the maximum ex post **social gain** from μ is

  $$g(\mu) \equiv \text{sup} \int v dx \text{ s.t. } x \text{ feasible for } \mu.$$

  Notice $g : M_+(A) \to \mathbb{R}$ is HD1 and superadditive, therefore concave. Let $q \in C_+(A)$. The **subdifferential** of g at μ is denoted

  $$\partial g(\mu) \subset C_+(A).$$



Define the set of feasible directions to move from μ as $D \equiv \{y : \mu + ty \in M_+(A)$ for some $t > 0\}$. Recall the *directional derivative* of g at μ in the direction y is

$$g'(\mu; y) \equiv \lim_{t \downarrow 0} \frac{g(\mu + ty) - g(\mu)}{t}.$$

Proposition (inf formula) *For any direction $y \in D$,*

$$g'(\mu; y) = \inf_{q \in \partial g(\mu)} q\mu,$$

where $q\mu \equiv \int q d\mu$.

TWO MARGINS OF ANALYSIS IN THE CONTINUUM, HENCE THE POSSIBILITY OF CALCULUS OVER INDIVIDUALS

1. The usual **commodity margin**. Example: If $A = R_+$, $\frac{\partial c(a,t)}{\partial a}$.

2. Because individuals are infinitesimal in the continuum, the **individual margin**. Examples: $g'(\mu; \delta_a) \equiv MP^+(a)$, or $g'(\mu; -\delta_a) \equiv MP^-(a)$, or $g'(\mu; -\delta_a + \delta_d) \equiv MP(a, d)$. Latter two assume a is an atom in supp μ, otherwise . . .

As an application of the calculus over individuals, the inf formula implies:

Lemma.

$MP^-(a) \leq q(a) \leq MP^+(a)$, *for all* $a \in$ supp μ.

CHARACTERIZATION OF THE CORE

Definition. *The imputation* $q^* \in C(\text{supp } \mu)$ *is in the ex post* **core** *for* μ *if* $q^*\mu = g(\mu)$ *and* $q^*\mu' \geq g(\mu')$ *for all coalitions* $\mu' \leq \mu$.

NOTE: For assignment games, being in the core is equivalent to stability.

Definition. *The pair* (x, p) *is* **Walrasian** *for* μ *if* x *is feasible for* μ, $p \in C_+(S^0)$ *with* $p(\mathbf{0}) \equiv 0$, *and*

- *For each* $b \in \text{supp } \mu$ *and each* $(b, s) \in \text{supp } x$, $v(b, s) - p(s) = v_b{}^*(p) \equiv \max_{s' \in S^0} v(b, s') - p(s')$
- *For each* $s \in \text{supp } \mu$ *and each* $(b, s) \in \text{supp } x$, $b \in B$ *implies* $p(s) = v_s{}^*(p) \equiv \max\{p(s), v(\mathbf{0}, s)\}$, *otherwise* $b = \mathbf{0}$.

Proposition (GOZ1 core characterization)
The following statements are equivalent:

1. *q^* is in the core of μ;*

2. *there exists a pair (x, p) that is Walrasian for μ with ${v_a}^*(p) = q^*(a)$ for all $a \in \text{supp } \mu$.*

3. *there exists a subgradient $q \in \partial g(\mu)$ with $q^* = q\,|_{\text{supp } \mu}$ and $p = q\,|_S$.*

PERFECT COMPETITION MEANS PEDS

Let $\rho : M_+(A) \to C_+(S^0)$ be a Walrasian price selection.

Definition. *Individuals face* **perfectly elastic demands and supplies** *(PEDS) in μ if for any Walrasian price selection ρ and any sequence of populations $\mu^n \to \mu$ with* supp $\mu^n \subset$ supp μ,

$$\rho(\mu^n) \to \rho(\mu)$$

(convergence in the norm topology).

