ECN 134 Financial Economics

Answer Key for Midterm

1. (8) A twenty-year zero coupon with face value \$1000 bond sells at \$400; what is the five-year spot rate?

Answer : $P_B = 400 = PV_B = \frac{1000}{(1+r)^{20}} <=> r = (\frac{1000}{400})^{1/20} - 1 = 0.0468$. Therefore, the 20-year spot rate is roughly 4.7%.

2. (10) A rational person will never save at negative real interest rate. True or false? Explain briefly.

Answer : False. If you think saving just as an opportunity for investment, this statement may be true. However, you may really need to save today because you know that you won't have any income and you can't consume anything in the future. So, in this case, you would even save at a negative interest rate. A very good example is saving for retirement. During the retirement, the future consumption is going to be zero unless you saved a part of your income while working. Hence, no matter whether interest rate is positive or negative you have to save in order to ensure the future consumption for your retirement period.

3. (14) i) State Fisher's separation theorem of the appendix to chapter 4 carefully. Why is it called "separation" theorem?

Answer : FST is the fundamental justification for why we compute the present values. The theorem states that with a perfect capital market (i.e. constant interest rate and lend & borrow freely) one can rank the opportunities of the investments independent of people's preferences. This is the reason why we looked at present values or future values (or in other words people's budget sets) and disregarded prefernces. So in this sense, the theorem seperates preferences from the investment decisions. See the appendix to chapter 4 in the textbook for a detailed treatment.

ii) Describe verbally and graphically a situation in which its assertion fails, and explain briefly why.

Answer: There are several cases where Fisher's separation theorem fails. One of them is a situation where one cannot borrow and lend at the same interest rate (class room example). This implies that the budget line has a kink at the endowment point. In this case we have to know about the consumer's preference to decide which investment is better. Another case is the problem 3 in PS 1, where we can not borrow freely.

4. (16) a) You want to buy a house that costs $400,000. You can make a downpayment of $100,000 and take a $300,000 mortgage at 7% interest. You hate to be in debt, though, an want to pay off the mortgage as soon as possible. Assuming you can come up with the same amount C each year to pay off the mortagage, how much do you need to pay annually to be debt-free within 10 years.

Answer : Now, your debt is $300,000 with the interest rete 7%. Using Annuity Formula, $300,000 = \frac{C}{0.07}(1-(\frac{1}{1+0.07})^{10})$ yields $C = 42,713.33$ Hence, you need to pay at least $\boxed{\$42,713.33}$ to be debt-free within 10 years.

b) But you can't really afford that much. On reflection, the maximum you could come up with each year is $28,000. How long would it take you to pay it back?

Answer :
$300,000 = \frac{28,000}{0.07}(1-(\frac{1}{1+0.07})^{t}) <=> (1+0.07)^t = 4 = 2^2$

1) By applying doubling rule, the "doubling year" $t_d = \frac{0.7}{0.07} = 10$. Thus, $t = 2 * t_d = 20$ Hence, after about $\boxed{\text{20 years}}$ you can pay back your debt.

2) By taking log on both sides, $t = 2 * \frac{\log 2}{\log 1.07} = 2 * 10.25 = 20.5$

c) Compare the total $ payments in a) and b); which one is larger? (Hint: you don't need to correctly answer the earlier parts to answer c) correctly)

Answer : $10 * 42,713.33 = 427,133.3 < 20 * 28,000 = 560,000$ Thus, in case of b) your payment is larger. Intuitively, this is because you pay off less in each period, you need to pay off the debt over a longer period, which implies by the "power of the compounding" your debt incresaes over time exponentially, hence you will ultimately pay more.

5. (12) Over the 500 months period from 1950-1991, the monthly return on German stocks was on average 1%, with a standard deviation of 4.1% . Assume that monthly returns are (approximately) normally distributed; a table of the normal distribution is attached at the back.

i) In (approximately) how many of these 500 months did the monthly return exceed 0% but not 6%?

