ECN 134

SOLUTION KEY #1

1. (i) $\mu = .06$ and $\sigma = .10$: Find $P(X < 0) = P\left(Z < \frac{0-.06}{.10}\right) = P(Z < -.6)$.

Turning to the normal tables we find that the probability associated with the critical value of $-.6$ is 27.43%. That is 27.43% of the time, and thus approximately $\boxed{13 \text{ years}}$, the return on corporate bonds was less than 0%.

(ii) $\mu = .11$ and $\sigma = .16$: Find out what the rate of return is for the worst five years of the S&P. That is: what was the rate of return for the worst 10% of the last 48 years. We must solve for a : $.10 = P(X < a) = P\left(Z < \frac{a-.11}{.16}\right)$. Turning to the normal tables we find that the critical value leading to a .10 outcome is -1.28 . Thus find a such that

$$\begin{aligned} -1.28 &= \frac{a - .11}{.16} \\ a &= -.09 \end{aligned}$$

This tells us (approximately) that in the worst 5 years of the last 48 years the S&P lost $\boxed{9\% \textit{ or more}}$.

2. r = 0.12.

i) NPV = -75,000 + 140,000/(1+r)^5 = -75,000 + 79,439.76 = $\boxed{4,439.76}$.

ii) $\boxed{\text{Yes}}$, since Venkatesh can borrow against the return from selling the land.

iii) Keeping future consumption unchanged means Venkatesh borrows against the entirety of his return, i.e., he borrows 140,000/(1+0.12)^5 = 79,439.76 at t=0. So C_0=Y_0 - 75,000 + 79,439.76 = Y_0 + 4439.76. The effect on C_0 is $\boxed{4439.76}$ = NPV.

iv) Venkatesh is willing to pay up to the PV of his returns on the land, or $ $\boxed{79,439.8}$.

3. Work (39,39) $\rightarrow$ $PV_W = 39+39/(1+r)$;

Law school (23,65) $\rightarrow$ $PV_L = 23 + 65/(1+r)$.

Note that if the budget constraint is linear, Hang-Ro will allocate half of PV to each periods consumption.

Situation I: r = 30%. Hence, slope = -(1+r) = -1.3 and $PV_W = 69$, $PV_L = 73$.

Her optimal consumption plan is;

Work: $C_0 = 69/2 = 34.5$, $C_1 = (34.5)\ (1.3) = 44.9$ $\rightarrow$Lending = 39- 34.5 = 4.5.

Law: $C_0 = 73/2 = 36.5$, $C_1 = (36.5)\ (1.3) = 47.5$ $\rightarrow$Borrowing = 36.5 -23 = 13.5.

Since the law school budget constraint is everywhere outside the work budget constraint, Hang-Ro will clearly choose Law school, independent of her preference.

Situation II: r = 50%. Hence slope = -1.5 and $PV_W = 65$, $PV_L = 66.3$.

i) In this situation, we need to know preferences. If preferences were different, she might choose L over W.

ii) Work: If borrowing was unrestricted, then

$$C_0 = 65/2 = 32.5\ ,\ C_1 = 32.5 \times 1.5 = 48.8.$$

This point is in fact on the relevant portion of the constraint, and Hang-Ro will consume at (32.5, 48.8), lending 39.5- 32.5 = 6.5, and attain a utility level of $32.5 \times 48.8 = \boxed{1586}$.

Law: If borrowing were unrestricted, then

$$C_0 = 66.3/2 = 33.2,\ C_1 = 33.2 \times 1.5 = 49.8.$$

Unfortunately, this point is infeasible. She will end up at the kink, borrowing 3 at t=0, and consuming $C_0 = 26$, $C_1 = 60.5$, thus enjoying a utility of $26 \times 60.5 = \boxed{1573}$. Now, she is slightly better off working right away.