The ex post population μ is **perfectly competitive** *if individuals faces perfectly elastic demands and supplies in μ.*

GOZ2 CHARACTERIZATION OF PC

Proposition *The following are equivalent:*

1. *all individuals face perfectly elastic demands and supplies in μ;*

2. *The core imputation q^* is unique in μ (so core bargaining is determinate);*

3. *every $q \in \partial g(\mu)$ satisfies $q \mid_{\text{supp } \mu} = q^*$ (so the restriction of g to* supp μ *is differentiable).*

4. *$q^*(a) = MP^-(a) = MP^+(a)$, for all $a \in$* supp μ *(so every individual gets his marginal product under perfect competition).*

PERFECT COMPETITION IS NOT AUTOMATIC IN THE CONTINUUM

Let (x, p) be a Walrasian equilibrium for μ. For any $a \in \text{supp}\ \mu$ and $d \in A$, let

$$\varrho(a, d) \in C_+(S^0)$$

be the conjecture of an individual with attribute a about what will happen to Walrasian prices if he deviates from a to d, given the distribution of attributes is μ.

TEMPTATION: "Perfect competition must be automatic in the continuum because people are nonatomic." Yielding to this temptation leads to **naive competitive conjectures**:

$$\varrho(a, d) = p, \text{ for all } a, d.$$

PROBLEM: One's analysis of continuum economies may be completely unrelated to one's analysis of large but finite economies, where individuals may be able to affect equilibrium prices a lot.

To avoid discontinuities at infinity, as in GOZ2,

> **we regard each "individual" in the continuum as an infinitesimal,**

the limit of a sequence of groups with positive mass approaching zero, — rather than as a measure zero point on the real line.

The dangers of the naive view and how we avoid them can be illustrated with a famous example.

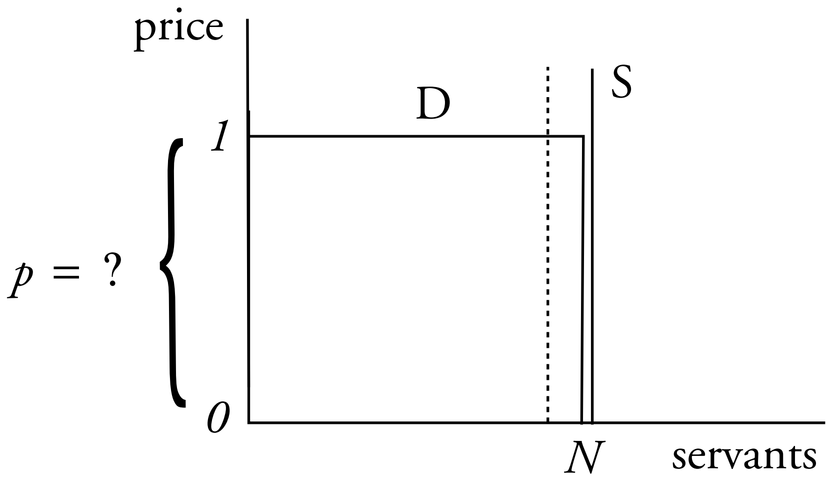
EDGEWORTH'S MASTER-SERVANT EXAMPLE

Edgeworth discovered that core bargaining may not be determinate even if there are arbitrarily many buyers and sellers in a market.

To begin, assume μ is finite (the counting measure) with N masters b_1 and N servants s_1. Assume

$$v(b_1, s_1) = 1,$$

and people have zero reservation values. The core is very large: any price $p \in [0, 1]$ is Walrasian.



Let b_0 and s_0 be dummy attributes interpreted as withdrawing from the market, with $v(b_0, s_1) = v(b_1, s_0) = 0$.

Observe if even one servant deviates from s_1 to s_0, the Walrasian price of servants will rise to $\bar{p} \equiv 1$. Similarly, if even one master deviates from b_1 to b_0, the Walrasian price will drop to $\underline{p} \equiv 0$. So for any N — no matter how large — some individuals can significantly affect Walrasian prices. The economy is not perfectly competitive; or equivalently, core bargaining is not determinate.