Answer : Here, the monthly return on German stocks are the random variable. Let denote X the the return on German stocks. Then $X\tilde{\ }N(1,(4.1)^2)$, hence the corresponding z-score has to be $z = \frac{X-1}{4.1}$.

$\Pr(0 < X < 6) = \Pr(\frac{0-1}{4.1} < z < \frac{6-1}{4.1}) = \Pr(z < \frac{6-1}{4.1}) - \Pr(z < \frac{0-1}{4.1}) = \Pr(z < 1.22) - \Pr(z < -0.24) = 0.8888 - 0.4052 = 0.4836$

$500 * 0.4836 (= 241.8)$ For about $\boxed{242}$ months, the monthly return exceeded 0% but not 6%.

ii) In the worst 20 months, German stocks lost x % or more of their value; x= __ _ ?

Answer : $\frac{20}{500} = 0.04 = \Pr(z < -1.75) = \Pr(\frac{X-1}{4.1} < -1.75) = \Pr(X < -6.175)$

Thus, in the worst 20 months, German stocks lost $\boxed{6.175\%}$ or more.

6. (20) Consider a 20-year bond with face value \$1000 and an 5% annual coupon.

i) If the annual interest rate was equal to 6% and constant, what would the bond's PV be?

Answer :
- PV of the face value of 1,000 ; $\frac{1{,}000}{(1+0.06)^{20}} = 311.8$
- PV of the annual coupon stream ; Your constant annual coupon stream will be $1,000 * 0.05 = 50$

$\frac{50}{0.06}(1 - (\frac{1}{1+0.06})^{20}) = 573.496$
- PV of your bond ; $311.8 + 573.496 = \boxed{885.296}$

ii) If the annual interest rate was equal to 5% and constant, what would its PV be?

Answer :

- PV of the face value of 1,000 ; $\frac{1{,}000}{(1+0.05)^{20}} = 376.889$
- PV of the annual coupon stream ; Your constant annual coupon stream will be $1,000 * 0.05 = 50$

$\frac{50}{0.05}(1 - (\frac{1}{1+0.05})^{20}) = 623.11$

-PV of your bond ; $376.889 + 623.11 = 1,000$

Without calculating, since the interest rate(=0.05) is the same as $\frac{c}{F} = \frac{50}{1{,}000} = 0.05$ we can know that the PV will be $\boxed{1{,}000}$

iii) The price of the bond is \$ 950. Compute its approximate yield to maturity by assuming that the PV is a linear function of the interest rate.

Answer :

By linear approximation, $\frac{1000-885}{0.05-0.06} = \frac{1000-950}{0.05-r}$ $r = 0.054$ Thus, the yield to maturity is roughly $\boxed{5.4\%}$.

7. (20) Exxon is forecast to earn \$4/share next year. Each year, Exxon will reinvest 75% of its earnings at a rate of return 6%. Its required rate of return is 8%.

i) What will Exxon's dividends be next year?

Answer : The dividend is the remaining fraction of earning for next year after reinvestment : $4 * (1 - 0.75) = \boxed{1}$

ii) What is Exxon's fair share price now?

Answer : Exxon's groth rate ; $g = 0.06 * 0.75 = 0.045$

$P = PV = \frac{\text{div}_1}{r-g} = \frac{1}{0.08-0,045} = \boxed{28.57}$

iii) What will Exxon's fair share price be in 10 years?

Answer : After 10 years, your dividend next year will be $1 * (1 + 0.045)^{10} = 1.55$

$P_{10} = PV_{10} = \frac{\text{div}_{11}}{r-g} = \frac{1*(1+0.045)^{10}}{0.08-0.045} = 28.57 * 1.55 = \boxed{44.37}$

iv) Recommend an easy way to raise Exxon's share price now!

Answer : Note that the rate of return from the reinvestmemt is less than the required rate of return. Thus the reinvestment is destroying the value. So the less the reinvestment, the higher the price will be. Thus, by lowering the retention ratio the share price will be higher.