ECONOMICS 134 (NEHRING)

SOLUTION KEY #4

Question 1

i) Bovine's fair stock price is $\frac{2.40}{0.06} = \boxed{\$40}$.

ii) Since RoM invests at its opportunity cost of capital, its fair stock price must also be $\boxed{\$40}$.

iii) RoM's dividends next year will be $0.25 \times 2.40 = \boxed{\$0.60}$.

iv) RoM's dividends will grow at the rate $g = \pi\rho = 0.06 \times 0.75 = \boxed{0.045}$.

v) RoM's dividends are given by the expression

$$D_t^{RoM} = 0.6 \times 1.045^{(t-1)}.$$

To surpass Bovine Cash's dividends, they will need to quadruple over time; that is, dividends will need to double twice. At g=4.5%, it takes approximately $\frac{0.7}{0.045} \approx 15.5$ years to double, and thus 31 years to quadruple. Hence RoM will surpass Bovine Cash's dividends in $\boxed{\text{about 32 years from now}}$.
Remark: the exact solution is found by solving the equation

$$0.6 \times 1.045^{(t-1)} = 2.4 \quad .$$

vi) The PV of all future dividends is $\frac{0.6}{0.6-0.45} = \boxed{\$40}$. This must come out to the same value as ii), since the fair stock price is *always* equal to the PV of all future dividends.

Question 2

The hypothetical prices of U.S. Treasury Strips with face value \$1000 are approximately as follows.

Maturity	Price
2-2009	$987
2-2012	$897
2-2018	$640

Using: $PV = P_b = \frac{F}{(1+r_t)^t}$
or:$(1 + r_t)^t = \frac{F}{P_b}$

or: $1 + r_t = \left(\frac{F}{P_b}\right)^{\frac{1}{t}}$

or: $r_{t=} \left(\frac{F}{P_b}\right)^{\frac{1}{t}} - 1$

i) For $r_1 : r_1 = \left(\frac{1000}{987}\right)^{\frac{1}{1}} - 1 = \boxed{1.32\%}$;

ii) For $r_4 : r_4 = \left(\frac{1000}{897}\right)^{\frac{1}{4}} - 1 = \boxed{2.76\%}$;

iii) For $r_{10} : r_{10} = \left(\frac{1000}{640}\right)^{\frac{1}{10}} - 1 = \boxed{4.56\%}$;

iv) Thus, the current yield curve is upward sloping.

Question 3

These are the formulae:

PV_{face}(F,r,t) = F/(a+r)^t,

PV_{coupon}(C,r,t) = C/r × [1 - 1/(1+r)^r].

These are the r = .07, 0.08, 0.09;

values F = 1000, C =70, t = 20;

r	PV_{coupon}	PV_{face}	PV_{bond}	$\frac{PV_{face}}{PV_{bond}}$
0.07	741.581	258.419	1000	0.258
0.08	687.27	214.548	901.819	0.238
0.09	638.998	178.431	817.429	0.218

The fraction of the repayment of the face value *decreases* in the interest rate, since the repayment is the very last cash flow received from the bond and thus discounted steeply, in particular at high interest rates.

iii) straightforward (omitted); if your graph is neat, you should read off an interest rate (yield to maturity) of approximately 8.6 % .

iv) Suppose the relationship is linear; then the yield to maturity is: (901 - 850) / (901 - 817) × 1 + 8 = $\boxed{8.607}$ percent. This yields a PV_{bond}(C,F,0.08607,t) = 849.1; so the approximation is pretty good!