Now suppose μ consists of a continuum of masters and servants with mass N each. If take naive point of view, whatever $p \in [0,1]$ one picks . . . .

Instead, following GOZ2, observe if any small positive mass of servants deviates to s_0 [leading to a new distribution of attributes μ_n] the price of servants will rise to $\bar{p}$ — no matter how small the mass. Similarly, if a small mass of masters deviates to b_0, ... In other words, regarding individuals as infinitesimal, the only rational conjectures are

$$\varrho(b_1, b_0)(s_1) = \underline{p} \text{ and } \varrho(s_1, s_0)(s_1) = \bar{p}.$$

Hence the continuum version of the example is not perfectly competitive either.

EXISTENCE OF PERFECT COMPETITION

A continuum of agents is the "natural habitat" for PC.

While perfect competition is exceptional in the finite assignment model:

Proposition (GOZ2) *In the continuum assignment model, perfectly competitive populations μ are generic (dense G_δ).*

MASTER-SERVANT EXAMPLE: All economies μ off the 45-degree line are perfectly competitive in $\mu(b_1) - \mu(s_1)$ space.

QUESTION: Does genericity carry over to the 2-stage game, in which μ is chosen rather than endowed? ANSWER: Only sometimes!

REMAINDER OF PAPER SKETCHED

1. Introduce the idea of **strong perfect competition**, that extends GOZ2 definition of perfect competition (PEDS) from deviations $d \in$ supp μ to deviations $d \notin$ supp μ, and corrects the mistake in GOZ2. (The fix requires a definition of PEDS that is more delicate for deviations outside the support of μ.)

   MAIN RESULTS RELATIVE TO GOZ2:

   - A characterization of strong PC: Under strong PC, each individual **always** appropriates his full social marginal product — **no matter what he does**; that is, even if he innovates a new attribute $d \notin$ supp μ. NOTE: This gives him very good social incentives to innovate.

- Genericity of strong PC: In the continuum assignment model, PC $\Rightarrow$ strong PC.

Returning to the 2-stage game, in an **investment equilibrium** each individual chooses his Stage 1 attribute $a \in A$ to maximize his expected net profit in Stage 2 given his rational expectations about the distribution of attributes μ and his rational conjectures $\varrho(a, d)$ about what will happen during Stage 2 core bargaining if he deviates to any $d \in A$.

If individuals anticipate strong ex post competition, each individual will have very good incentives to invest because **he always fully appropriates his social contribution**, no matter what attribute he decides to produce — even new attributes. Hence, MAIN RESULT: If people anticipate μ will be PC, there will be no fear of holdups. Indeed ex ante investments will tend to be Pareto efficient. There is one exception: if there is 2-sided investment, there remains the possibility of **coordination failures** between innovating buyers and sellers.

Hence

1-sided investment + PC $\Rightarrow$
efficient investment

2-sided investment + PC $\Rightarrow$
efficiency (sometimes)

Conditions are given when coordination problems will not arise even under 2-sided investment.

THE SUPERMODULAR ASSIGNMENT MODEL

CMP2 study a special 1-dimensional case of the assignment model in which

- the attribute sets B and S are subsets of the real line,
- the value function $v(b, s)$ is supermodular, and
- value is only created by matches (zero reservation values).

RESULT: In the supermodular investment model, master-servant bargaining problems are generic even in the continuum. So PC will be rare.

The reason is easy to explain. Assume any attribute that generates potential gains from trade has a production cost bounded away from zero . . . .

CAN YOU SPOT THE MASTER-SERVANT PROBLEMS?

