Question 1

i) Bovine’s fair stock price is \( \frac{2.40}{0.06} = \$40 \).

ii) Since RoM invests at its opportunity cost of capital, its fair stock price must also be \( \$40 \).

iii) RoM’s dividends next year will be \( 0.25 \times 2.40 = \$0.60 \).

iv) RoM’s dividends will grow at the rate \( g = \pi \rho = 0.06 \times 0.75 = 0.045 \).

v) RoM’s dividends are given by the expression

\[ D_{RoM}^t = 0.6 \times 1.045^{(t-1)}. \]

To surpass Bovine Cash’s dividends, they will need to quadruple over time; that is, dividends will need to double twice. At \( g=4.5\% \), it takes approximately \( \frac{0.7}{0.045} \approx 15.5 \) years to double, and thus 31 years to quadruple. Hence RoM will surpass Bovine Cash’s dividends in about 32 years from now.

Remark: the exact solution is found by solving the equation

\[ 0.6 \times 1.045^{(t-1)} = 2.4. \]

vi) The PV of all future dividends is \( \frac{0.6}{0.06-0.45} = \$40 \). This must come out to the same value as ii), since the fair stock price is always equal to the PV of all future dividends.

Question 2

The hypothetical prices of U.S. Treasury Strips with face value \$1000 are approximately as follows.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-2009</td>
<td>$987</td>
</tr>
<tr>
<td>2-2012</td>
<td>$897</td>
</tr>
<tr>
<td>2-2018</td>
<td>$640</td>
</tr>
</tbody>
</table>

Using: \( PV = P_b = \frac{F}{(1+r_t)^t} \)

or: \( (1 + r_t)^t = \frac{F}{P_b} \)
or: $1 + r_t = \left( \frac{F}{P_b} \right)^\frac{1}{t}$
or: $r_t = \left( \frac{F}{P_b} \right)^\frac{1}{t} - 1$

i) For $r_1 : r_1 = \left( \frac{1000}{987} \right)^\frac{1}{20} - 1 = 1.32\%$.

ii) For $r_4 : r_4 = \left( \frac{1000}{997} \right)^\frac{1}{20} - 1 = 2.76\%$.

iii) For $r_{10} : r_{10} = \left( \frac{1000}{989} \right)^\frac{1}{20} - 1 = 4.56\%$.

iv) Thus, the current yield curve is upward sloping.

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**Question 3**

These are the formulae:

$PV_{\text{face}}(F,r,t) = \frac{F}{a+r}^t,$

$PV_{\text{coupon}}(C,r,t) = \frac{C}{r} \times \left[ 1 - \frac{1}{(1+r)^t} \right].$

These are the $r = .07, 0.08, 0.09$;

values $F = 1000, C =70, t = 20;$

<table>
<thead>
<tr>
<th>$r$</th>
<th>$PV_{\text{coupon}}$</th>
<th>$PV_{\text{face}}$</th>
<th>$PV_{\text{bond}}$</th>
<th>$\frac{PV_{\text{face}}}{PV_{\text{bond}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07</td>
<td>741.581</td>
<td>258.419</td>
<td>1000</td>
<td>0.258</td>
</tr>
<tr>
<td>0.08</td>
<td>687.27</td>
<td>214.548</td>
<td>901.819</td>
<td>0.238</td>
</tr>
<tr>
<td>0.09</td>
<td>638.998</td>
<td>178.431</td>
<td>817.429</td>
<td>0.218</td>
</tr>
</tbody>
</table>

The fraction of the repayment of the face value *decreases* in the interest rate, since the repayment is the very last cash flow received from the bond and thus discounted steeply, in particular at high interest rates.

iii) straightforward (omitted); if your graph is neat, you should read off an interest rate (yield to maturity) of approximately 8.6 %.

iv) Suppose the relationship is linear; then the yield to maturity is: \( \frac{901\ -\ 850}{901\ -\ 817} \times 1 + 8 = 8.607\% \) percent. This yields a $PV_{\text{bond}}(C,F,0.08607,t) = 849.1$; so the approximation is pretty good!