## Econ 134 - Financial Economics <br> Nehring <br> Problem Set \# 4

Two out of three problems will be graded for serious effort, for a maximum of 50 points. In addition, you will also be graded on the correctness of some of two of your answers, for an additional $2 \times 10$ points; again, I won't tell you in advance for which. Make sure to box all your answer to receive credit. Maximal total score: $50+20=70$ points.

1. (30) i) Bovine Cash leads a very placid existence: it pays always pays out all of its earnings as dividends; as a result, its earnings per share remain at $\$ 2.40$ forever. Bovine Cash's required rate of return is $6 \%$. What is Bovine Cash's fair stock price?
ii) Likewise, Run-of-the-Mill's, Inc. (RoM) predicted earnings next year are $\$ 2.40$; its required rate of return is also $6 \%$. However, RoM retains most of its earnings and keeps aquiring smaller, equally glamorous firm; the return on its investments is equal to $6 \%$. What is Run-of-the-Mill's fair stock price?
iii) Specifically, Run-of-the-Mill always distributes $25 \%$ of its earnings, and reinvests the remaining $75 \%$ (with a return of $6 \%$ as above). What is RoM's dividend next year?
iv) How fast will RoM's dividends grow?
v) When will RoM's dividends surpass those of Bovine Cash? Hint: you may use the doubling rule to give an approximate answer.
vi) What is the PV of all future dividends? Compare to ii) and explain briefly.
2. (20) Suppose that (counterfactually) the current ask prices of U.S. Treasury Strips with face value $\$ 1000$ were approximately as follows.

| Maturity | Price |
| :--- | :--- |
| $10-2009$ | $\$ 987$ |
| $10-2012$ | $\$ 897$ |
| $10-2018$ | $\$ 640$ |

i) Compute the 1-,
ii) 4- and
iii) 10-year spot rates; make sure to show your work.
iv) Based on this fragmentary information, sketch the (hypothetical) current yield curve.
3. (30) Consider a bond that pays a "coupon" of $\$ 70$ p.a. for 20 years starting next year. At maturity, after 20 years, the bond-holder will receive back, on top of the last coupon payment, the face value of the bond, $\$ 1000$.
i) For the annual interest rates $\mathrm{r}=7 \%, \mathrm{r}=8 \%$, and $\mathrm{r}=9 \%$, compute the present value of

- the coupon stream,
- the repayment of the face value,
- the bond (sum of the two)
ii) For the annual interest rates $r=7 \%, r=8 \%$, and $r=9 \%$, compute the fraction of the present value of the bond represented by the PV of the repayment of the face value. Give an intuitive explanation of the pattern you observe.
iii) Based on your computations in i), represent graphically the PV of the bond as a function of the interest rate over the range of $7 \%$ to $9 \%$.

Based on your (inexact, of course, but neat!) graph, determine at what interest rate the PV of the bond (=market price) would equal 850; that interest rate is called the "yield to maturity".
iv) To be able to do a simple computation, pretend that the PV of the bond is a linear function over the range from $8 \%$ to $9 \%$. Under that assumption, compute the interest rate that would make the PV equal to $\$ 850$ (Hint: this involves solving a linear equation).

To check the correctness of your computation and to verify the usefulness of the linear approximation, compute the PV of the bond given the interest rate you just obtained; it must lie within 2 dollars of $\$ 850$ !

Remark: There is only a very brief passage on the yield to maturity in CF, p. 135. However, the "yield to maturity" (of a bonds) is mathematically the same thing as the "internal rate of return" of an investment project. This is discussed in CF on p. 152-153, and includes a picture of the kind asked for in part iii). Look it up!