Consider a supermodular model in which the value function is

$$v(b, s) = bs.$$

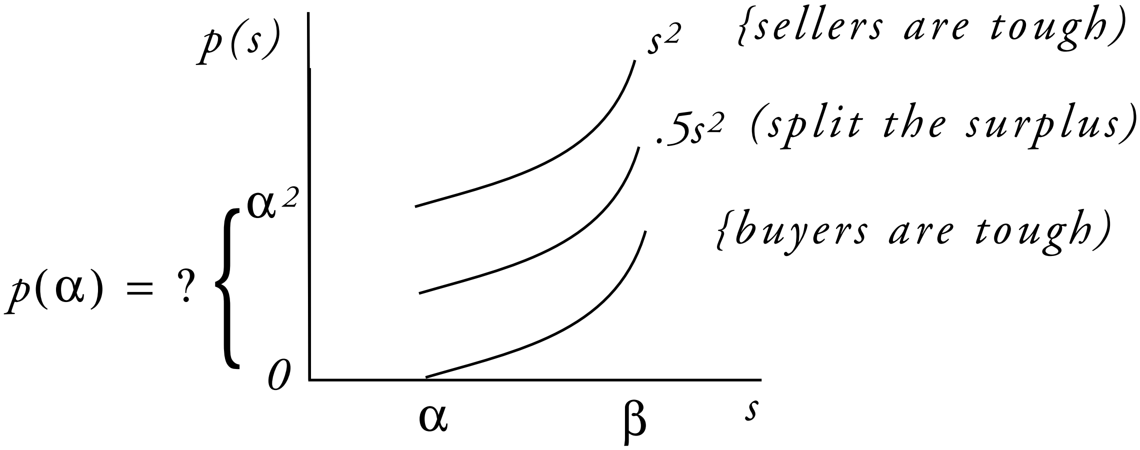
In the ex ante population $\mathcal{E}$, buyers' and sellers' types are uniformly distributed on the interval $[.2, .3]$. So we can imagine there is one buyer and one seller of each type.

The cost functions are $c(b, i) = b^5/(5i)$ for all b, i; similarly, $c(s, j) = s^5/(5j)$ for all s, j. Net gains are maximized if the buyer of type i and seller of type $j = i$ choose attributes

$$b(i) = s(i) = \sqrt[3]{i}, \quad \forall i \in [.2, .3].$$

So efficient investment requires ex post attributes to be distributed uniformly on $[\alpha, \beta] \equiv [\sqrt[3]{.2}, \sqrt[3]{.3}]$.

Let μ be this efficient distribution. There are many ex post core imputations for μ, hence many possible ex post price functions (1 degree of freedom). So, efficiency $\Rightarrow$ **not PC**.



Hence efficient investment depends on individuals' relative bargaining abilities.

Split the surplus leads to efficiency. But if $p(\alpha) < c(\alpha, .2)$ — e.g., because buyer α is a very tough bargainer — there will be holdup problems. Anticipating this, μ will not be chosen, e.g., some sellers may under-invest.

This jazzed-up master-servant example has many **inefficient** investment equilibria, including the trivial equilibrium in which no seller invests fearing a holdup, hence no buyer invests either.

The paper's model includes the supermodular model as a special case. This model is restrictive: not enough outside options to avoid bilateral monopoly bargaining problems. In richer assignment models with more outside options, e.g., the housing model in the paper, PC is generic; hence there is no fear of holdups, and the incentive to invest is very good.

Winding Down

A foundation for general equilibrium.

Unlike the Walrasian model with complete markets — which features both price-taking and market-taking, — we've looked at a model with both price-making and market-making:

price-making: ex post Walrasian prices are determined by competitive bargaining, without the help of an exogenous Walrasian auctioneer.

market-making: the set of markets $s \in S$ that will be operating in equilibrium is determined by sellers' perceived profit from innovating new attributes. Markets may or may not be complete

Nevertheless, conditions are given under which PC leads to efficient price making (ex post Walrasian) and efficient market making (all markets with potential gains from trade are open, even though typically ex post markets will be incomplete, i.e., not all $s \in S$ produced).

The key is that under PC, each individual will be able to fully appropriate the full social value of whatever he undertakes.