**ECONOMICS 134 (NEHRING)**

**SOLUTION KEY #3**

**Question 1.**

i) Suppose we were in year three, then use the perpetuity formula:

$8/0.16 = \boxed{50}$. This is the value of the stream in year three.

ii) Then the same stream must be additionally discounted by 1/(1+r) in year two (discount once):

$$50 \ / \ (1+0.16) = 43.1.$$

Similarly, the stream must be worth :

$$50/(1+0.16)^2 = 37.16 \text{ in year one}$$
$$\text{and } 50/(1+0.16)^3 = \boxed{32.04} \text{ in year zero.}$$

In year four, the ex-dividend price will be 50 again.

**Question 2.**

i) The dividends grow by 14% for the next 20 years, and then by 6% every year after that, forever:

In 1996 the dividend was \$100. Note: we do not count this in our PV calculations, we only use this as a reference point from which we make our calculations.

Dividend in 1 year: $100 * (1.14) = \boxed{114.0}$

Dividend in 2 years: $100 * (1.14)^2 = 129.96$

Dividend in 10 years: $100 * (1.14)^{10} = \boxed{370.72}$

Dividend in 20 years: $100 * (1.14)^{20} = \boxed{1374.3}$

Dividend in 21 years: $100 * (1.14)^{20} * (1.06)^1 = \boxed{1456.8}$

Note: These are the actual dividends paid in the corresponding years, not their PV.

ii) Use growing annuity formula: $PV(GrowingAnnuity) = c\left[\frac{1}{r-g} - \frac{1}{r-g} * \left(\frac{1+g}{1+r}\right)^T\right]$

Note: This give us the PV of the growing annuity the year before the payments start. In this case, the dividends start in year 1, so the formula will give us the value in year 0, which is what we want.

$c\left[\frac{1}{r-g} - \frac{1}{r-g} * \left(\frac{1+g}{1+r}\right)^T\right] = 114\left[\frac{1}{.12-.14} - \frac{1}{.12-.14} * \left(\frac{1+.14}{1+.12}\right)^{20}\right] = 114 \times 21.237 = \boxed{2421.02}$

iii) Use the growing perpetuity formula: $PV(GrowingPerpetuity) = \frac{c}{r-g}$

Note: The formula gives the PV for the period before the first payment of the growing perpetuity. In our problem, the growing perpetuity starts in year 21, so the formula will give us the value of the growing perpetuity in year 20. Thus, to get the PV in year 0, we must further discount the value from the formula which is given in year 20 dollars, to year 0 by multiplying by: $\left(\frac{1}{1.12}\right)^{20}$.

PV of the growing perpetuity: $\left[\frac{c}{r-g}\right]\left(\frac{1}{1+r}\right)^{20} = \left[\frac{1456.8}{.12-.06}\right] * \left(\frac{1}{1.12}\right)^{20} = .10366\,7 \times 24280.0 = \boxed{2517.03}$

iv) $\left(\frac{2421.02+2517.03}{100}\right) = \boxed{49.3805}$. This is the "Differential Growth Factor." As stated in the problem, you can use this number to multiply by 1996 dividends to get the PV of the stock.

v) Given the answer to the last part of this question, we multiply the 1996 dividends by the "Differential Growth Factor" to get the total PV of Coca-Cola stock.

$1.25 * 49.3805 = \boxed{\$61.7256 \text{ billion.}}$

This is less than half of the market value!

vi) In this part, we use "gross dividends" in our stock valuation procedure. Here the fair price turns out to be:

$2.657 * 49.3805 = \boxed{\$131.204 \text{ billion.}}$

Pretty close approximation!

vii) Now we recompute the PV of the dividend stream, then recompute the "Differential Growth Factor", and finally recompute the value of the stock using "gross dividends":

Present value of growing annuity: $114 \left[\frac{1}{.12-.14} - \frac{1}{.12-.14} * \left(\frac{1+.14}{1+.12}\right)^{10}\right] = 114.0 \times 9.6813 = 1103.67$

Note: dividends in year 11 will be: $100 * (1.14)^{10} * (1.06) = 392.965$

Present value of growing perpetuity: $\left[\frac{392.965}{.12-.06}\right] * \left(\frac{1}{1.12}\right)^{10} = .321973 \times 6549.42 = 2108.74$

Total PV: $1103.67 + 2108.74 = 3212.41$

Differential Growth Factor: $\frac{3212.41}{100} = 32.1241$

Value or "fair price" of stock: $2.657 * 32.1241 = \boxed{\$85.3537 \text{ billion.}}$

Much less than the market value!

viii) 2 out of 64 ounces of total fluid intake is a very large number. Indeed, Coca-Cola has already 48% market share of the world soft-drink market. How many more soft-drinks will people ever drink? While the above calculations indicate that Coca-Cola will need to sell a lot more soft-drinks in the future to justify its current share price, not that much room for growth seems to be left.

Could it be that Wall Street bets on rapid global warming ??

**Question 3**

i) The PV of Methusalem's coupon stream is £$\frac{20000}{0.04}$ $= 500,000$. His consumption plan must respect his intertemporal budget constraint. Hence, if he wants his consumption to grow at an annual rate of 2%, his first year consumption $C_1$ must satisfy

$$500,000 = \frac{C_1}{0.04 - 0.02}.$$

Solving for $C_1$, one obtains $\boxed{C_1 = \pounds 10,000}$.

ii) Methusalem will consume less than his perpetuity income until time $t^*$ at which $C_{t^*} = 20,000$, that is: until his consumption will have doubled. Using the doubling rule, this will happen about $\frac{0.7}{0.02} = \boxed{35 \text{ years}}$ after year 1, i.e. at $t^* = 36$.

iii) Methusalem has to keep the PV of his consumption stream below £500,000, in particular *finite.* So, if he is willing to start with very little consumption initially, he is able afford consumption plans that grow at any rate $\boxed{\text{strictly less than } 4\%}$. However, consumption plans that grow at a rate of 4% or more have infinite PV, hence Methusalem cannot afford them.